Lecture 16
Special relativity III: space + time = space-time or what is now, then, and there ?

Einstein's new relativity

- Galileo:
  - The laws of mechanics are the same in all inertial frames of reference
  - time and space are the same in all inertial frames of reference
- Einstein:
  - The laws of physics are the same in all inertial frames of reference
  - the speed of light in the vacuum is the same in all inertial frames of reference

$\Rightarrow$ time spans and distances are relative

Boost factor

- Boost factor:

$$\Gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

- time dilation: $\Delta t_R = \Gamma \times \Delta t_P$
- length contraction: $d_P = d_R / \Gamma$

Energy: $E = m c^2$

- Newton:
  - kinetic energy: $E_{kin} = \frac{1}{2} m v^2$
  - $v=0 \Rightarrow E_{kin} = 0$
- Einstein:
  - $E = \Gamma m_0 c^2$
  - $v=0 \Rightarrow \Gamma = 1 \Rightarrow E = m_0 c^2$ "rest energy"
  - $E_{kin} = (\Gamma - 1) m_0 c^2$
- Example:

  The rest energy of an average person can cover the world energy need for 45 days !!!!

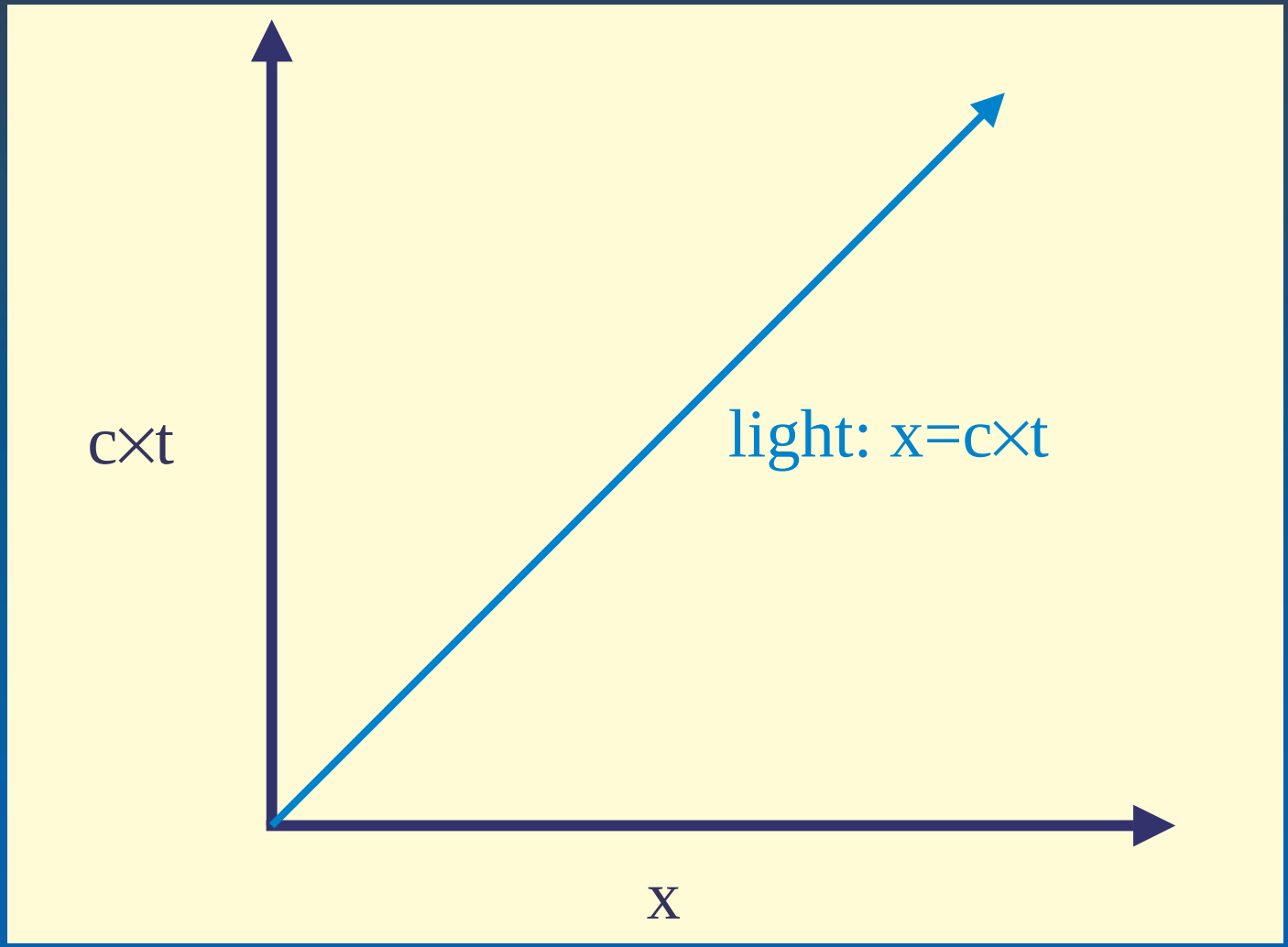
This is messy, so let's clean up a bit

- proper time: time measured by a clock at rest with respect to a specific observer
  $\Rightarrow$ clock at the fastest possible rate
- proper length: length of an object as measured in its own rest frame
  $\Rightarrow$ largest possible length
- time and length in other inertial frames can be calculated by the so-called Lorentz transformation (i.e. multiplying with or dividing by the boost factor Γ)

