HOW MAGNETIC IS THE SOLAR TACHOCLINE?

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ABSTRACT

Models explaining the presence of sunspots by fluxtube rising from the bottom of the convection zone require toroidal field strengths of about 10^5 G to achieve emergence at low latitudes. We investigate the stability of toroidal fields in the tachocline being subject to a solar-like differential rotation and nonaxisymmetric perturbations. We find that toroidal magnetic fields above roughly 100 G are unstable and lead to a turbulent state which will quickly destruct the large-scale fields.

Key words: Sun: magnetic fields; instabilities.

1. INTRODUCTION

The stability of the tachocline is an issue with which a large series of studies has dealt in both the hydrodynamic and the magnetohydrodynamic context. The hydrodynamic stability of purely latitudinal shear was studied in [1], where instability was found for a pole-equator difference of more than 28%. The computations have been improved by [2] with a more realistic tachocline rotation for which the stability limit increases to 54% differential rotation.

Toroidal magnetic fields alone are also subject to an instability described in [3], even without differential rotation. Two-dimensional models with a combination of toroidal fields and latitudinal differential rotation required a magnetic diffusivity of 10^{10} cm^2/s in order to achieve stability of the fields on a time-scale of 2 yr [4]. We are now presenting a stability analysis of toroidal magnetic fields in the tachocline, combined with a solar-like rotation profile, that is depending also on radius, not only on latitude.

The problem of the cyclic magnetic-field generation in the sun is not yet solved satisfactorily. If the tachocline is the place where most of the magnetic flux is stored, which gives rise to solar activity, the field strength must be strong enough for the emergence of sunspots at the surface, and these only at low heliographic latitudes. Field strengths of at least 10^5 Gauss have been proposed to be necessary for the emergence of flux at the surface on latitudes of less than 40° [5, 6]. Somewhat lower field strengths were obtained in models of loops rising flux tubes [7, 8], but all necessary magnetic fields were beyond 10^4 Gauss.

Instead of using the flux-tube approximation, we are approaching the stability issue with non-ideal MHD and latitudinally and radially extended field belts. The influence of the thickness of the field belts and of the magnetic Prandtl number, which is the ratio of viscosity over magnetic diffusivity, can be studied.

1.1. The model

A spherical shell is considered which extends from an inner radius $r_\mathrm{i} = 0.5$ to an outer radius $r_\mathrm{o} = 0.7$. The latitudinal coverage goes from the northern to the southern pole, $\theta = 0$ to 2π. Solutions which are symmetric and antisymmetric with respect to the equator can equally be investigated.

We impose a background rotation profile of the form

$$\Omega = \begin{cases} \Omega_\mathrm{eq}\Big[1 - \alpha\cos^2\theta - \alpha\left(\frac{1}{4} - \cos^2\theta\right)\frac{r_\mathrm{o}-r}{r_\mathrm{o}-r_\mathrm{t}}\Big] \\ \text{if } r > r_\mathrm{t} \\ \Omega_\mathrm{eq}\big[1 - \frac{\alpha}{4}\big] \\ \text{if } r \le r_\mathrm{t} \end{cases} \tag{1}$$

where Ω_eq is the angular velocity at the equator, r is the radius, r_t is the radius of the bottom of the tachocline below which no differential rotation is prescribed, θ is the colatitude, and α is a parameter describing the strength of the differential rotation. At $\alpha = 1$, the pole is at rest. The background velocity is thus $\boldsymbol{U} = (0, 0, r\sin\theta\Omega)$.

The background magnetic field is purely toroidal and is distributed by

$$B_\phi = \sin^2\theta\cos\theta\sin^2\left[\pi\left(\frac{r-r_\mathrm{i}}{r_\mathrm{o}-r_\mathrm{i}}\right)^q\right], \tag{2}$$

where q controls the radial thickness of the toroidal field belt. We set $q = 2$ in most cases, giving a thickness of about 0.1 solar radii. Figure 1 illustrates the configuration. The maximum B_ϕ sits at a radius of $r = 0.641$ and a colatitude of $\theta = 55^\circ$ in the northern hemisphere. If $q = 4$, the maximum is at $r = 0.668$. The full radial

