Lecture 30
The big-bang model III: The life of a Universe

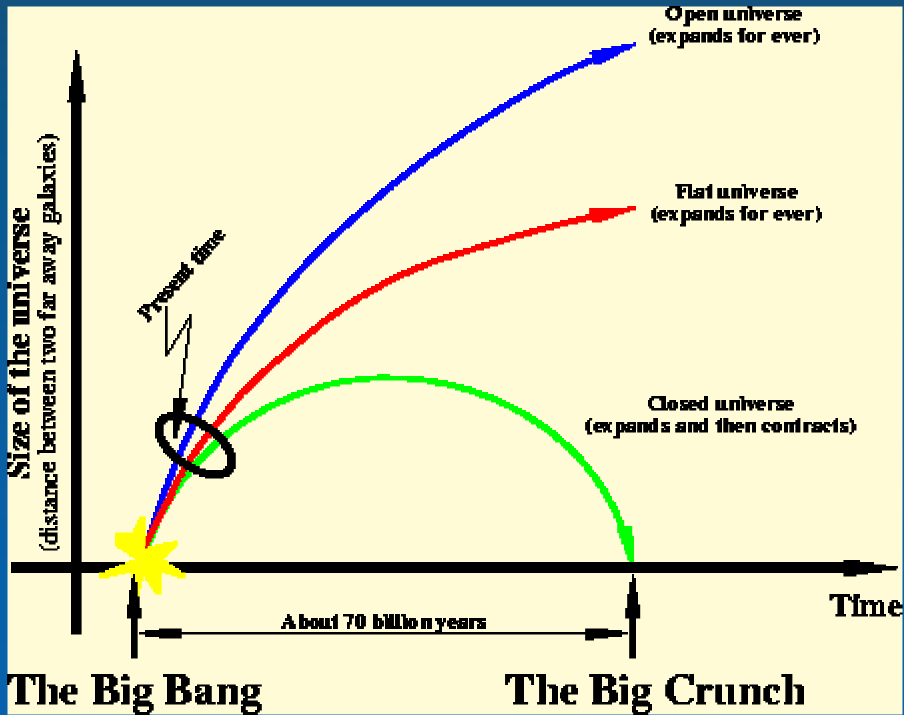
Expansion velocity of the Universe

- Friedmann equation

$$v^2 = \frac{8\pi G}{3}\rho R^2 - kc^2$$

- k is the curvature constant
  * $k=0$: flat space, forever expanding
  * $k>0$: spherical geometry, eventually recollapsing
  * $k<0$: hyperbolic geometry, forever expanding


k>0
k=0
k<0


Open universe
(expands for ever)
Flat universe
(expands for ever)
Closed universe
(expands and then contracts)
Present time
Size of the universe
(distance between two far away galaxies)
Time
About 70 billion years
The Big Bang
The Big Crunch

Density parameter Ω_0

- $\rho_{crit} = 8\times10^{-30}\ g/cm^3$ ≈ 1 atom per 200 liter
- density parameter Ω_0

