Lecture 23
Black holes I:
the Universe's ultimate lock box!

Gravitational time dilation and redshift

- Can be measured by experiments on Earth (challenging, but feasible)
- Better: White Dwarfs (very compact objects; mass comparable to that of the Sun, radius comparable to that of the Earth), because they have a stronger gravitational field
- Even better: Neutron Stars and Pulsars (very compact objects; mass comparable to that of the Sun, radius only 10-100 km), because they have a very strong gravitational field

Flash-back: Newtonian gravity

- What velocity is required to leave the gravitational field of a planet or star?

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

- Example: Earth
  - Radius: $R = 6470 \text{ km} = 6.47\times10^6 \text{ m}$
  - Mass: $M = 5.97 \times10^{24} \text{ kg}$

  $\Rightarrow$ escape velocity: $v_{esc} = 11.1 \text{ km/s}$

Flash-back: Newtonian gravity

- What velocity is required to leave the gravitational field of a planet or star?

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

- Example: Sun
  - Radius: $R = 700\,000$ km $= 7\times10^8$ m
  - Mass: $M = 2\times10^{30}$ kg

  $\Rightarrow$ escape velocity: $v_{esc} = 617$ km/s

Flash-back: Newtonian gravity

- What velocity is required to leave the gravitational field of a planet or star?

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

- Example: a solar mass White Dwarf
  - Radius: $R = 5000 \text{ km} = 5\times10^6 \text{ m}$
  - Mass: $M = 2\times10^{30} \text{ kg}$

  $\Rightarrow$ escape velocity: $v_{esc} = 7300 \text{ km/s}$

Flash-back: Newtonian gravity

- What velocity is required to leave the gravitational field of a planet or star?

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

- Example: a solar mass neutron star
  - Radius: $R = 10$ km $= 10^4$ m
  - Mass: $M = 2\times10^{30}$ kg

  $\Rightarrow$ escape velocity: $v_{esc} = 163\,000$ km/s $\approx$ ½ c

Flash-back: Newtonian gravity

- Can an object be so small that even light cannot escape ? $\Rightarrow$ Black Hole

$$R_S = \frac{2GM}{c^2}$$

R_S: "Schwarzschild Radius"

