Lecture 29
The big-bang model II:
The cosmological constant:
Einstein's greatest blunder
or tomorrow's Nobel prize ?

Let's switch to general relativity

- Friedmann equation

$$v^2 = \frac{8\pi G}{3}\rho R^2 - kc^2$$

- k is the curvature constant

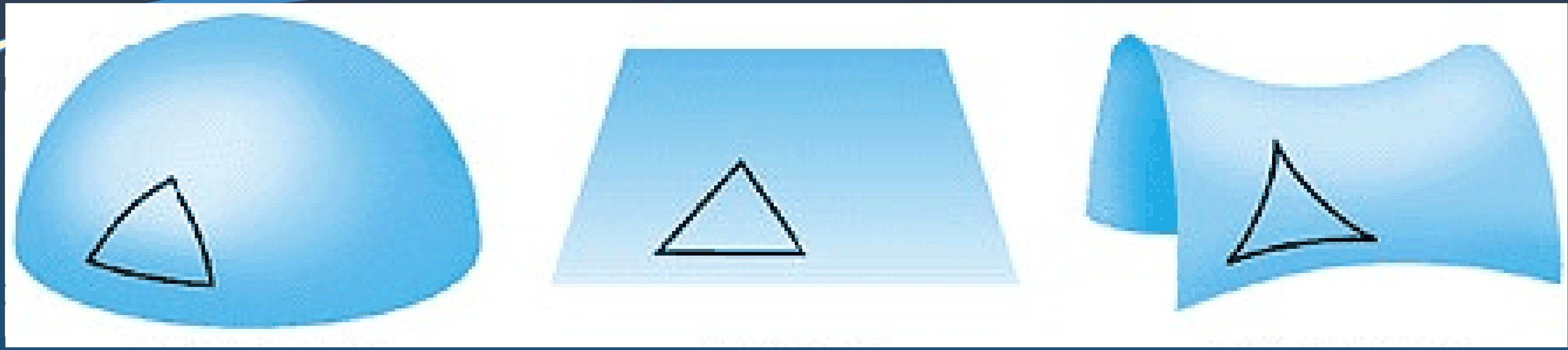
Let's switch to general relativity

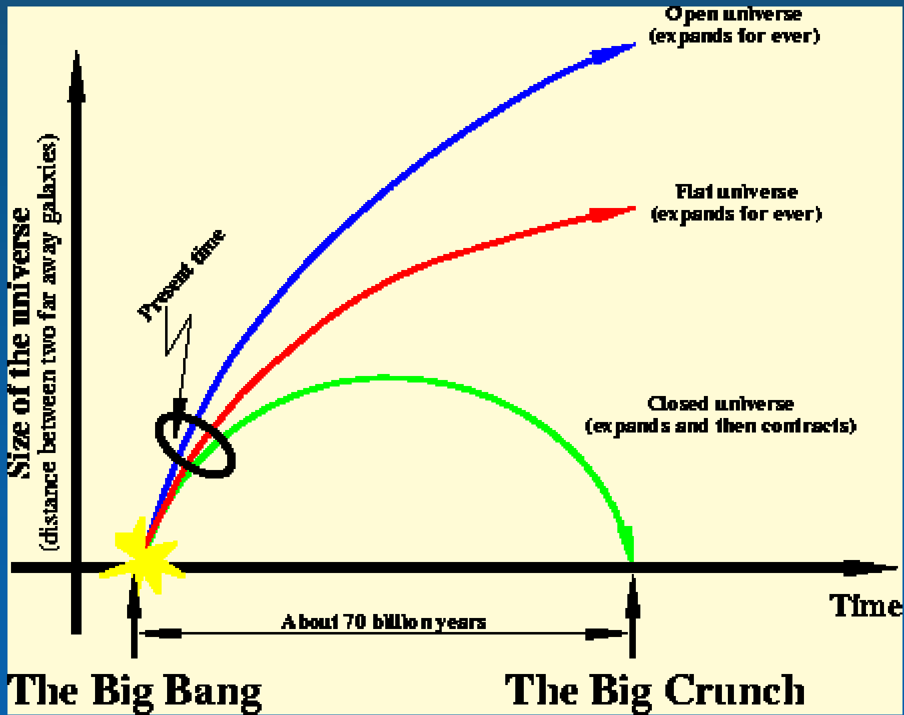
- Friedmann equation

$$v^2 = \frac{8\pi G}{3}\rho R^2 - kc^2$$

- k is the curvature constant
  - * $k=0$: flat space, forever expanding
  - * $k>0$: spherical geometry, eventually recollapsing
  - * $k<0$: hyperbolic geometry, forever expanding

k>0 k=0 k<0





Open universe
(expands for ever)
Flat universe
(expands for ever)
Closed universe
(expands and then contracts)
Present time
Size of the universe
(distance between two far away galaxies)
About 70 billion years
Time
The Big Bang
The Big Crunch

Can we predict the fate of the Universe ?