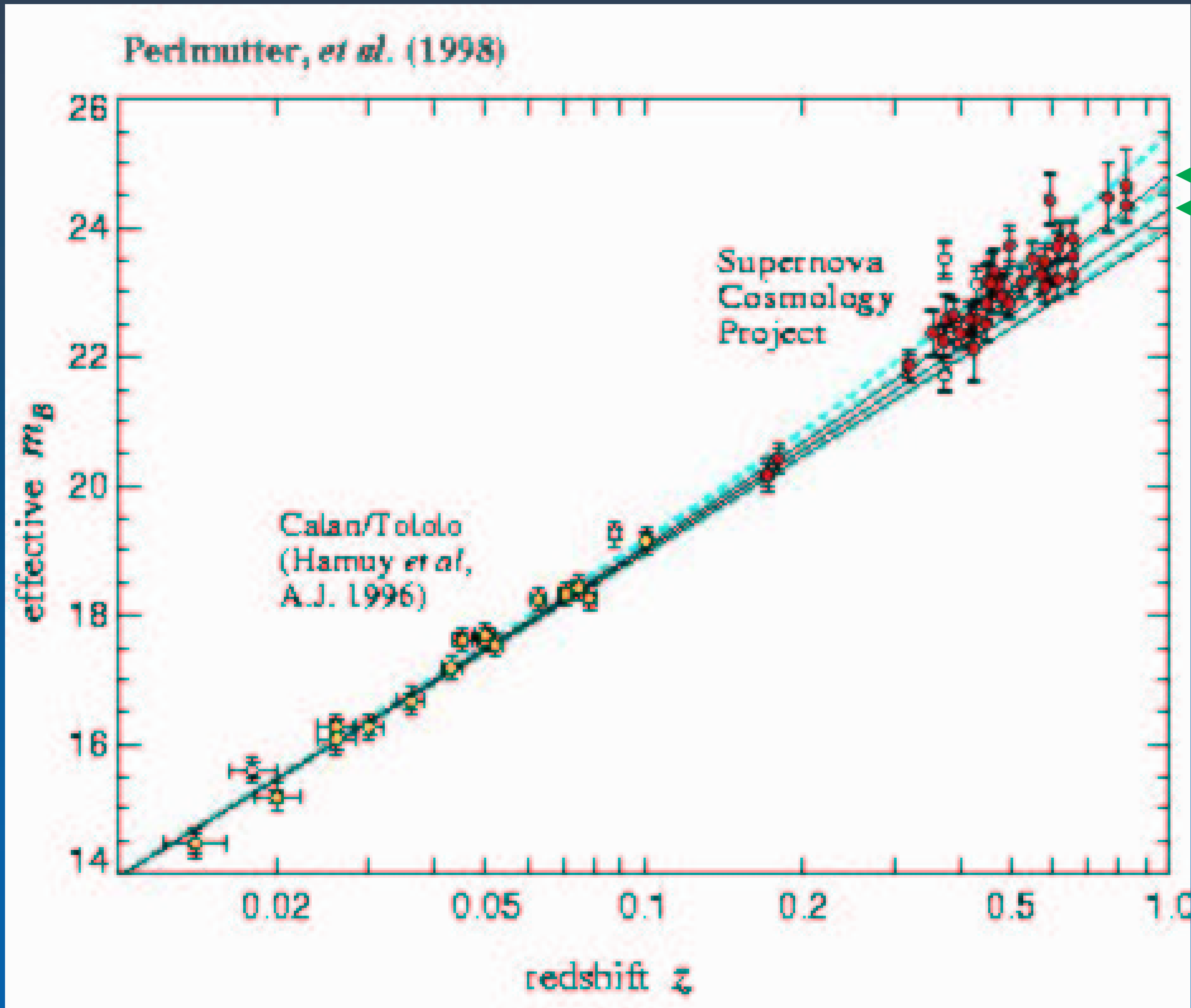
$$\Omega_0 = \frac{\rho}{\rho_{crit}} = \frac{3H_0^2\rho}{8\pi G}$$

  * $\Omega_0 = 1$: flat space, forever expanding (open)
  * $\Omega_0 > 1$: spherical geometry, recollapsing (closed)
  * $\Omega_0 < 1$: hyperbolic geometry, forever expanding
- currently favored model: $\Omega_0 = 0.3$

How can we measure Ω_0 ?

- Count all the mass we can “see”
  - * tricky, some of the mass may be hidden …
- Measure the rate at which the expansion of the universe is slowing down
  - * a more massive universe will slow down faster
- Measure the geometry of the universe
  - * is it spherical, hyperbolic or flat ?

Science discovery of the year 1998



fainter

$q_0 = 0$

$q_0 = 0.5$

Data indicates:

$q_0 < 0$

⇒ Expansion is accelerating

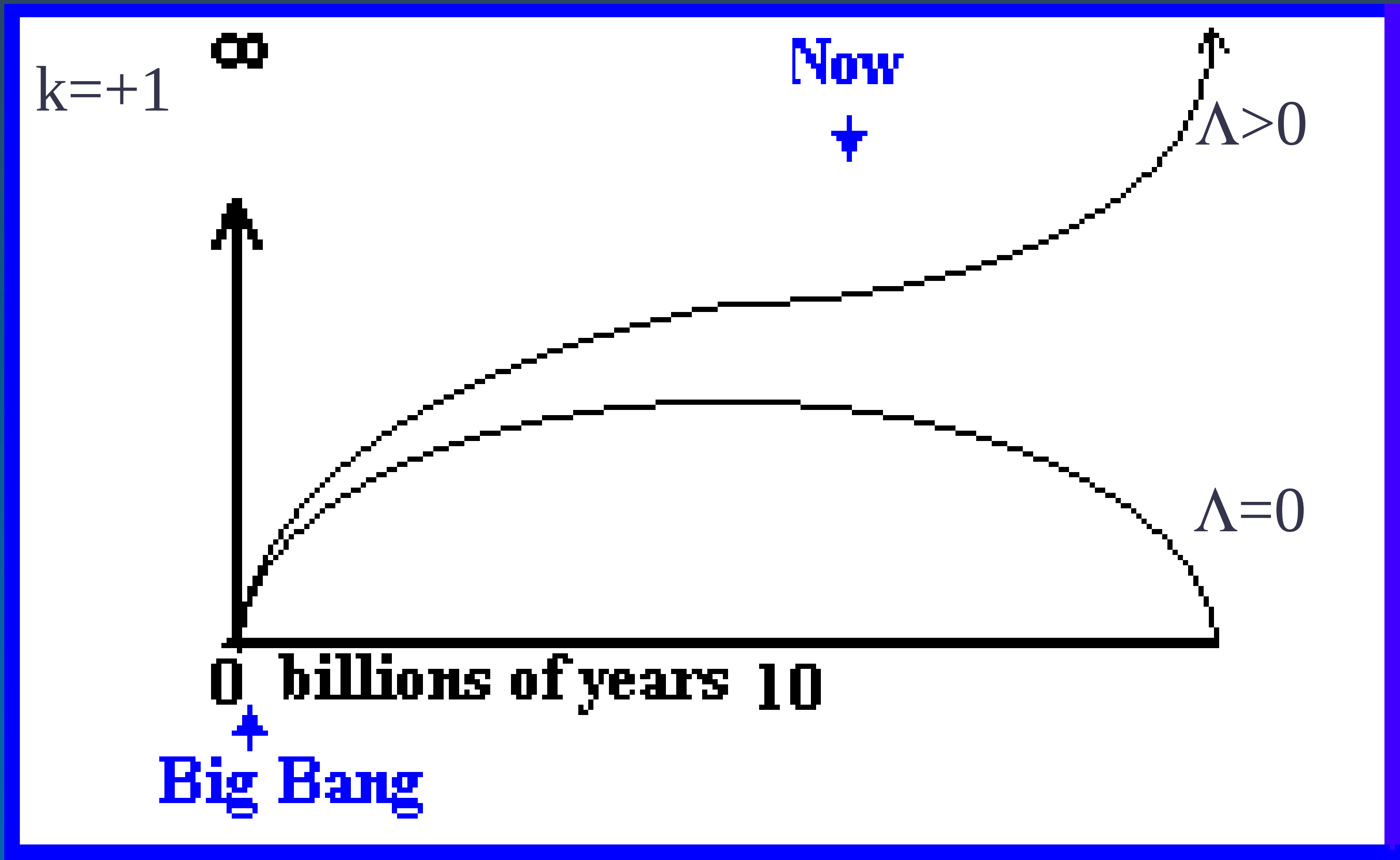
more distant

Friedmann's equation for Λ>0

$$v^2 = \frac{8\pi G}{3}\rho R^2 - kc^2 + \frac{\Lambda R^2}{3}$$

- k is the curvature constant
  - * $k=0$: flat space
  - * $k>0$: spherical geometry
  - * $k<0$: hyperbolic geometry
- but for sufficiently large Λ a spherically curved universe may expand forever

The fate of the Universe for Λ>0



Cosmology: the quest for three numbers

- The Hubble constant H_0
  - $\Rightarrow$ how fast is the universe expanding
- The density parameter Ω_0
  - $\Rightarrow$ how much mass is in the universe
- The cosmological constant Ω_Λ
  - $\Rightarrow$ the vacuum energy of the universe
- current observational situation:
  - $H_0 = 65$ km/s/Mpc
  - $\Omega_0 = 0.3$; $\Omega_\Lambda = 0.7 \Rightarrow$ flat space

How old is the Universe?

