Lecture 28
The big-bang model I: Making the Universe comprehensible

A metric of an expanding Universe

- Robertson-Walker metric

$$\Delta s^2 = (c\Delta t)^2 - R^2(t)\left(\frac{\Delta r^2}{1-kr^2} + r^2\Delta\theta^2 + r^2\sin^2\theta\,\Delta\phi^2\right)$$

- *R(t)* is the scale factor
- *k* is the curvature constant
  - *k=0*: flat space
  - *k>0*: spherical geometry
  - *k<0*: hyperbolic geometry

Cosmological redshift

- While a photon travels from a distance source to an observer on Earth, the Universe expands in size from R_{then} to R_{now}.
- Not only the Universe itself expands, but also the wavelength of the photon λ.

$$\lambda_{received} = \frac{R_{now}}{R_{then}} \lambda_{emitted}$$

Cosmological redshift

- General definition of redshift:

$$z = \frac{\lambda_{received} - \lambda_{emitted}}{\lambda_{emitted}}$$

$\Rightarrow$ for cosmological redshift:

$$1 + z = \frac{\lambda_{received}}{\lambda_{emitted}} = \frac{R_{now}}{R_{then}}$$

Cosmological redshift

- Examples:
  - $z=1 \Rightarrow R_{then}/R_{now} = 0.5$
    - at z=1, the universe had 50% of its present day size
    - emitted blue light (400 nm) is shifted all the way through the optical spectrum and is received as red light (800 nm)
  - $z=4 \Rightarrow R_{then}/R_{now} = 0.2$
    - at z=4, the universe had 20% of its present day size
    - emitted blue light (400 nm) is shifted deep into the infrared and is received at 2000 nm
  - most distant astrophysical object discovered so far: z=5.8

A large redshift z implies ...

- The spectrum is strongly shifted toward red or even infrared colors
- The object is very far away
- We see the object at an epoch when the universe was much younger than the present day universe
- most distant astrophysical object discovered so far: z=5.8
- z>5.8: “dark ages”