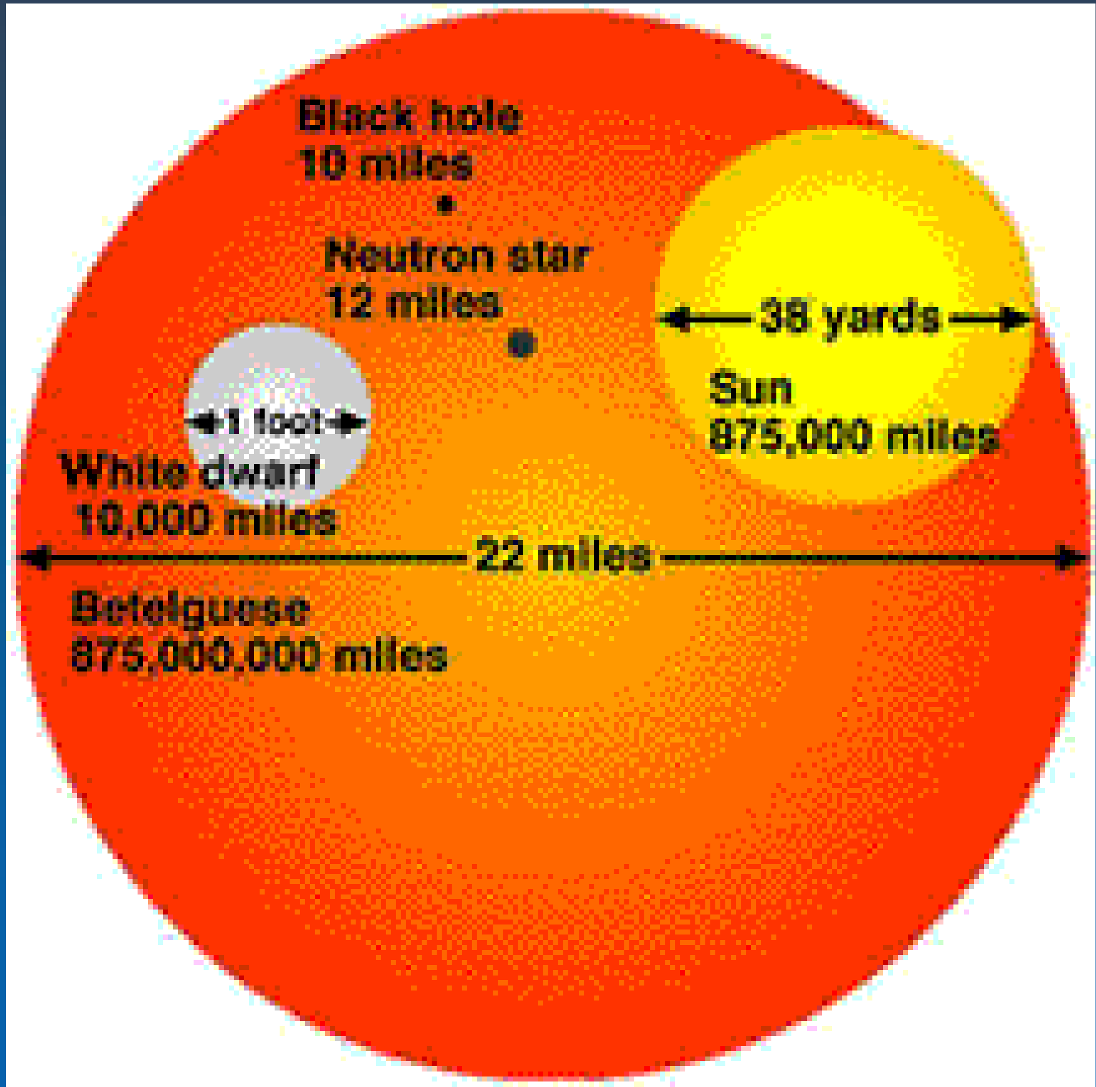
- Example: for a solar mass
  - Mass: $M = 2 \times 10^{30}$ kg

$\Rightarrow$ Schwarzschild Radius: $R_S = 3$ km

Some definitions ...

- The Schwarzschild radius R_S of an object of mass M is the radius, at which the escape speed is equal to the speed of light.
- The event horizon is a sphere of radius R_S. Nothing within the event horizon, not even light, can escape to the world outside the event horizon.
- A Black Hole is an object whose radius is smaller than its event horizon.

Sizes of objects



Let's do it within the context of general relativity — space-time

- space-time distance (flat space):

$$\Delta s^2 = c^2 \Delta t^2 - \Delta R^2$$

time space

- Fourth coordinate: ct
- time coordinate has different sign than spatial coordinates

Let's do it within the context of general relativity — space-time

- space-time distance (curved space of a point mass):

$$\Delta s^2 = \left(1 - \frac{R_S}{R}\right) c^2 \Delta t^2 - \frac{1}{1 - R_S / R} \Delta R^2$$