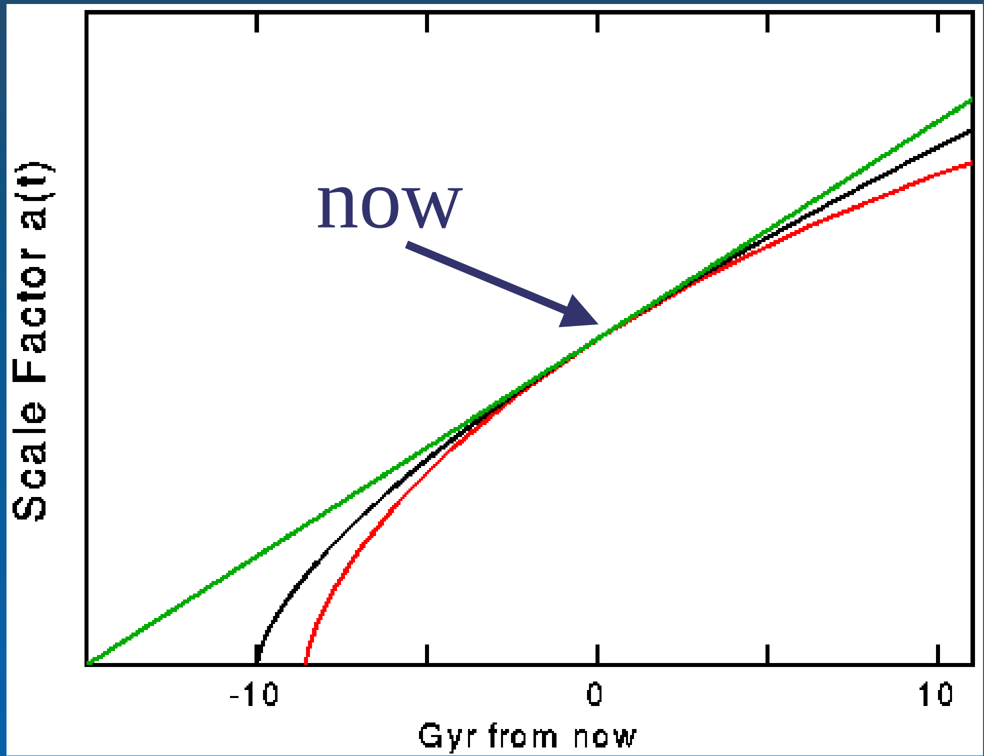
- A galaxy at distance *d* recedes at velocity $v = H_0 \times d$.
- When was the position of this galaxy identical to that of our galaxy? Answer:

$$t_{Hubble} = \frac{d}{v} = \frac{1}{H_0}$$

- t_{Hubble}: Hubble time. For $H_0 = 65$ km/s/Mpc: $t_{Hubble} = 15$ Gyr

The age of the Universe revisited

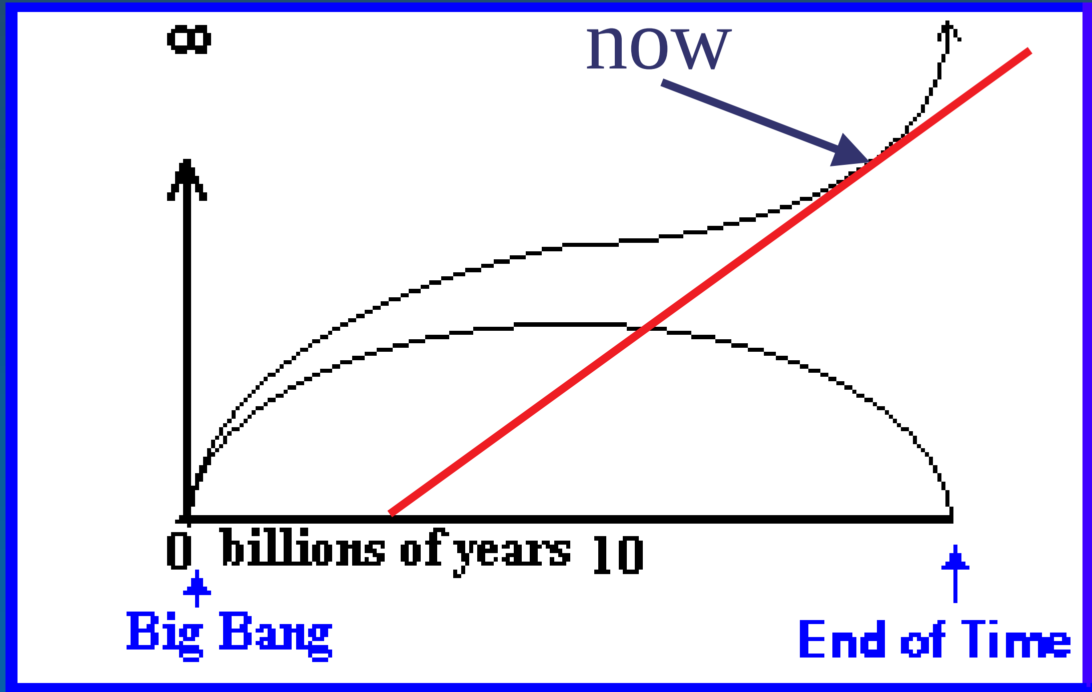
- So far, we have assumed that the expansion velocity is not changing ($q_0=0$, empty universe)
- How does this estimate change, if the expansion decelerates, i.e. $q_0>0$?