- Friedmann equation:

$$H_0^2 = \frac{v^2}{R^2} = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2}$$

- $k=0$:

$$\rho_{crit} = \frac{3H_0^2}{8\pi G}$$

Can we predict the fate of the Universe ?

- If the density ρ of the Universe
  - * $\rho = \rho_{crit}$: flat space, forever expanding
  - * $\rho > \rho_{crit}$: spherical geometry, recollapsing
  - * $\rho < \rho_{crit}$: hyperbolic geometry, forever expanding
- so what is the density of the universe?
  - * We don't know precisely
  - * $\rho > \rho_{crit}$ very unlikely
  - * currently favored model: $\rho \approx 0.3\rho_{crit}$

How big is ρ_{crit} ?

- $\rho_{crit} = 8\times10^{-30}$ g/cm^3 ≈ 1 atom per 200 liter
- density parameter Ω_0

$$\Omega_0 = \frac{\rho}{\rho_{crit}} = \frac{3H_0^2\rho}{8\pi G}$$

  * $\Omega_0 = 1$: flat space, forever expanding (open)
  * $\Omega_0 > 1$: spherical geometry, recollapsing (closed)
  * $\Omega_0 < 1$: hyperbolic geometry, forever expanding
- currently favored model: $\Omega_0 = 0.3$

How can we measure Ω_0 ?

- Count all the mass we can “see”
  - * tricky, some of the mass may be hidden …
- Measure the rate at which the expansion of the universe is slowing down
  - * a more massive universe will slow down faster
- Measure the geometry of the universe
  - * is it spherical, hyperbolic or flat ?

Let's try to measure the deceleration

- Acceleration according to Newton:

$$a = -G\frac{M}{R^2} = -\frac{4\pi G\rho}{3}R$$

- deceleration parameter

$$q_0 = -\frac{aR}{v^2} = \frac{\Omega_0}{2}$$

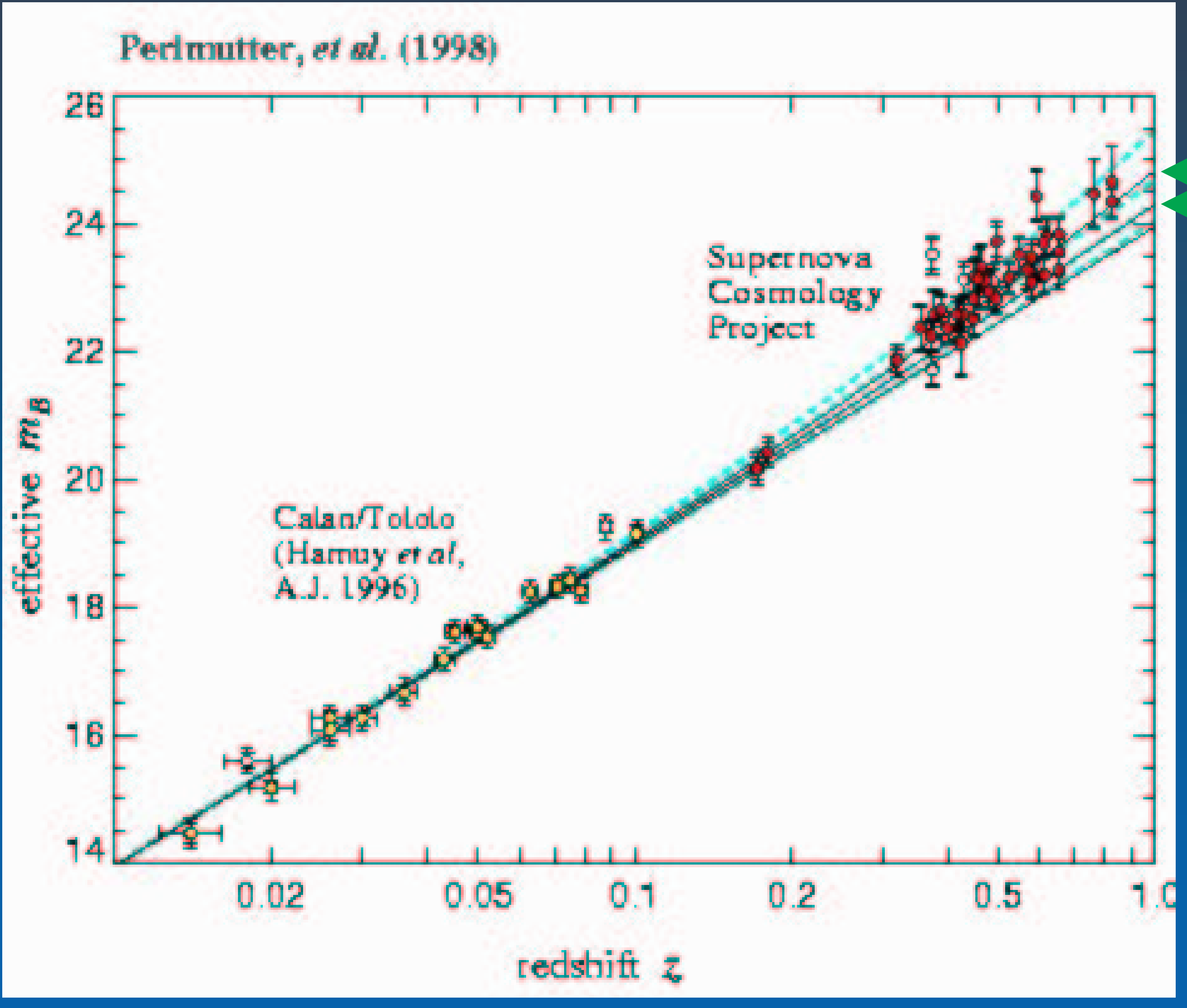
So what's the meaning of q_0 ?