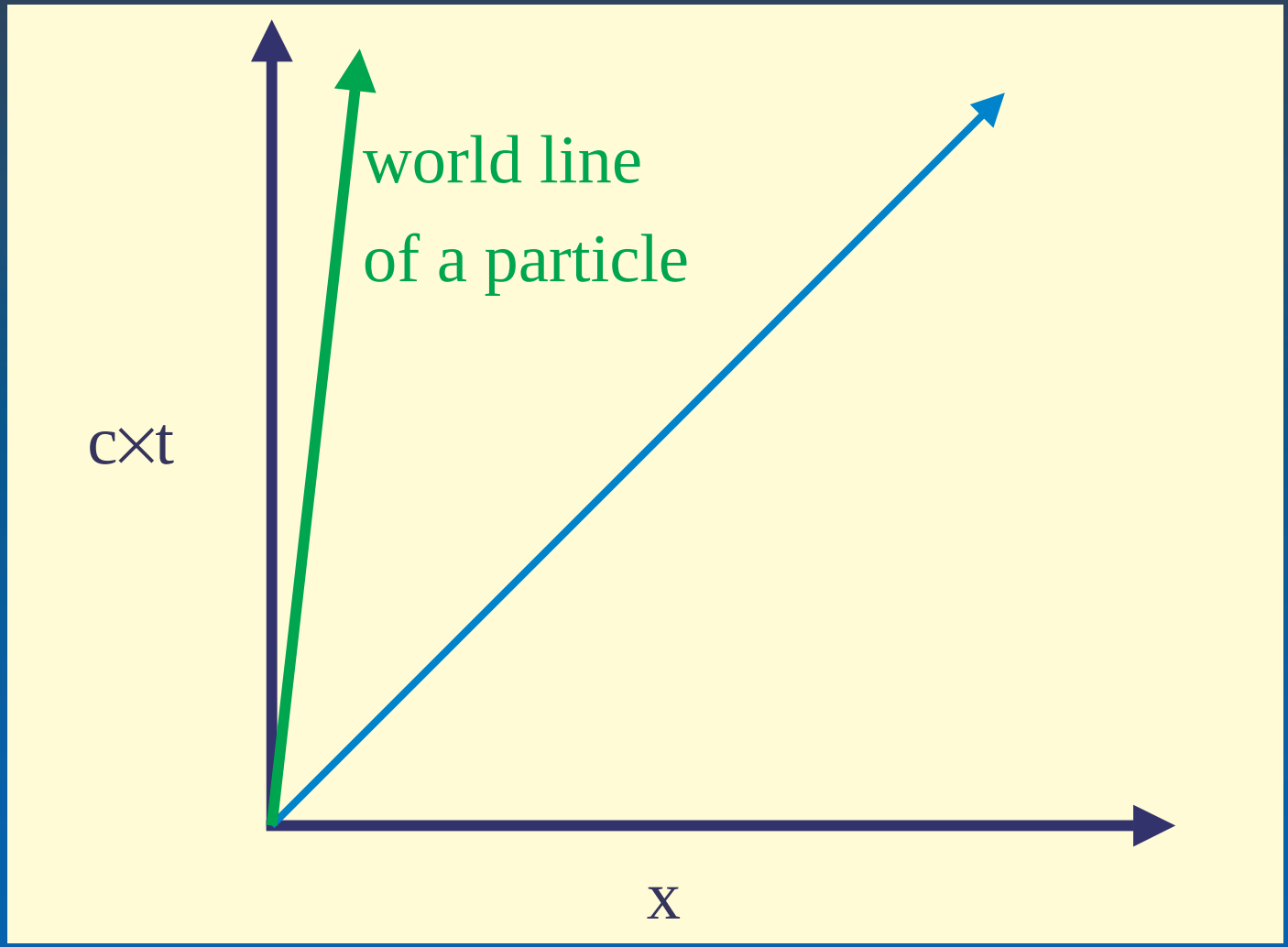
Minkowski's space-time

- As we have seen, time intervals, lengths, and simultaneity is relative and depend on the relative velocity of the observer.
- velocity connects time and space
- Let's stop separating space and time, let's rather talk about space-time
- space-time is 4 dimensional,
  3 spatial + 1 time dimension
  but is space and time really the same thing ?