A metric of an expanding Universe

- Robertson-Walker metric

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- *R(t)* is the scale factor
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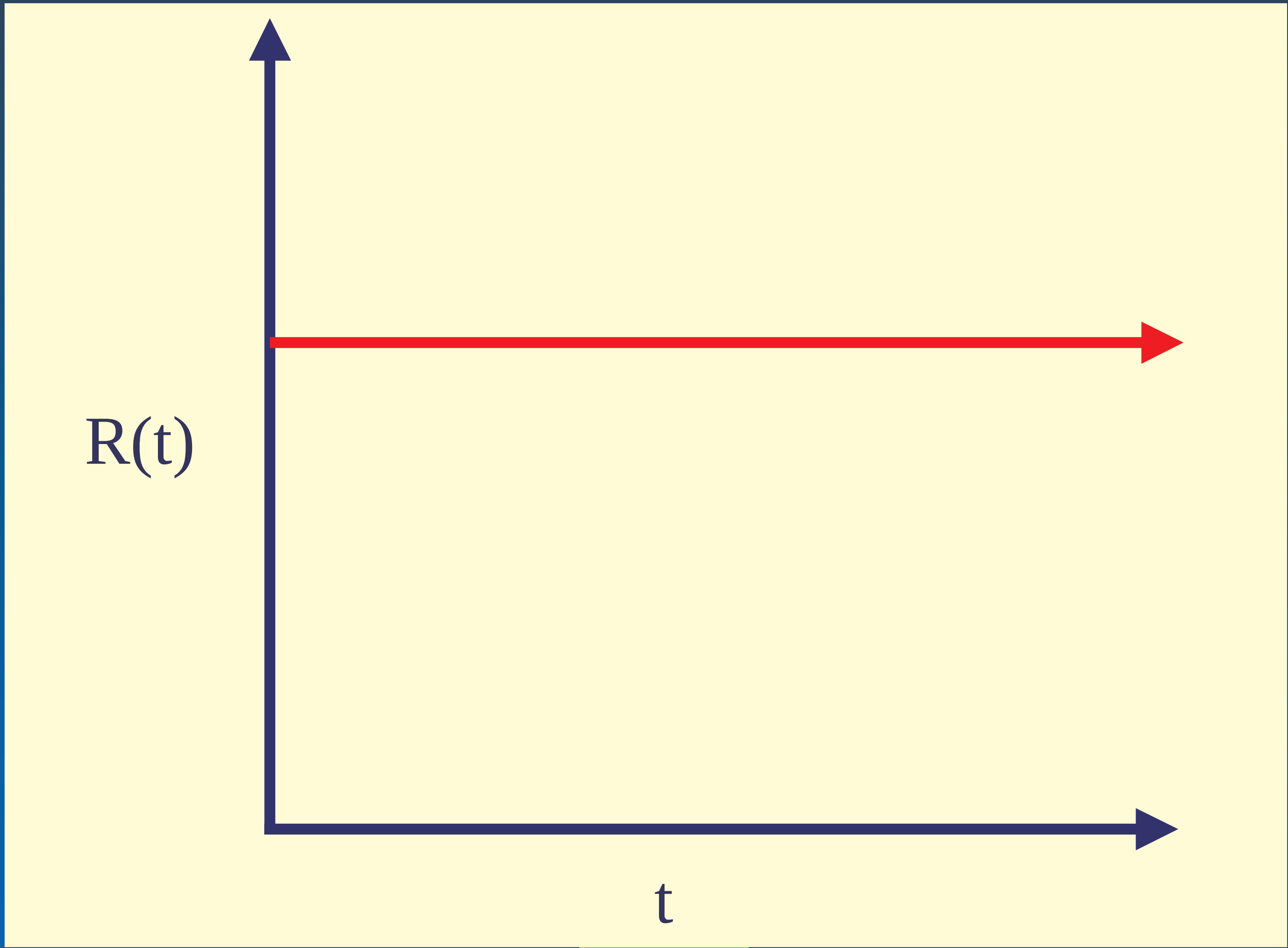
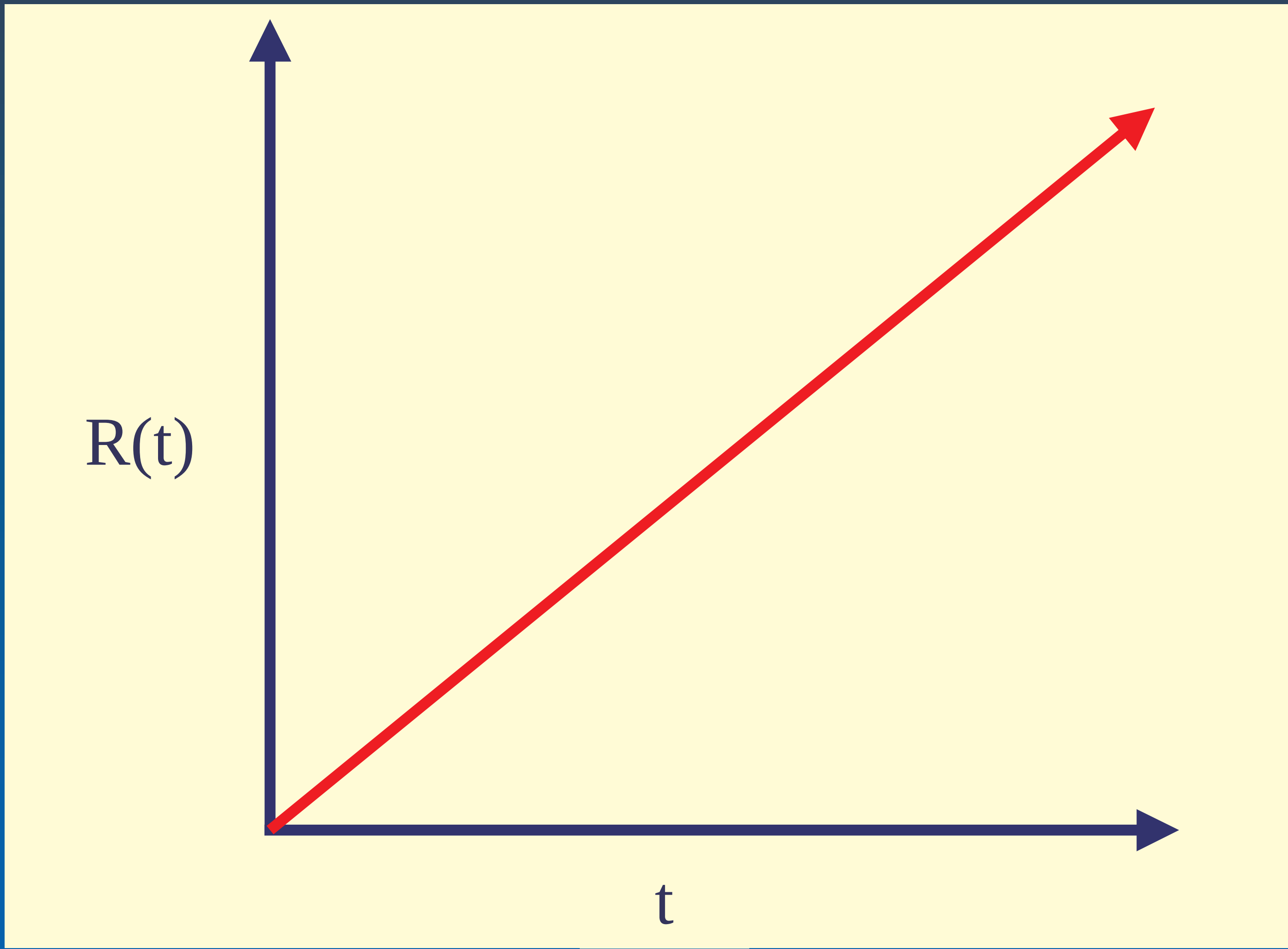
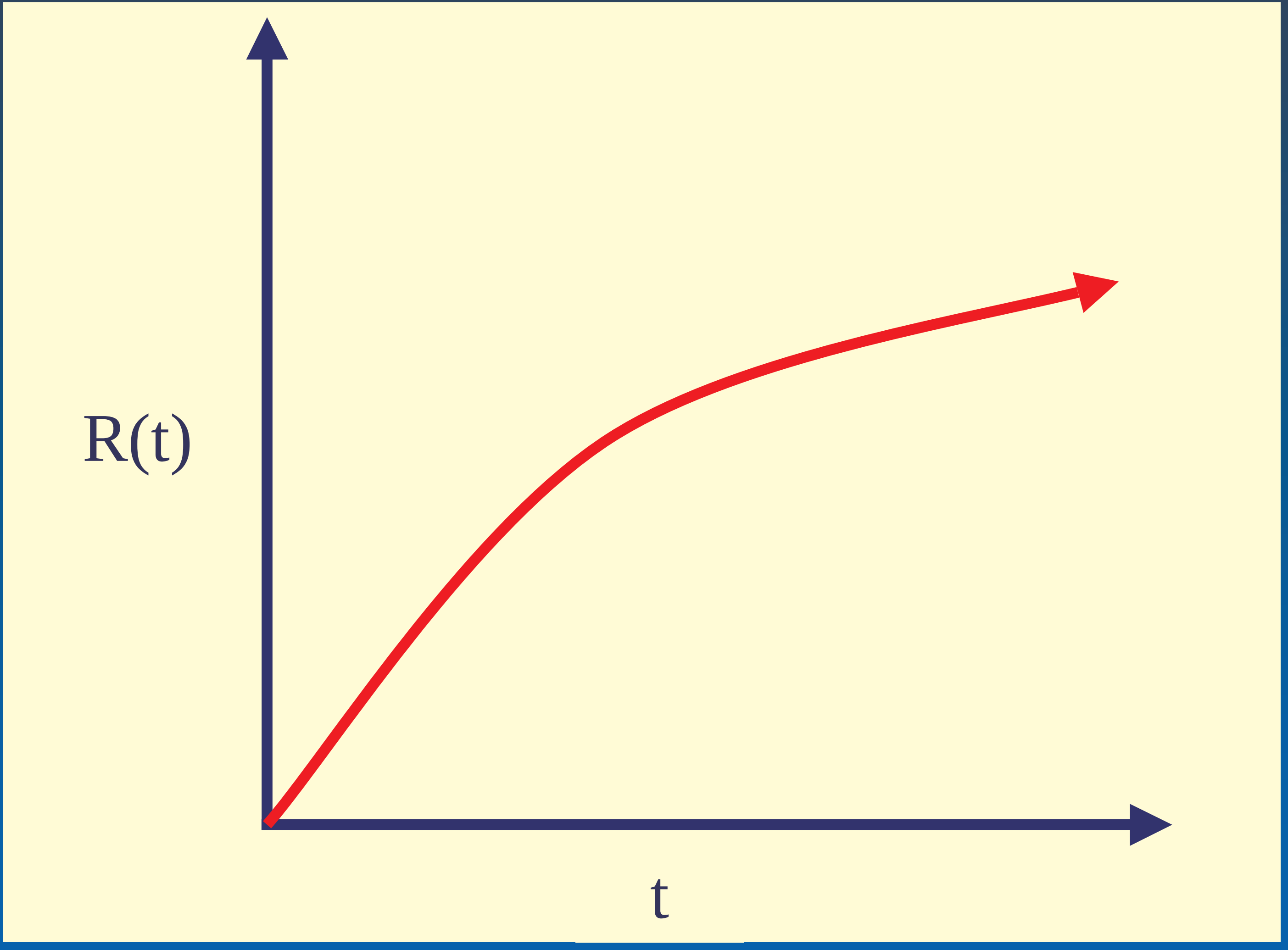
Example: static universe