- deceleration parameter q_0
  - * $q_0>0.5$: deceleration is so strong that eventually the universe stops expanding and starts collapsing
  - * $0<q_0<0.5$: deceleration is too weak to stop expansion
- What's the difference between q_0, Ω_0 and k ?
  - * k: curvature of the universe
  - * Ω_0: mass content of the universe
  - * q_0: kinematics of the universe

So let's measure q_0 !

- How do we do that?
  * Measure the rate of expansion at different times, i.e. measure and compare the expansion based on nearby galaxies and based on high redshift galaxies
- Gravity is slowing down expansion $\Rightarrow$ expansion rate should be higher at high redshift.

So let’s measure q_0 !



fainter

$q_0 = 0$

$q_0 = 0.5$

Data indicates:

$q_0 < 0$

$\Rightarrow$ Expansion is accelerating

more distant

Science discovery of the year 1998

- The expansion of the universe is accelerating !!!
- But gravity is always attractive, so it only can decelerate

$\Rightarrow$ Revival of the cosmological constant Λ

Friedmann's equation for $\Lambda>0$

$$v^2 = \frac{8\pi G}{3}\rho R^2 - kc^2 + \frac{\Lambda R^2}{3}$$

- k is the curvature constant
  * $k=0$: flat space
  * $k>0$: spherical geometry
  * $k<0$: hyperbolic geometry
- but for sufficiently large Λ a spherically curved universe may expand forever

Deceleration parameter *q* for Λ>0

- Acceleration according to Newton:

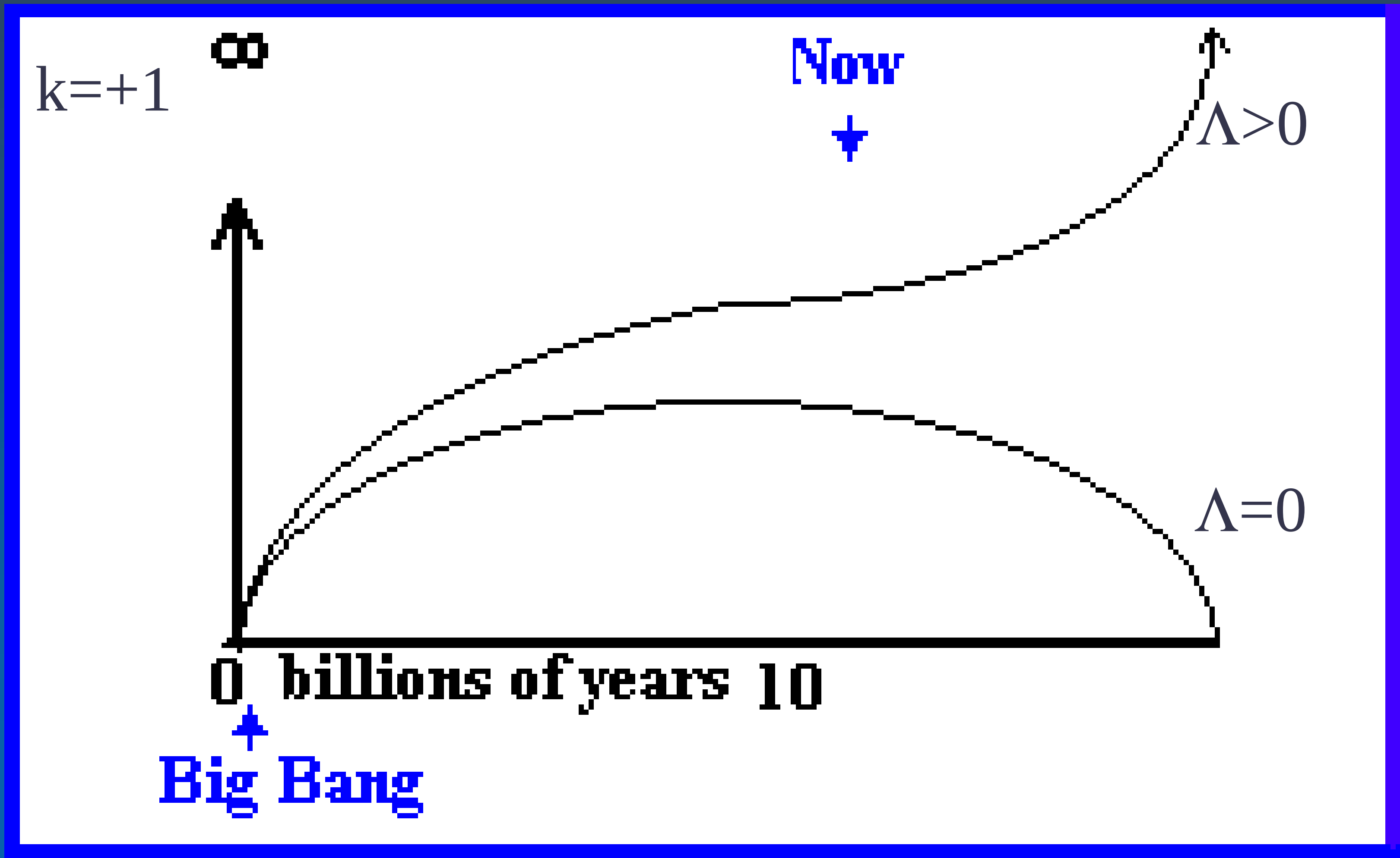
$$a = -\frac{4\pi G\rho}{3}R + \frac{\Lambda}{3}R$$

- deceleration parameter

$$q_0 = -\frac{aR}{v^2} = \frac{\Omega_0}{2} - \Omega_\Lambda$$

with $$\Omega_\Lambda = \frac{\Lambda}{3H_0^2}$$

The fate of the Universe for Λ>0



Is the fate of the Universe well determined ?

- deceleration:
  - * $\frac{1}{2}\Omega_0 - \Omega_\Lambda > 0$: decelerating
  - * $\frac{1}{2}\Omega_0 - \Omega_\Lambda < 0$: accelerating
- curvature
  - * $\Omega_0 + \Omega_\Lambda = 1$: flat
  - * $\Omega_0 + \Omega_\Lambda < 1$: hyperbolic
  - * $\Omega_0 + \Omega_\Lambda > 1$: spherical
- two equations for two variables $\Rightarrow$ well posed problem

Cosmology: the quest for three numbers

- The Hubble constant H_0
  - $\Rightarrow$ how fast is the universe expanding
- The density parameter Ω_0
  - $\Rightarrow$ how much mass is in the universe
- The cosmological constant Ω_Λ
  - $\Rightarrow$ the vacuum energy of the universe
- current observational situation:
  - $H_0 = 65$ km/s/Mpc
  - $\Omega_0 = 0.3$; $\Omega_\Lambda = 0.7 \Rightarrow$ flat space