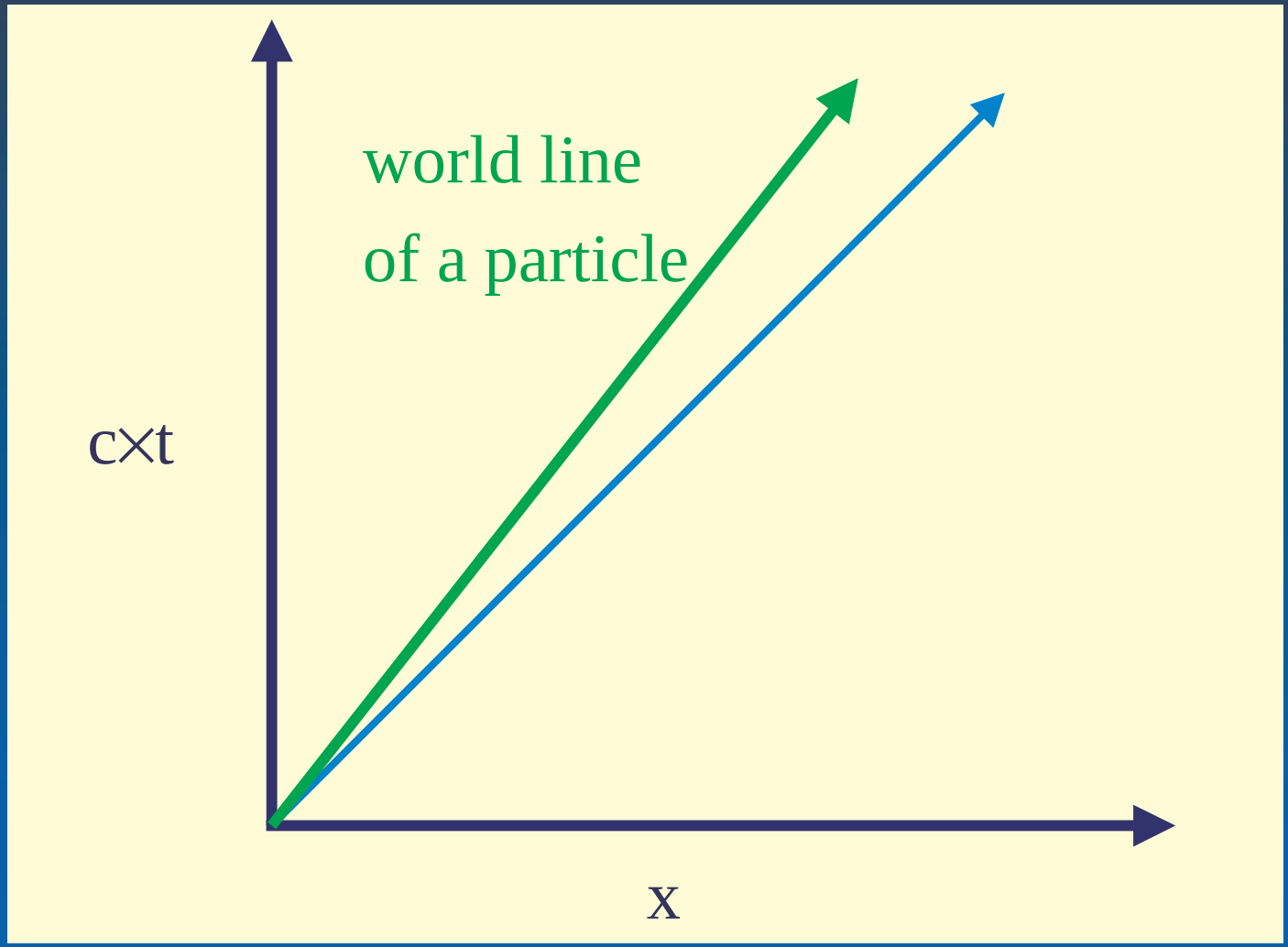
Minkowski diagram



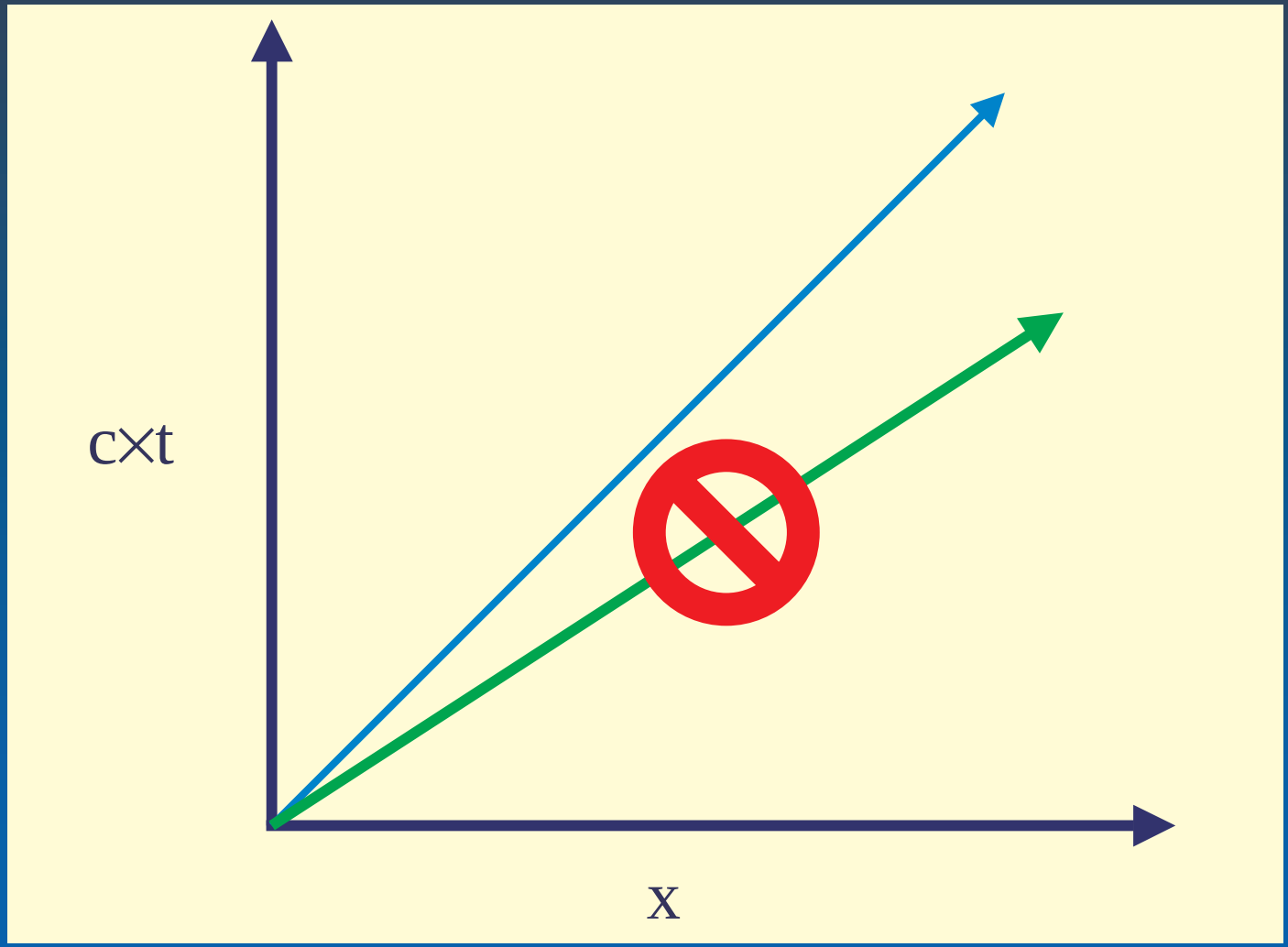