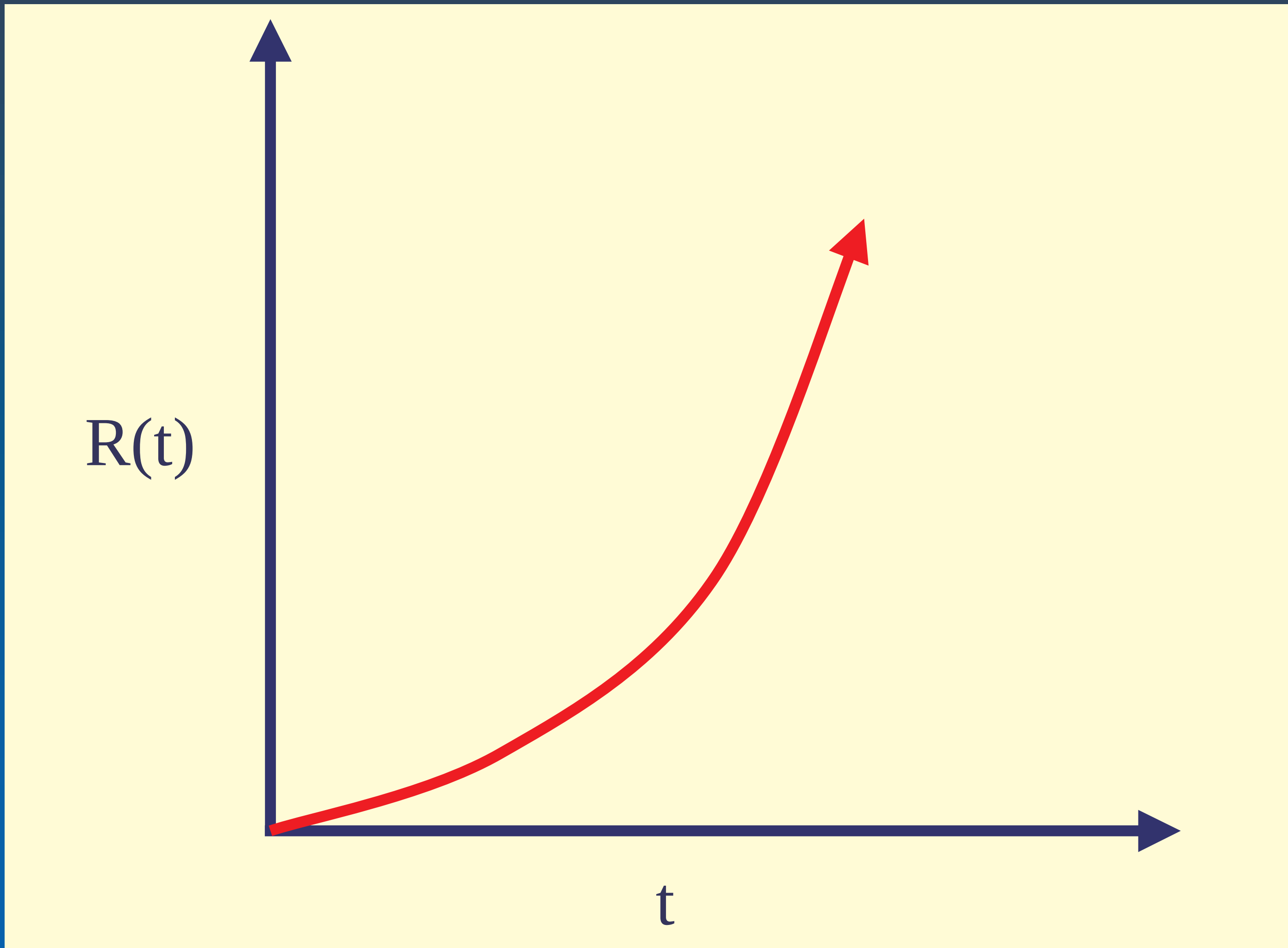
Example: expanding at a constant rate



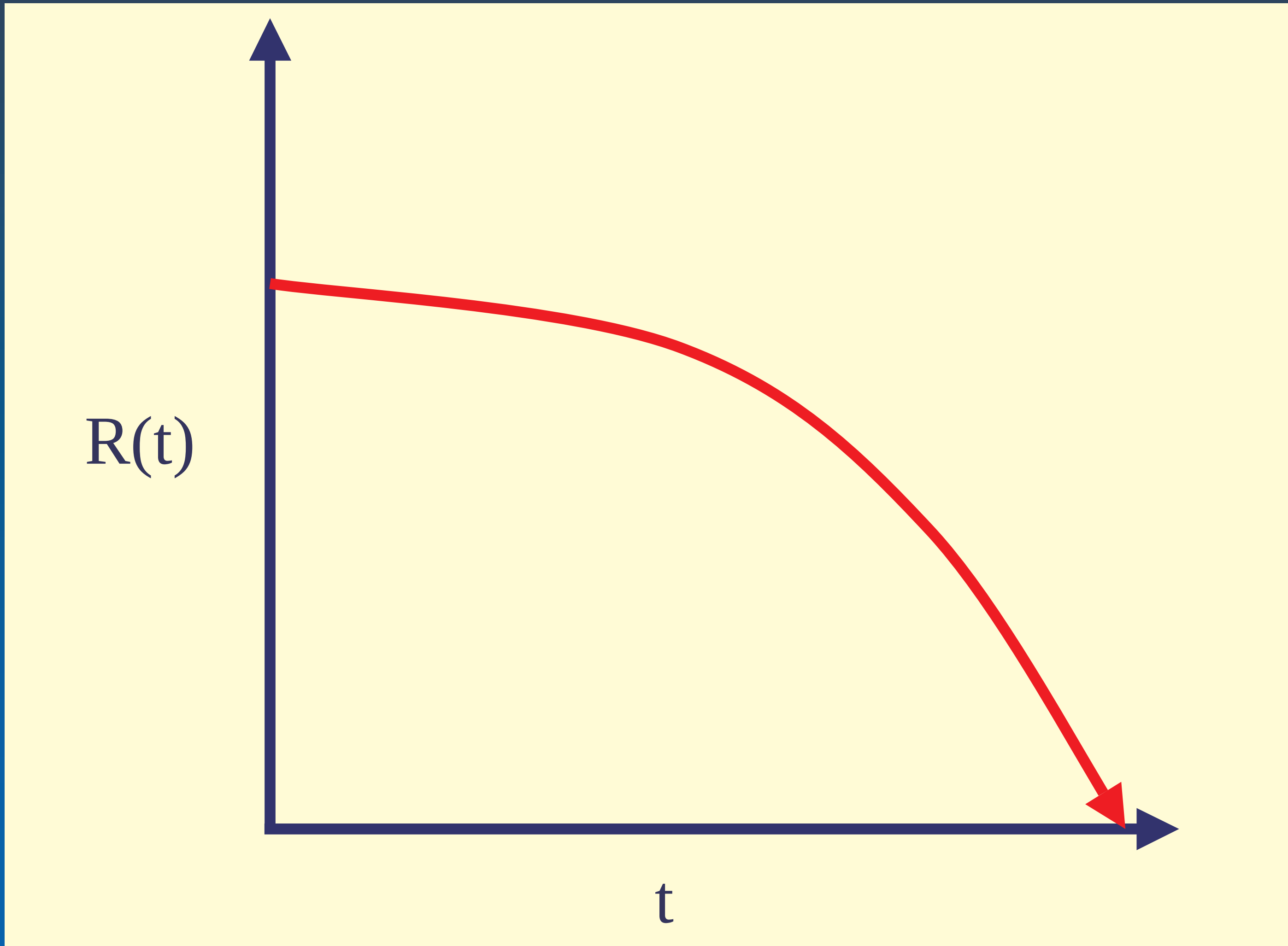
Example: expansion is slowing down



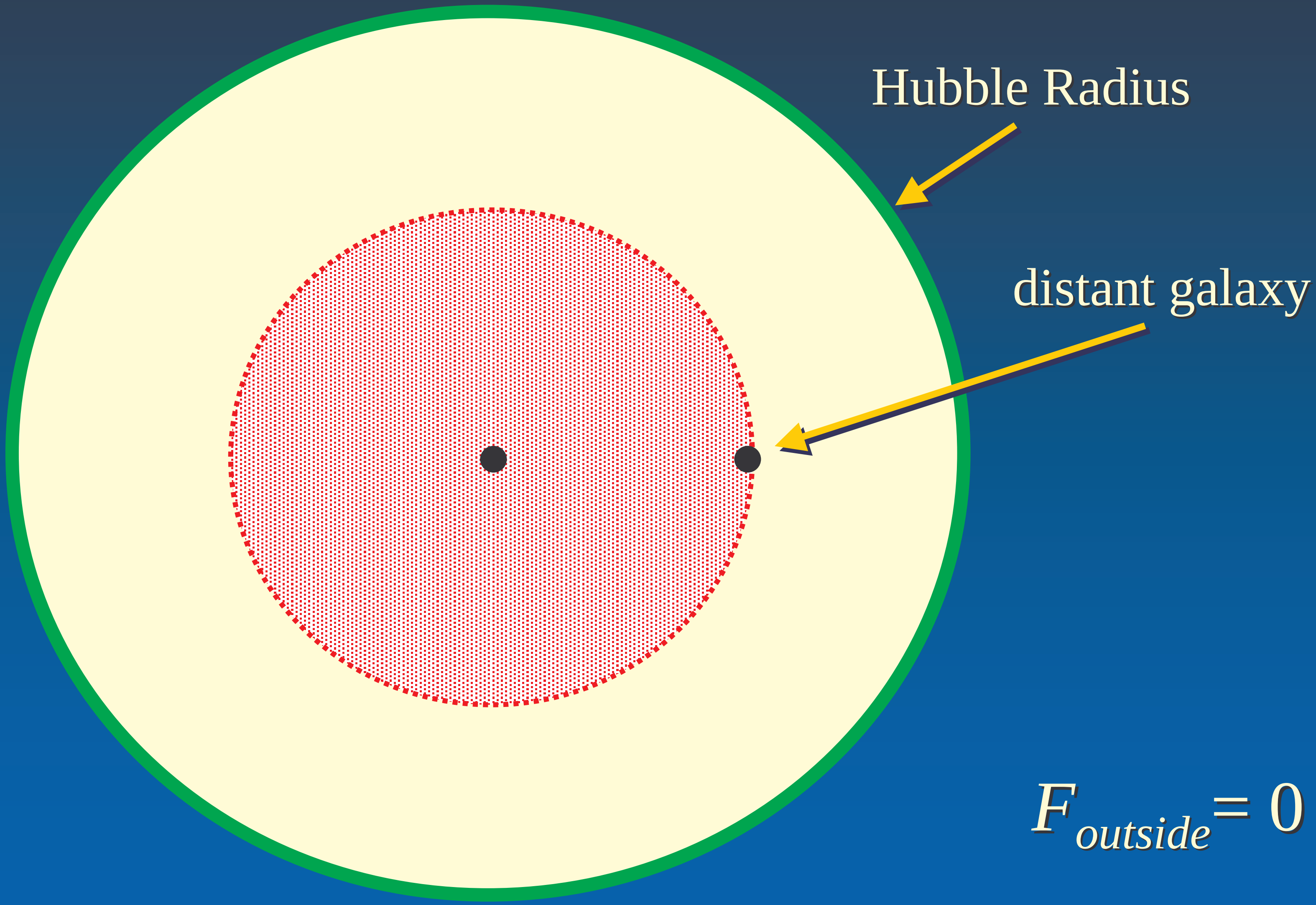
Example: expansion is accelerating



Example: collapsing

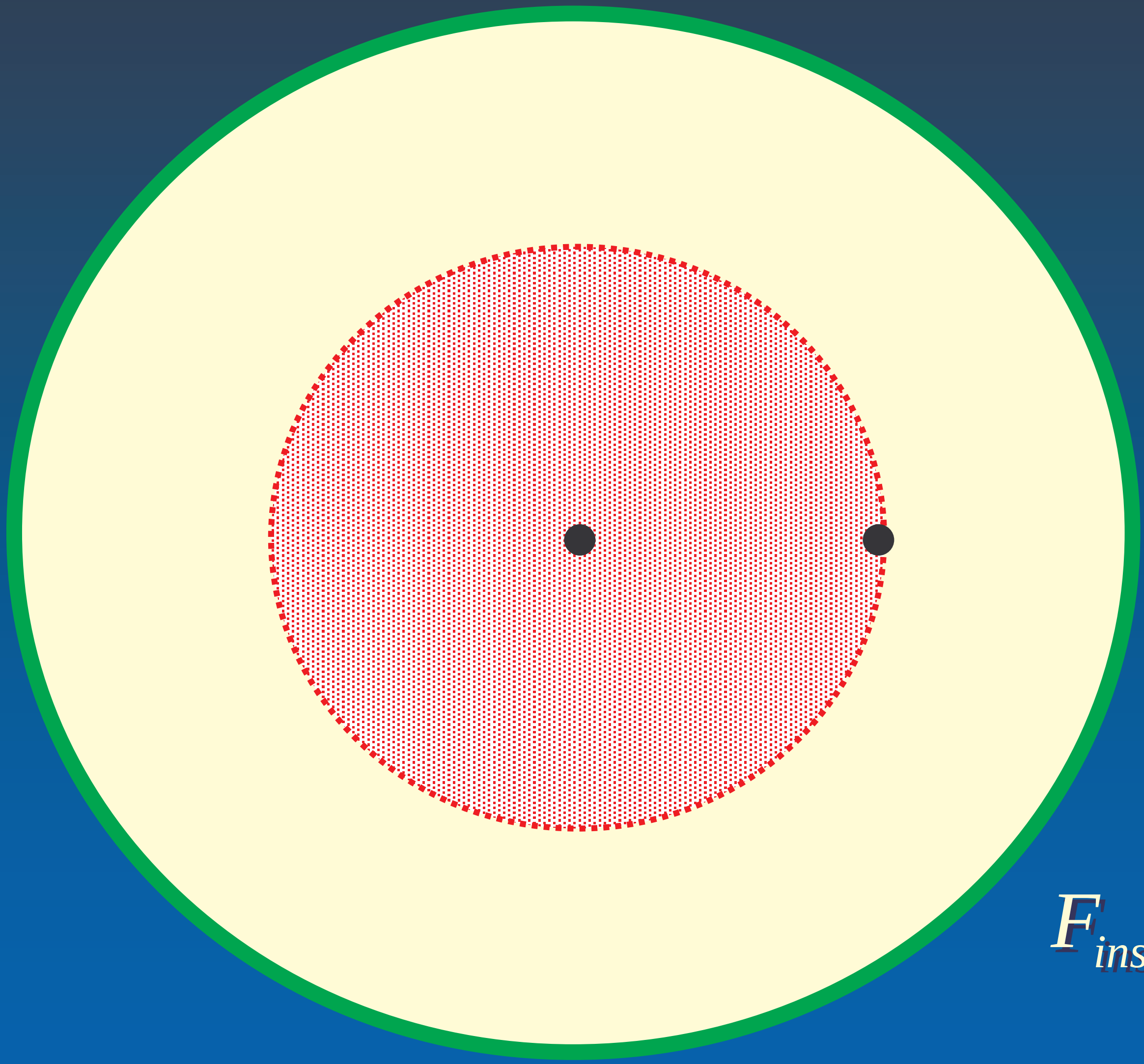


Can we calculate R(t) ?



$$F_{outside} = 0$$