time **space**

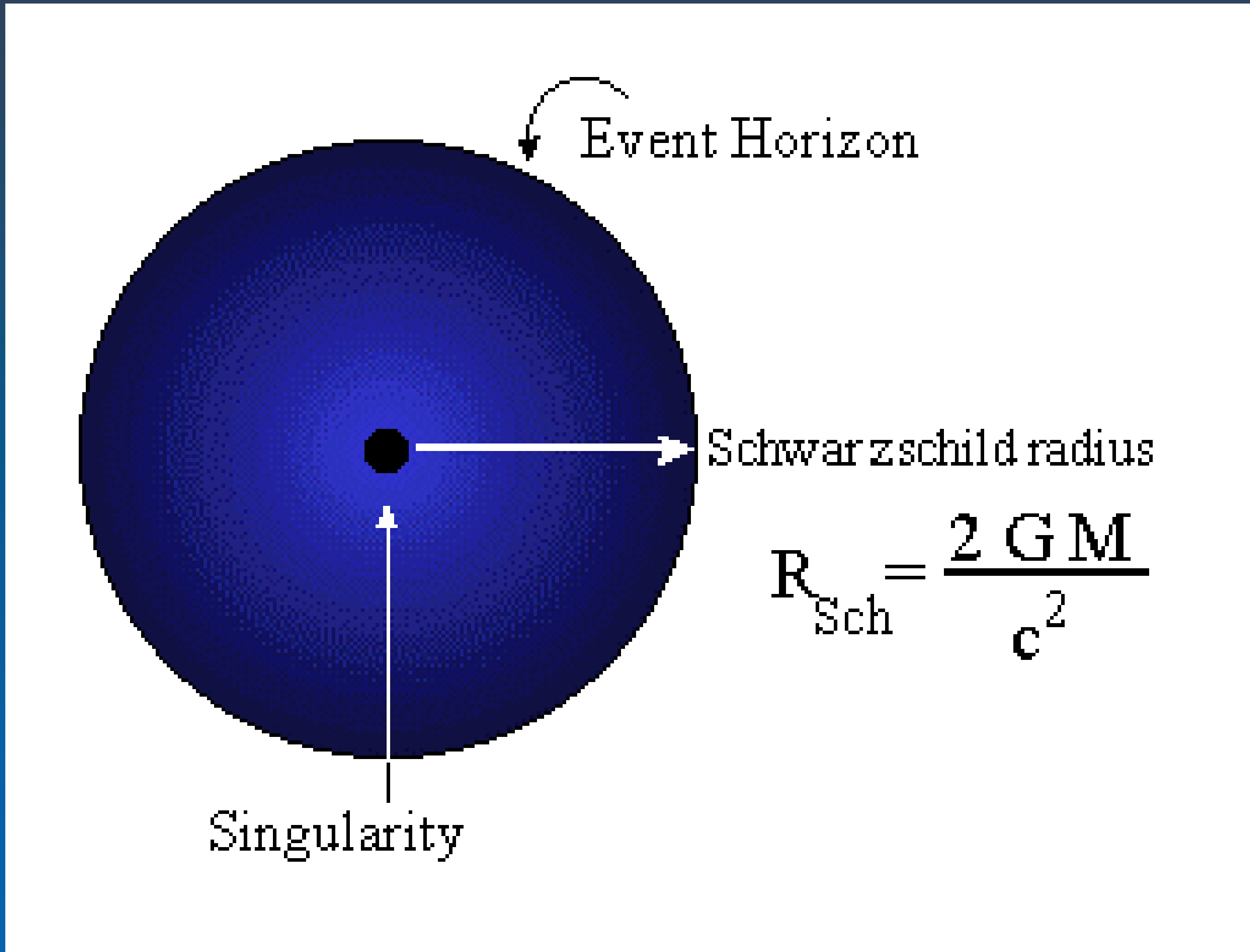
What happens if $R \to R_S$

$$\Delta s^2 = \left(1 - \frac{R_S}{R}\right) c^2 \Delta t^2 - \frac{1}{1 - R_S / R} \Delta R^2$$

time   space

- $R > R_S$: everything o.k.: time: +, space: – but gravitational time dilation and length contraction
- $R \to R_S$: time $\to 0$ space $\to \infty$
- $R < R_S$: signs change!! time: –, space: +
  $\Rightarrow$ "space passes", everything falls to the center
  $\Rightarrow$ infinite density at the center, singularity

Structure of a Black Hole



What happens to an astronaut who falls into a black hole?

- Far outside: nothing special
- Falling in: long before the astronaut reaches the event horizon, he/she is torn apart by tidal forces
- For an outside observer:
  - astronaut becomes more and more redshifted
  - The astronaut's clock goes slower and slower
  - An outside observer never sees the astronaut crossing the event horizon.

What happens, if an astronaut falls into a black hole?

