Lecture 7
Newton's law of gravity: Is there a difference between an apple and the Moon ?

Newton’s three laws

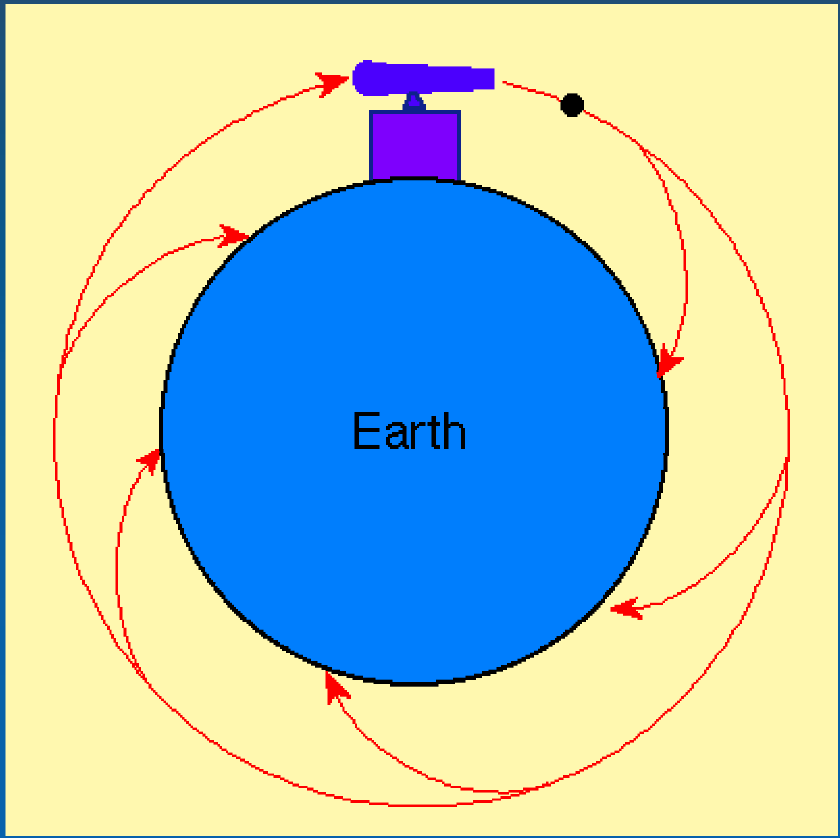
Newton’s first law: A body at rest or in the state of uniform motion will remain at rest or in uniform motion, unless acted upon by a net external force.

Newton’s second law: The acceleration of an object is equal to the net force applied to it, divided by its mass.

Newton’s third law: For every action, there is an equal and opposite reaction.

The story with the apple

- Observation 1: The Moon orbits the Earth on a near circular orbit. Newton 1 $\Rightarrow$ Moon is constantly being accelerated. It is continuously falling towards the Earth.



The story with the apple

- Observation 1: The Moon orbits the Earth on a near circular orbit. Newton 1 $\Rightarrow$ Moon is constantly being accelerated. It is continuously falling towards the Earth.
- Observation 2: An apple falls from a tree.
- Newton's ingenious idea: The very same force [gravity] that makes an apple fall from the tree also keeps the Moon on its orbit around the Earth.

Let’s fill in the details ...

- At what rate is the Moon falling towards the Earth ? ~ 1/3600 times as big as the rate at which things fall at the surface of the Earth. $\Rightarrow$ at the Moon’s distance, gravity is weaker.
- How much more distant is the Moon ? Moon’s distance is about 60 Earth radii. But 60*60=3600 ! $\Rightarrow$ gravity decreases with the inverse square of the distance.

Newton's law of gravity

$$F = G\frac{Mm}{r^2}$$

- M: mass of one object [e.g. Earth]
- m: mass of the other object [e.g. apple, Moon]
- r: distance between the two objects
- F: Force with which the two objects are attracting each other
- G: gravitational constant [6.67×10^{-11} N m^2/kg^2]

Newton II + law of gravity
⇒ equivalence principle

$$F = m \times a = \frac{GmM}{r^2}$$

$$\Rightarrow a = \frac{GM}{r^2}$$

- Acceleration does not depend on m, the mass of the object. All objects fall at the same rate.
- Left hand side: “m” inertia of the object
- Right hand side: “m” gravitational attraction of object
  ⇒ equivalence of inertial and gravitating mass

From Newton to Kepler

- If Newton's laws are indeed much more general, then it should be possible to derive [and thus understand] Kepler's laws.
- Kepler:

$$\frac{R^3}{P^2} = C$$

but

- – what is the origin of such a relation ?
- – what determines the constant C ?

From Newton to Kepler

- Newton: by combining Newton's axioms of motion with the inverse square law for gravity, one obtains [after a few pages of derivations]:

$$\frac{R^3}{P^2} = \frac{G(M+m)}{4\pi^2}$$

For many astronomical systems: M>>m

$\Rightarrow$ M+m $\approx$ M

$$\frac{R^3}{P^2} = \frac{G(M+m)}{4\pi^2} \approx \frac{GM}{4\pi^2}$$

- The constant in Kepler's 3rd is determined by the mass of the central object.
- The importance of this formula in astronomy can be hardly overestimated, it allows to measure the mass of astronomical systems.

Example 1: The mass of the Sun

- Warning: make sure to use consistent units!
- Orbital period of the Earth around the Sun:
  $1\ \text{yr} = 3.15\times10^{7}\ \text{sec}$
- Distance of the Earth to the Sun:
  $1\text{AU} = 1.50\times10^{11}\ \text{m}$
  $\Rightarrow$ mass of the Sun:
  $M = 2\times10^{30}\ \text{kg}$

Example 2: The mass of the Moon

- Orbital period of the Moon around the Earth: 1 month $= 2.4\times10^{6}$ sec
- Distance of the Moon to the Earth: $R=3.84\times10^{8}$ m
  $\Rightarrow$ mass of the Earth:
  $M = 6\times10^{24}$ kg

Example 3: how to measure the mass of Planets

- e.g. Jupiter: measure distance between Jupiter and one of its moons, measure the orbital period of that moon, calculate Jupiter's mass.
- e.g. Venus: bad luck, Venus has no moons. Possible solution: send a satellite into Venus orbit.
- Other application: mass determination of stars, star clusters, galaxies, galaxy clusters ...