World lines — slowly moving



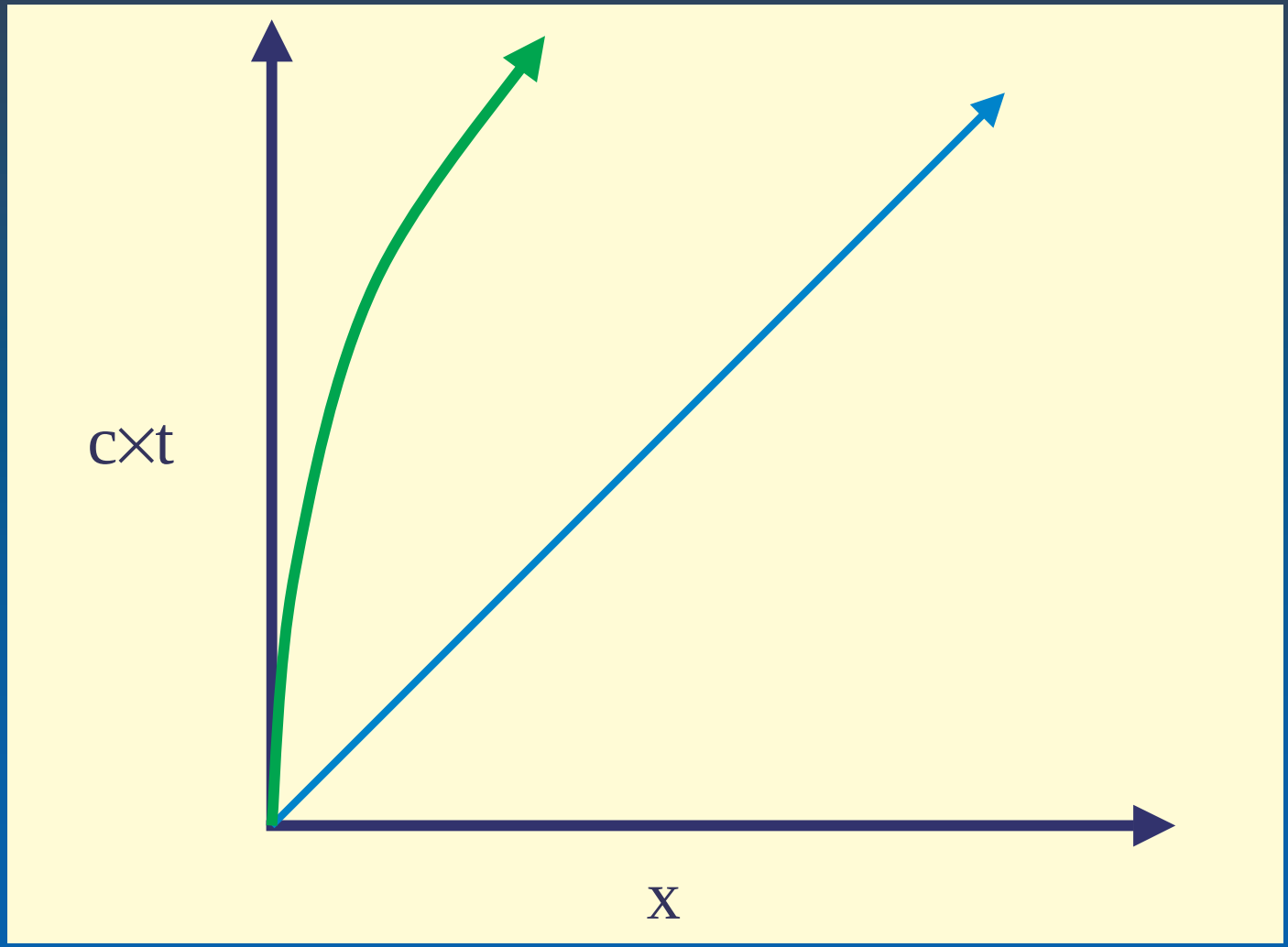