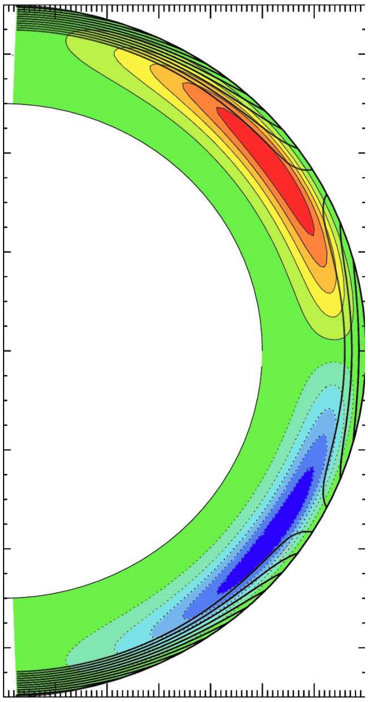

Figure 1. Vertical cross-section of the computational domain. The colour coding shows the strength of the toroidal magnetic field where red is positive and blue is negative. The other contours show the angular velocity which is highest at the equator and lowest at the pole.

widths at half-maximum of these profiles are $d = 0.073$, $d = 0.044$, and $d = 0.025$ for $q = 2$, 4, and 8, respectively. The entire background magnetic field is thus defined by $\boldsymbol{B} = (0, 0, B_\phi)$.

The equations are the linearized Navier-Stokes and induction equations in an incompressible medium with constant density,

$$\begin{aligned} \frac{\partial \boldsymbol{u}}{\partial t} &= \mathrm{Rm}\big[\boldsymbol{u}\times\nabla\times\boldsymbol{U} + \boldsymbol{U}\times\nabla\times\boldsymbol{u} - \nabla(\boldsymbol{u}\cdot\boldsymbol{U})\big] \\ &+ S\big[(\nabla\times\boldsymbol{b})\times\boldsymbol{B} + (\nabla\times\boldsymbol{B})\times\boldsymbol{b}\big] \\ &- \nabla p + \mathrm{Pm}\triangle\boldsymbol{u}, \qquad (3) \\ \frac{\partial \boldsymbol{b}}{\partial t} &= \nabla\times(\mathrm{Rm}\,\boldsymbol{U}\times\boldsymbol{b} + S\,\boldsymbol{u}\times\boldsymbol{B}) - \triangle\boldsymbol{b}\,, \qquad (4) \end{aligned}$$

where the velocity perturbation $\boldsymbol{u}$ and the magnetic field perturbation $\boldsymbol{b}$ are the quantities which are evolved in time. The system is integrated by the spherical spectral MHD code by Hollerbach [9]. The equations are normalized by the solar radius R and the magnetic diffusivity η. Times are thus in diffusion times $\tau = R^2/\eta$, velocities in units of η/R, and magnetic fields in units of $\sqrt{\mu\rho}\,\eta/R$ with μ being the permeability and ρ being the density which is constant here. The normalization leads to the dimensionless magnetic Prandtl number $\mathrm{Pm} = \nu/\eta$, where ν is the viscosity and η is the magnetic diffusivity. The

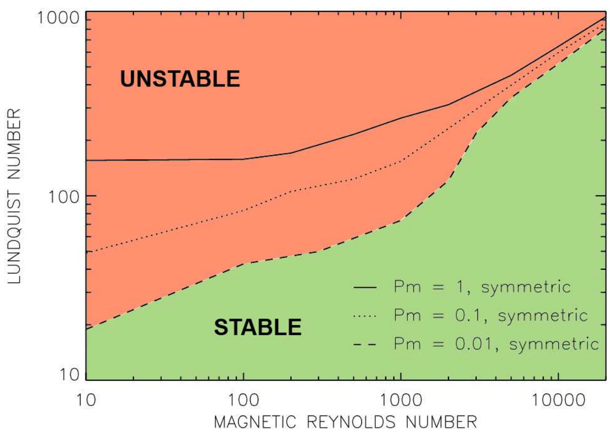

Figure 2. Critical Lundquist numbers for the instability of toroidal magnetic field belts in a rigidly rotating sphere as a function of magnetic Reynolds number. The instability is investigated for the $m = 1$ mode with a flow which is symmetric with respect to the equator. Three different magnetic Prandtl numbers are given, the lowest one is closest to the solar value of roughly 0.005.

other dimensionless numbers are the Lundquist number

$$S = \frac{RB_0}{\sqrt{\mu\rho}\,\eta}, \qquad (5)$$

where B_0 is the maximum toroidal magnetic field in cgs units, and the magnetic Reynolds number

$$\mathrm{Rm} = \frac{R^2\Omega_{\mathrm{eq}}}{\eta} \qquad (6)$$

as a free parameter which is essentially a variation of the diffusivity η since the solar radius R and Ω_{eq} are sufficiently well known.