Can we calculate R(t) ?



$$F_{inside} = -G\frac{M_{inside}m_{gal}}{R^2}$$

What is the future of that galaxy ?

- Critical velocity: escape speed

$$v_{esc} = \sqrt{\frac{2GM_{inside}}{R}}$$

- $v < v_{esc}$: galaxy eventually stops and falls back
- $v > v_{esc}$: galaxy will move away forever

Let's rewrite that a bit ...

$$v^2 = \frac{2GM_{inside}}{R} + 2\varepsilon_\infty$$

- $\varepsilon_\infty < 0 \Rightarrow v < v_{esc}$: galaxy eventually stops and falls back
- $\varepsilon_\infty > 0 \Rightarrow v > v_{esc}$: galaxy will move away forever

Let's rewrite that a bit ...

- Homogeneous sphere of density ρ :

$$M_{inside} = \frac{4\pi}{3} \rho R^3$$

- so for the velocity:

$$v^2 = \frac{8\pi G}{3} \rho R^2 + 2\varepsilon_\infty$$

- but what is ε_∞?

Let's switch to general relativity

- Friedmann equation

$$v^2 = \frac{8\pi G}{3}\rho R^2 - kc^2$$

- same *k* as in the Robertson-Walker metric

$$\Delta s^2 = (c\Delta t)^2 - R^2(t)\left(\frac{\Delta r^2}{1-kr^2} + r^2\Delta\theta^2 + r^2\sin^2\theta\,\Delta\phi^2\right)$$