World lines — fast moving



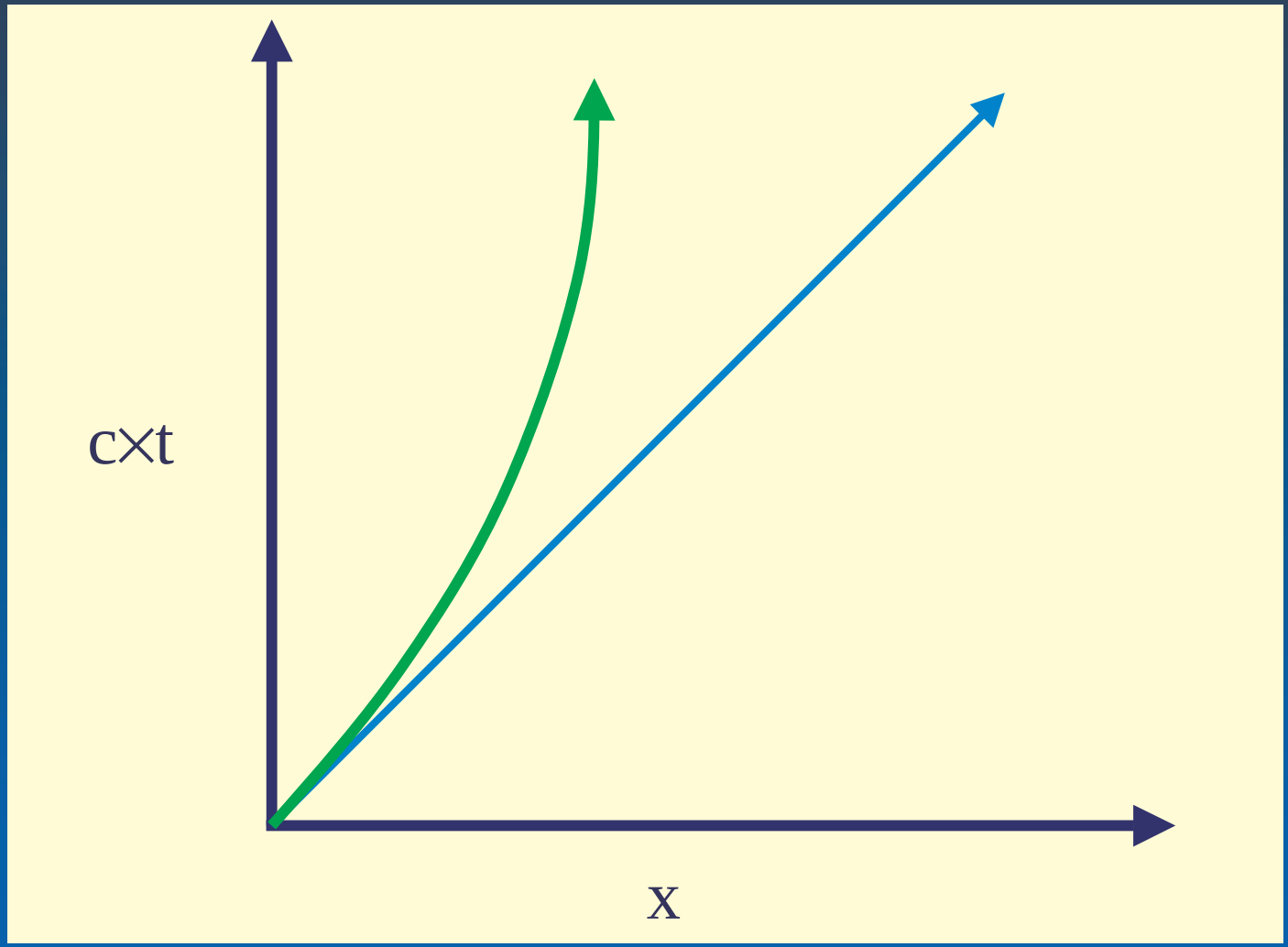
Faster than speed of light ?