The MHD code decomposes the azimuthal structure in Fourier modes m. In this Paper, we show the results for the $m = 1$ mode, that is the lowest nonaxisymmetric mode. Work on higher modes is in progress.

2. RESULTS

2.1. Zero differential rotation

First computations employed a rigidly rotating spherical shell in order to study the Tayler instability alone. As a result of these, flows symmetric with respect to the equator were found to be the more easily excited modes. We have therefore limited our studies to the symmetric modes when adding differential rotation. With the given background fields, $\boldsymbol{U}$ and $\boldsymbol{B}$, the nonaxisymmetric magnetic field contribution is antisymmetric with respect to the equator.

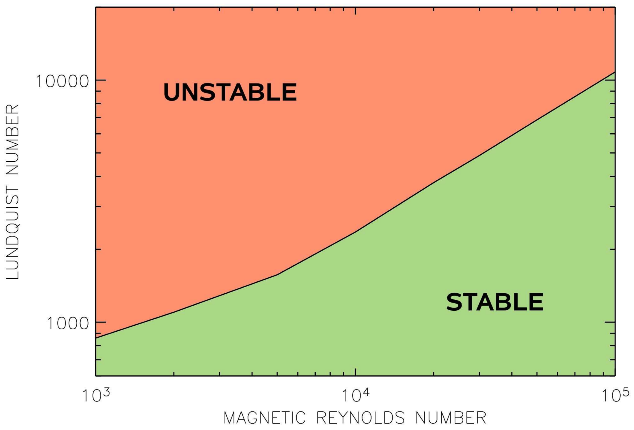

Figure 3. Critical Lundquist numbers for the instability of toroidal magnetic field belts as shown in Figure 1 as a function of magnetic Reynolds number. The differential rotation amplitude is $\alpha = 0.2$, the magnetic Prandtl number is $\mathrm{Pm} = 1$. The instability line is that of the $m = 1$ mode with a flow symmetric with respect to the equator.

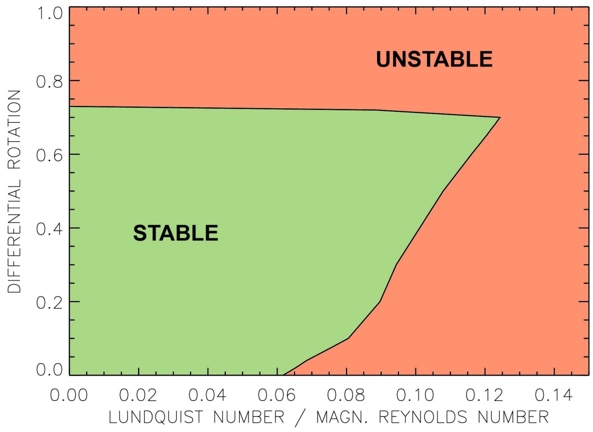

Figure 4. Stability diagram for the combination of differential rotation in terms of α and strength of the toroidal magnetic field in terms of Lundquist number over magnetic Reynolds number which is $\mathrm{Rm} = 10\,000$. The magnetic Prandtl number is again $\mathrm{Pm} = 1$.

Another issue is the influence of the thickness of the toroidal field belts in radius. The radial extent is controlled by the parameter q in (2). In a rigidly rotating sphere, the stability limits did not change significantly for $q = 4$, 6, and 8. The following computations are all done with $q = 2$.

The dependence of the critical Lundquist number on the magnetic Reynolds number is shown in Figure 2 for three different magnetic Prandtl numbers, $\mathrm{Pm} = 1$, 0.1, and 0.01. Statements about the stability of the solar tachocline require computations at very high Reynolds numbers. Since values of the radiative zone are not accessible computationally, we have to rely our results on an extrapolation of a parameter study. Magnetic Reynolds numbers of 20 000 were achieved for Figure 2.

2.2. Differential rotation

When adding a tachocline-like differential rotation to the configuration, we obtained a stability line for $\mathrm{Pm} = 1$ as shown in Figure 3. With high-resolution runs, very high magnetic Reynolds numbers of 10^5 were achieved, and the relation between the critical Lundquist number and the magnetic Reynolds number appears to be a power law. The exponent of this power law is 0.66, as derived from the critical Lundquist numbers for $\mathrm{Rm} \geq 5000$. An extrapolation to a solar magnetic Reynolds number of about 10^{12} leads to a maximum toroidal magnetic field strength of about 100 Gauss. The magnetic Reynolds number is a for a non-turbulent tachocline with a diffusivity of $\eta = 3000$ cm^2/s. The density was assumed to be $\rho = 0.25$ g/cm^3.