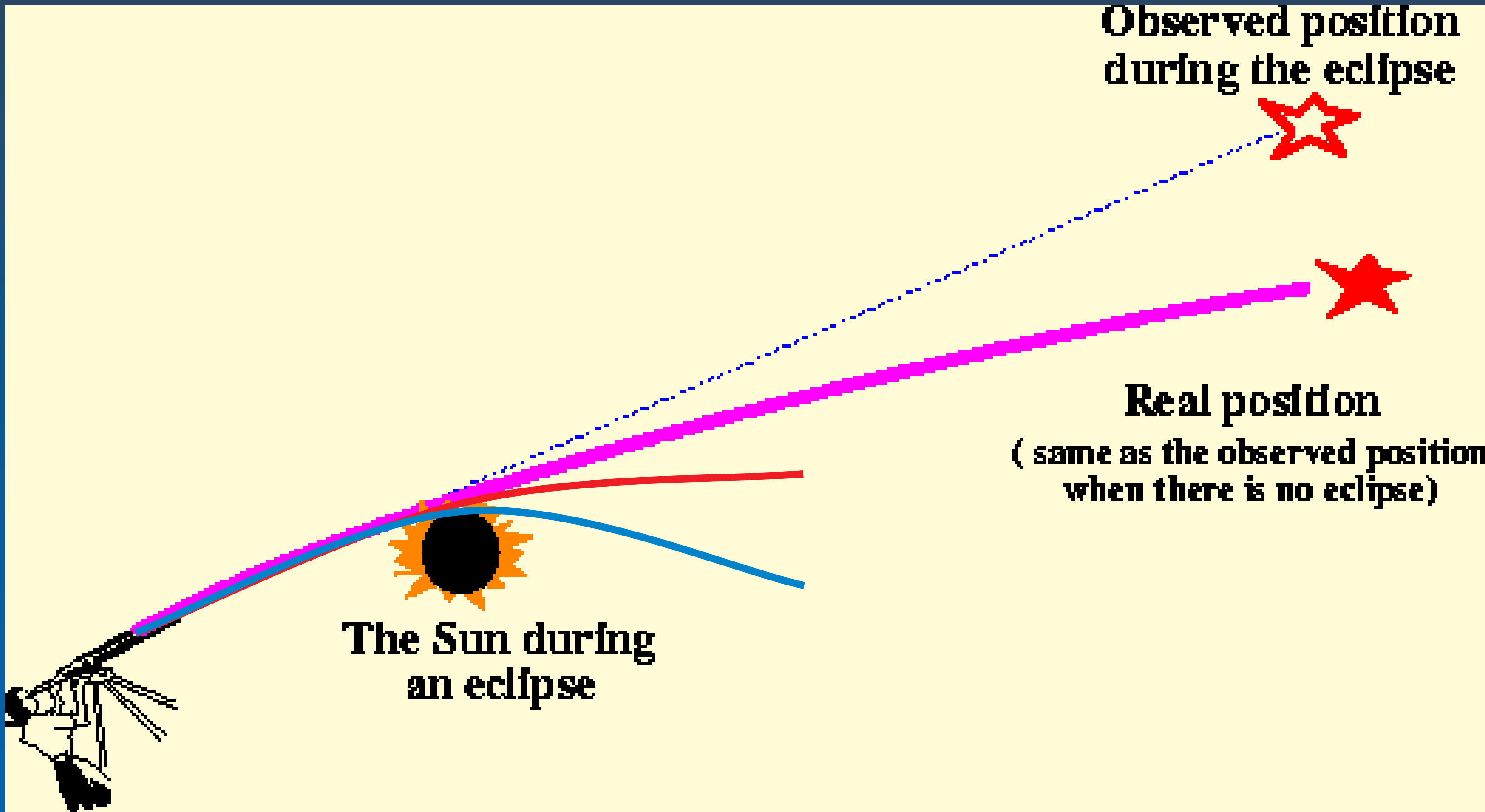
- For the astronaut:
  - He/she reaches and crosses the event horizon in a finite time.
  - Nothing special happens while crossing the event horizon (except some highly distorted pictures of the local environment)
  - After crossing the event horizon, the astronaut has 10 microseconds to enjoy the view before he/she reaches the singularity at the center.