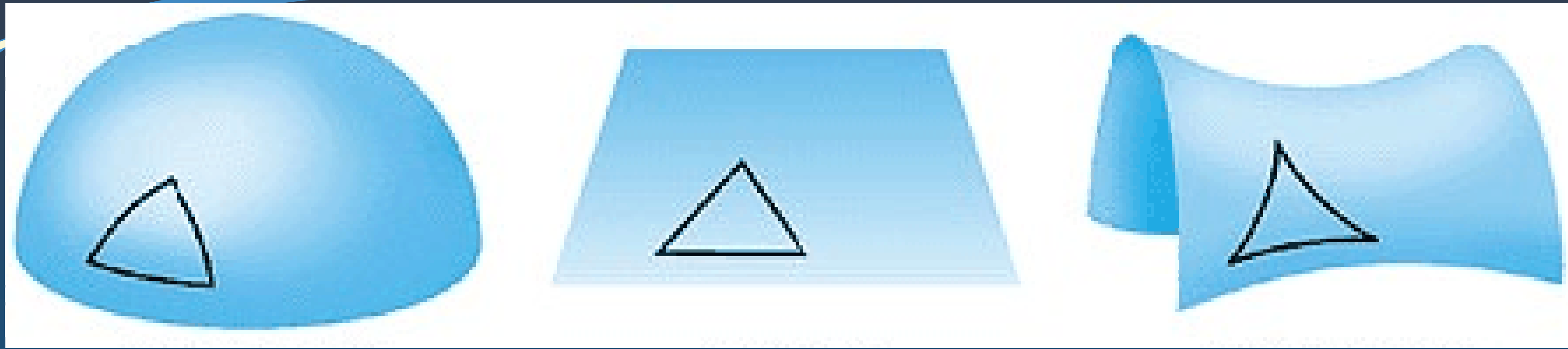
Let's switch to general relativity

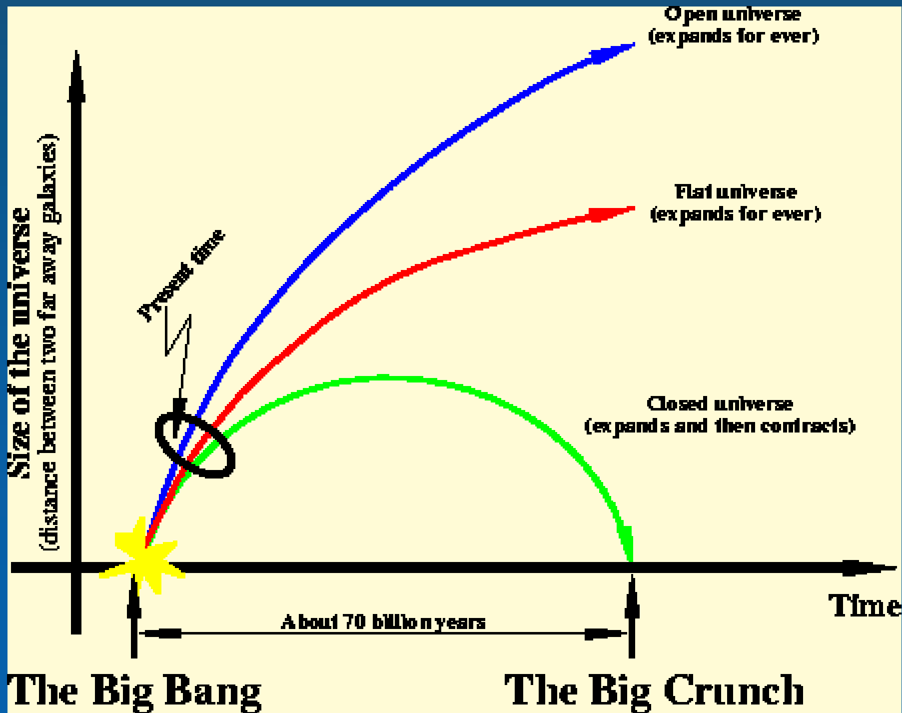
- Friedmann equation

$$v^2 = \frac{8\pi G}{3}\rho R^2 - kc^2$$

- k is the curvature constant
  - $k=0$: flat space, forever expanding
  - $k>0$: spherical geometry, eventually recollapsing
  - $k<0$: hyperbolic geometry, forever expanding

k>0 *k=0* *k<0*




Open universe
(expands for ever)
Flat universe
(expands for ever)
Closed universe
(expands and then contracts)
Present time
Size of the universe
(distance between two far away galaxies)
About 70 billion years
Time
The Big Bang
The Big Crunch

Can we predict the fate of the Universe ?

- Friedmann equation:

$$H_0^2 = \frac{v^2}{R^2} = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2}$$

- *k=0*:

$$\rho_{crit} = \frac{3H_0^2}{8\pi G}$$

Can we predict the fate of the Universe ?

- If the density ρ of the Universe
  - $\rho = \rho_{crit}$: flat space, forever expanding
  - $\rho > \rho_{crit}$: spherical geometry, recollapsing
  - $\rho < \rho_{crit}$: hyperbolic geometry, forever expanding
- so what is the density of the universe?
  - We don't know precisely
  - $\rho > \rho_{crit}$ very unlikely
  - currently favored model: $\rho \approx 0.3\rho_{crit}$