- An $\Omega_0>0$, $\Lambda=0$ universe is younger than 15 Gyr

The age of the Universe revisited

- So far, we only have considered decelerating universes
- How does this estimate change, if the expansion accelerates, i.e. $q_0 < 0$?



- An $\Lambda > 0$ universe can be older than 15 Gyr

The age of the Universe revisited

- $\Omega_0=0$, $\Lambda=0$: $t_{Hubble} = 1/H_0$ = 15 Gyr
- $\Omega_0=1$, $\Lambda=0$: $t_{Hubble} = 2/(3H_0)$= 10 Gyr
- open universes with $0<\Omega_0<1$, $\Lambda=0$ are between 10 and 15 Gyr old
- closed universes with $\Omega_0>1$, $\Lambda=0$ are less than 10 Gyr old
- $\Lambda>0$ increases, $\Lambda<0$ decreases the age of the universe
- $\Omega_0=0.3$, $\Lambda=0.7$: $t_{Hubble} = 0.96/H_0$ = 14.5 Gyr

Can we measure the age of the Universe ?

- not directly
- but we can constrain the age of the Universe. It must not be younger than the oldest star in the Universe.
- How do we measure the age of stars?
  * radioactive dating
  * stellar evolution models
- Result: age of the oldest star ~12-14 Gyr
- Ω_0>~1 strongly disfavored

The life of a universe – key facts

- Unless Λ is sufficiently large (which is inconsistent with observations) all cosmological models start with a big bang.
- An universe doesn't change its geometry. A flat universe has always been and will always be flat, a spherical universe is always spherical and so on.
- Two basic solutions:
  * eventual collapse for large Ω_0 or negative Λ
  * eternal expansion otherwise

Some common misconceptions

- The picture that the Universe expands into a preexisting space like an explosion
- The question "what was before the big bang?"
- Remember: spacetime is part of the solution to Einstein's equation
- Space and time are created in the big bang

So is the big crunch the same as the big bang run in reverse ?

- No. The Universe has meanwhile formed stars, black holes, galaxies etc.
- Second law of thermodynamics:
  The entropy (disorder) of a system at best stays the same but usually increases with time, in any process. There is no perpetual motion machine.
- Second law of thermodynamics defines an arrow of time.

Friedmann's equation for $\Lambda=0$, $\Omega_0<1$

$$H = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2}$$

Expansion rate of the Universe

Falls off like the cube of R

Falls off like the square of R

- At early epochs, the first term dominates

  $\Rightarrow$ the early universe appears to be almost flat

- At late epochs, the second term dominates

  $\Rightarrow$ the late universe appears to be almost empty

Friedmann's equation for $\Lambda>0$, $\Omega_0<1$

$$H = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2} + \frac{\Lambda}{3}$$

Expansion rate of the Universe

Falls off like the cube of R

Falls off like the square of R

constant

- At early epochs, the first term dominates

  ⇒ the early universe appears to be almost flat

- At late epochs, the third term dominates

  ⇒ the late universe appears to be exponentially expanding

A puzzling detail

- $\Lambda=0$: for most of its age, the universe looks either to be flat or to be empty
- $\Lambda>0$: for most of its age, the universe looks either to be flat or to be exponentially expanding
- Isn't it strange that we appear to live in that short period between those two extremes ?

$\Rightarrow$ Flatness problem