The interaction between the hydrodynamic instability at strong enough shear and the magnetic instability at strong enough magnetic fields is shown in Figure 4. The magnetic Reynolds number for this graph was $\mathrm{Rm} = 10\,000$. Since we compute the stability for a rotation profile depending on both r and θ, the hydrodynamic stability is increased as compared to Watson's work who found 28% for purely latitudinal shear [1]. For very low but finite solar viscosity, the limit for the differential rotation was 53% according to [2]. In our case here, the magnetic Reynolds number is not high enough, so that the limit is still at about 73%. This is not a severe problem here as the solar tachocline is hydrodynamically stable for any of these values, according to this type of stability analysis.

If a weak toroidal magnetic field is added to the differentially rotating sphere, the instability line does not change drastically. A weak field does not alter the hydrodynamic stability limit much. On the horizontal axis of the diagram, the Tayler instability can be seen at a ratio of Lundquist number to magnetic Reynolds number of 0.062 or at $S = 620$. If from there differential rotation is added, the change in the stability limit is also not drastically changed. Note that the magnetic axis is not logarithmic here. In fact, the differential rotation improves stability of nonaxisymmetric modes such as the $m = 1$ mode studied here. The maximum stable toroidal field has $S = 1250$.

The position of the maximum toroidal magnetic field can also be changed in latitude. This shows how different signs of the radial differential rotation (negative in high latitudes, positive in low latitudes) changes the stability of the toroidal field. Figure 5 shows the stability lines for locations at $19°$, $35°$, $45°$, and $60°$ latitude. High-latitude fields are more unstable than low-latitude fields. If Ω decreases with radius, a toroidal field thus becomes unstable at lower field strengths than in the case of an Ω increasing with radius.

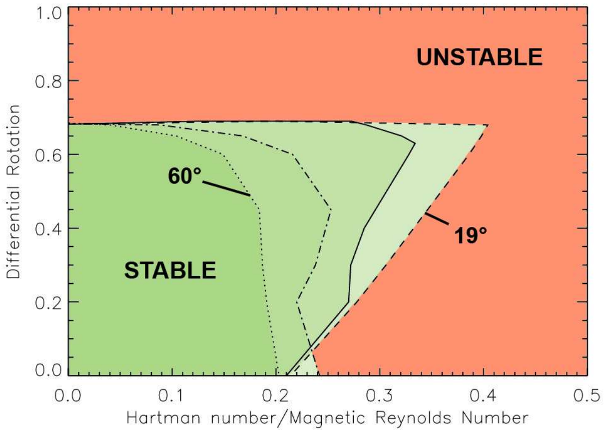

Figure 5. Stability diagram for different latitudinal positions of the maximum toroidal field. The positions of the maximum B_ϕ for the four lines are $60°$, $45°$, $35°$, and $19°$ (from left to right). The magnetic Reynolds number is again $\mathrm{Rm} = 10\,000$.

This looks compatible with the low-latitude emergence of sunspots. Magnetic fields are amplified in both low and high latitudes, but only the low-latitude fields reach possibly large enough values to provide surface-reaching flux. However, the difference between low and high latitudes is only a factor of about 2. The mere 100 Gauss stability limit could be 200 Gauss below $20°$ latitude, but this is still far away from the values obtained by flux tube models, explaining the emergence of sunspot from tachocline fields.

First fully 3D, nonlinear computations with 30 azimuthal modes show that the large-scale toroidal field breaks up into small-scale fields, and there is no adjustment to a "re-configured" large-scale field surviving the instability.

3. CONCLUSION

The stability of toroidal magnetic fields in a differentially rotating spherical shell was investigated. The configuration is meant as a representation of the differential rotation in the solar tachocline combined with strong toroidal fields as required for low-latitude sunspot emergence in various models. Our computations show that toroidal magnetic field belts are stable only up to field strengths of roughly 100 Gauss. Upon amplifying the fields by differential rotation or a dynamo process, they will become unstable as soon as they reach about 100 Gauss.

The conclusion will have severe implications for the storage of strong magnetic fields in the tachocline. Improved models, especially employing combinations of toroidal with poloidal magnetic fields, are therefore planned to get to a final answer on the maximum possible field in the tachocline.

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