Cosmic censorship

- Singularity: a point at which space-time diverges
  - infinite forces are acting
  - laws of physics break down
  - quantum gravity may help ?
  - no problem as long as a singularity is shielded from the outside world by an event horizon
- Hypothesis: Every singularity is surrounded by an event horizon.

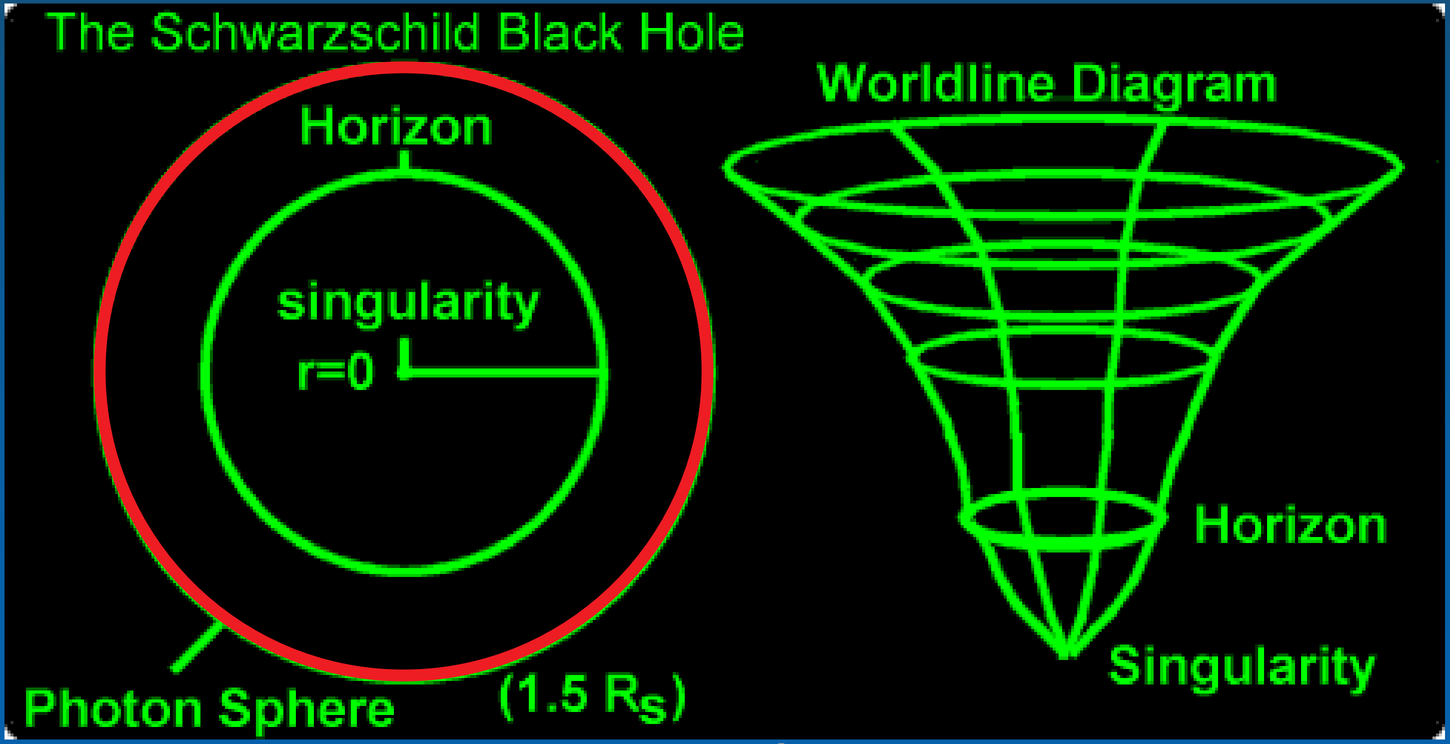
  There are no naked singularities

Near a black hole: bending of light

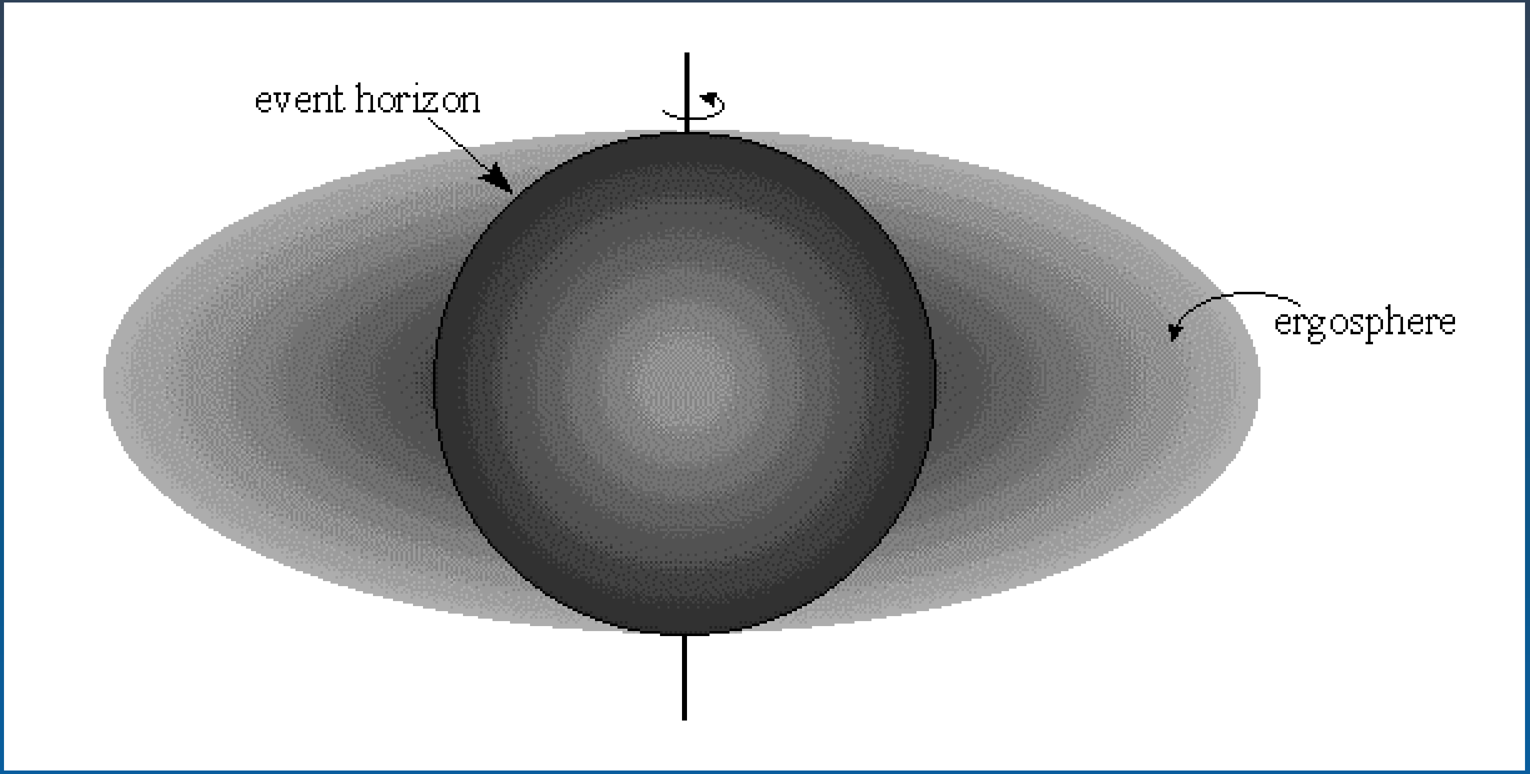


The Photon sphere

The photon sphere is a sphere of radius 1.5 R_S. On the photon sphere, light orbits a black hole on a circular orbit.



Structure of a rotating black hole



Within the ergosphere (or static sphere) nothing can remain at rest. Space-time is dragged around the hole

No-Hair theorem

- Properties of a black hole:
  - it has a mass
  - it has an electric charge
  - it has a spin (angular momentum)
  - that's it. Like an elementary particle, but much more massive

Black holes have no hair

Questions:

- Do they really exist ?
- How do we observe something that does not emit light?