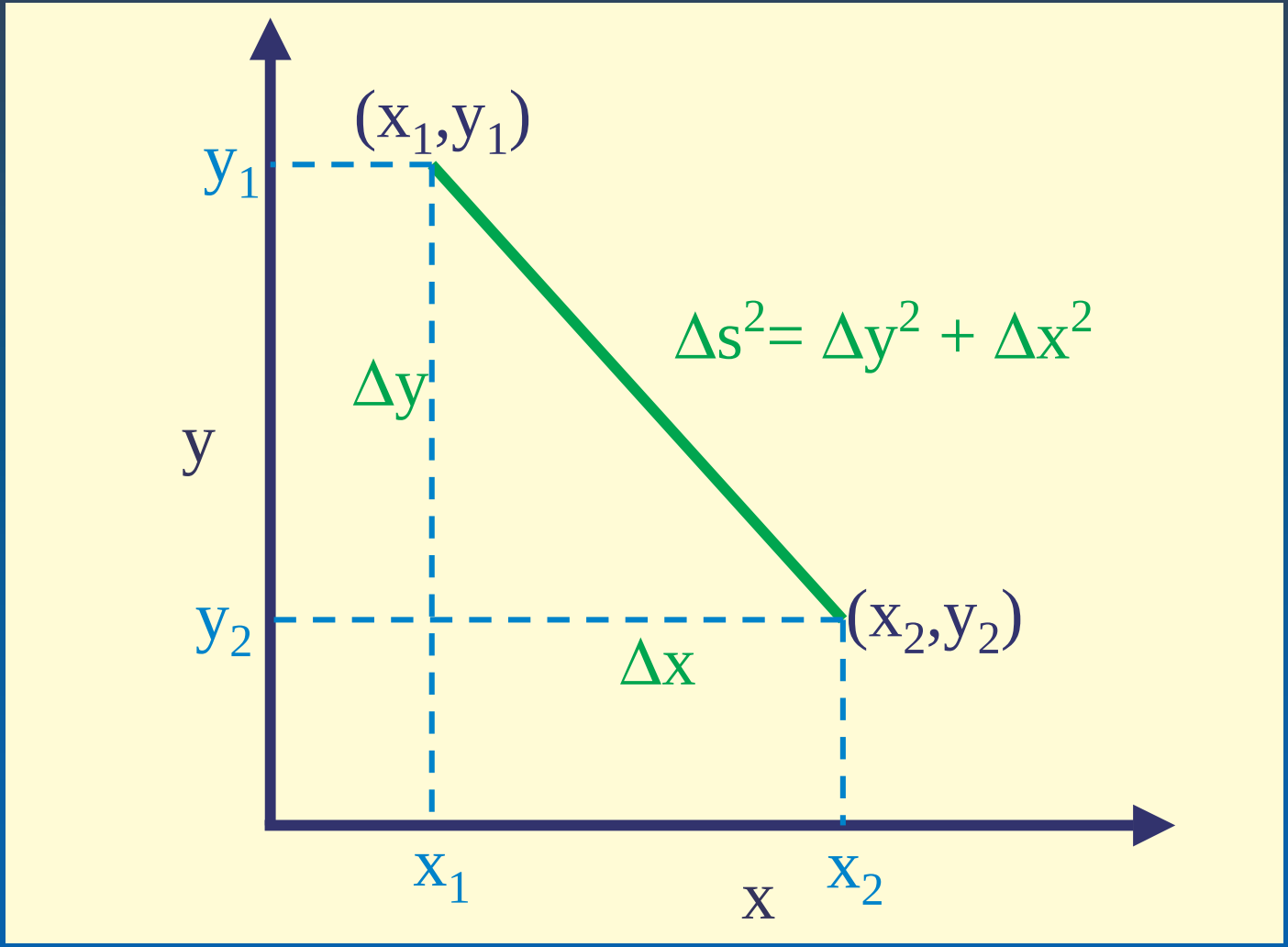
World lines — accelerated



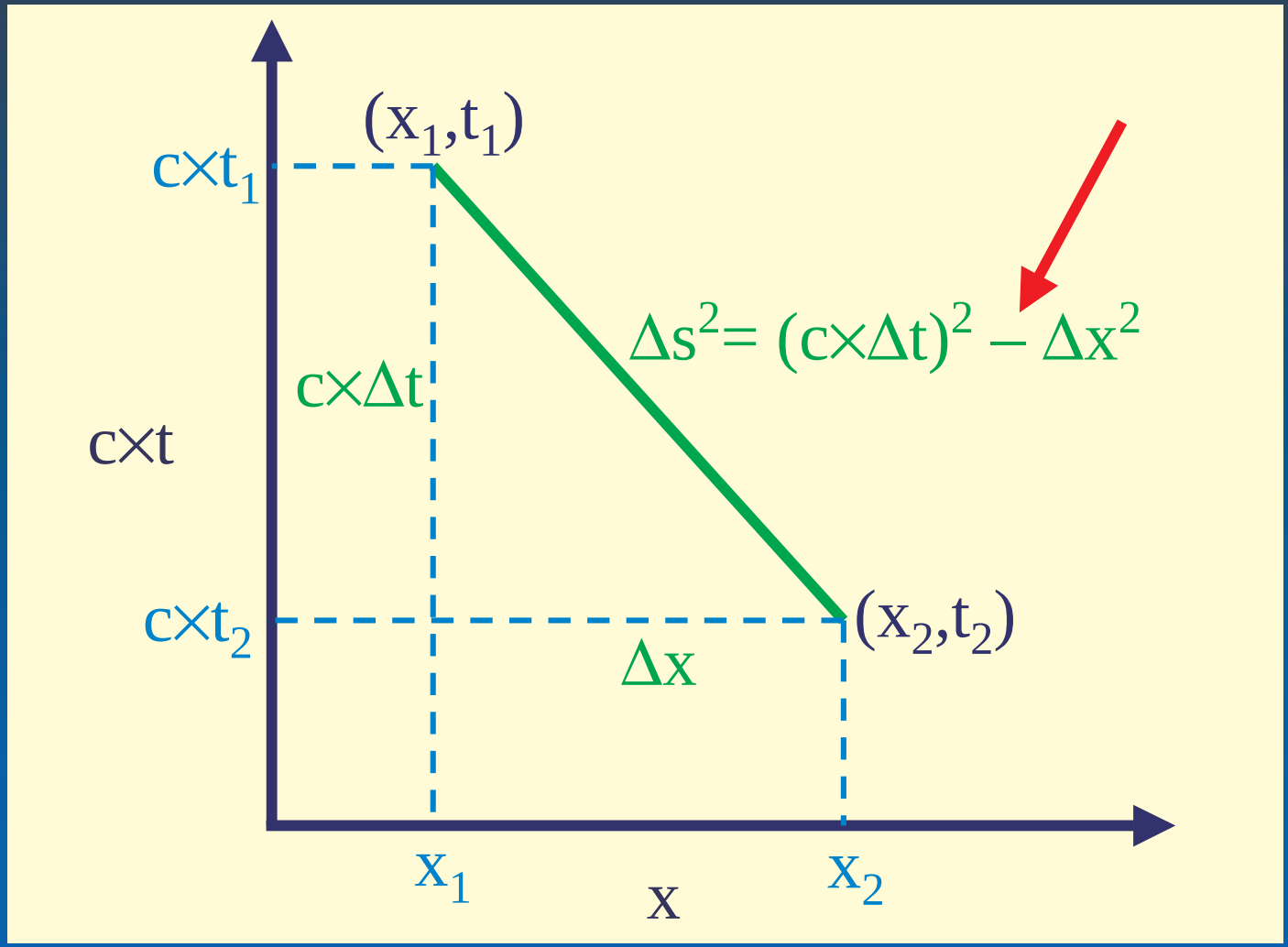
World lines — decelerated



geometrical interval



Space-time interval



Space-time interval

$$\Delta s^2 = (c \times \Delta t)^2 - \Delta x^2$$

- – sign: difference between space and time
- Δs^2 is invariant under Lorentz transformation
- for particle moving at speed of light:

  $\Delta x = c \times \Delta t \Rightarrow \Delta s^2 = 0$

  $\Rightarrow$ **light like (null) distance**

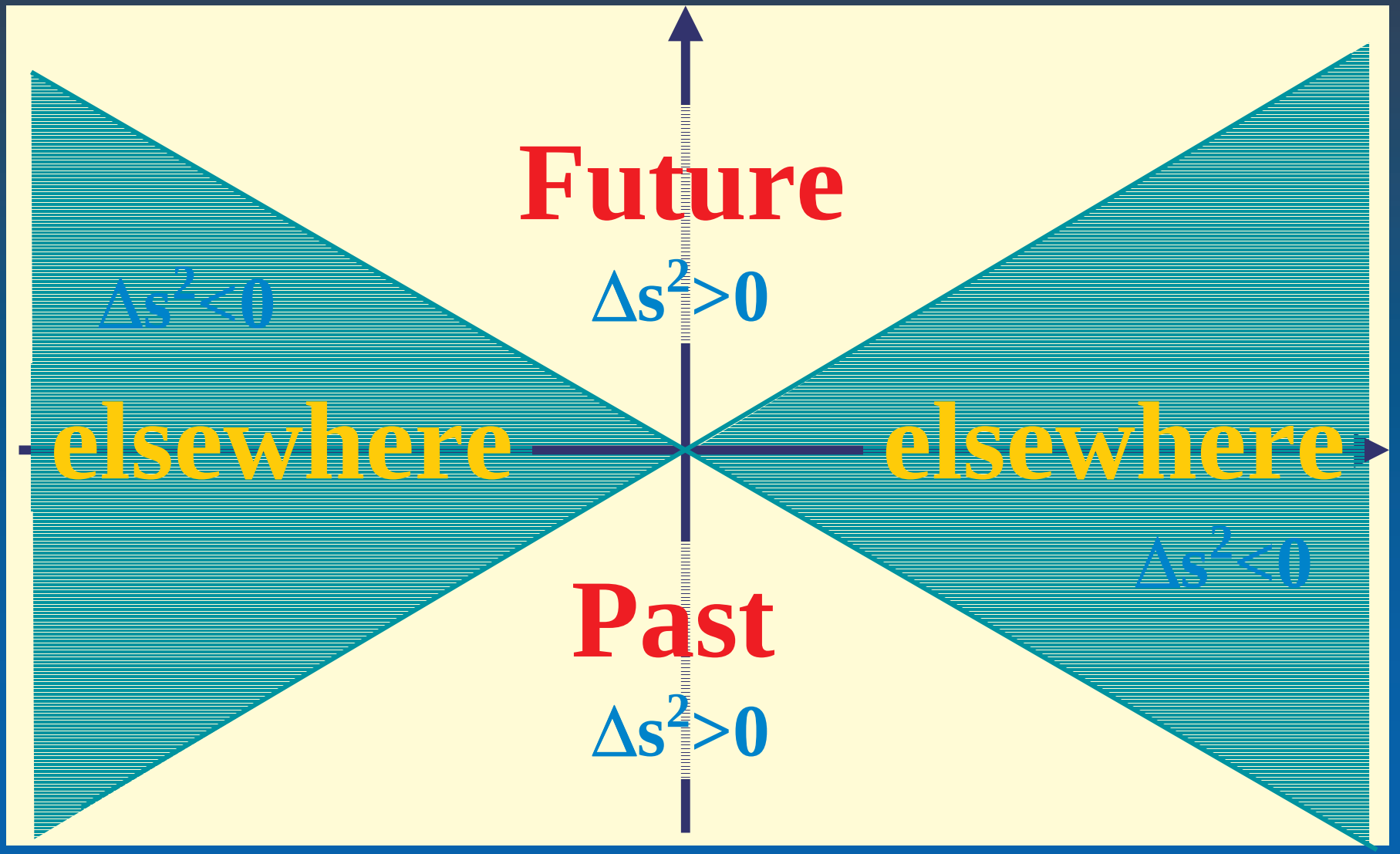
Character of space-time intervals

- $\Delta s^2 > 0 \Rightarrow c \times \Delta t > \Delta x$
  - spatial distance can be traveled by speed of light
  - there exist an inertial frame, in which the two events happen at the same position
  - but they never happen simultaneously

  $\Rightarrow$ time like distance
- $\Delta s^2 < 0 \Rightarrow c \times \Delta t < \Delta x$
  - spatial distance cannot be traveled by speed of light
  - there exist an inertial frame, in which the two events happen simultaneously
  - but they never happen at the same place

  $\Rightarrow$ space like distance

Future, past, and elsewhere



Principle of causality

- All observers agree that B is in the past of A and C is in the future
- Some see A happen first, some see D happen first
- Cause must always precede the effect
  ⇒ A must not influence D and vice versa
  ⇒ nothing can move faster than speed of light