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ASTRONOMY AND ASTROPHYSICS

Viscosity-alpha and dynamo-alpha for magnetic-driven turbulence in density-stratified Keplerian disks

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Abstract. For a given isotropic and homogeneous field of *magnetic* fluctuations both the viscosity-α as well as the dynamo-α have been computed for accretion disks on the basis of a quasilinear approximation with shear flow and magnetic buoyancy included. The resulting viscosity-α proves to be positive for sufficiently strong shear (i.e. the angular momentum transport is *outwards*) while the sign of the dynamo-α depends on the hemisphere. Again, for sufficiently strong shear it changes its sign, it is now *negative* for the upper disk plane and positive for the lower one.

Also the current helicity $\langle \boldsymbol{j}' \cdot \boldsymbol{B}' \rangle$ changes its sign for strong shear. For a Kepler shear flow in the upper disk plane it is positive while it is negative in the lower disk plane. Our current helicity of the fluctuations and the α-effect are almost always out of phase, the signs of all the quantities are in perfect correspondence to numerical simulations of Brandenburg (1999). The kinetic helicity has the *same sign* as the α-effect – not, as often assumed, the opposite one.

The relation between the dynamo-α and the viscosity-α reveals the dynamo-α amplitude as rather small compared with the turbulence intensity. This is in contrast to earlier SOCA-results but again in confirmation with numerical simulations.

Key words: *magnetohydrodynamics* (MHD) – Turbulence

1. Introduction

There is now evidence that the accretion disk dynamo works with an α-effect with negative sign above the disk plane and positive below. This is insofar of high importance as in $\alpha\Omega$-dynamos the sign of the α-effect directs the resulting geometry.

The most easily excited mode is quadrupolar for $C_\alpha < 0$ (Torkelsson & Brandenburg 1994). Rüdiger et al. (1995) working only with positive α-effect (in the upper disk plane) found the quadrupolar symmetry dominating.

In Rekowski et al. (2000) the different geometries of the dynamo-generated magnetic fields are demonstrated. For *negative* α, however, a stationary dipolar structure of the magnetic field results. The additional magnetic torque at the disk surface changes the profile of the effective temperature significantly to a profile which is more flat. The magnetic torque becomes of the same order as the radial viscous torque. The inclination angle of the poloidal field exceeds 30° even for a magnetic Prandtl number of order unity, and also the criterion for poloidal collimation after Spruit et al. (1997) is fulfilled. The dynamo-generated magnetic field configuration thus supports

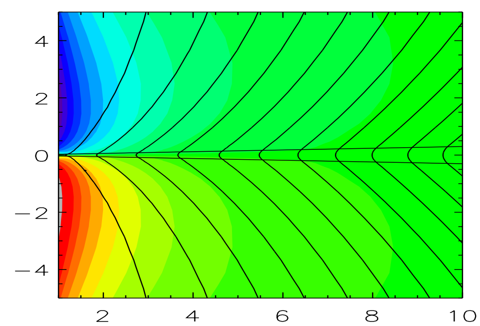

Fig. 1. The magnetic geometry for accretion-disk dynamos with negative α-effect (in the upper disk plane) is dipolar. The figure is taken from Rekowski et al. (2000)

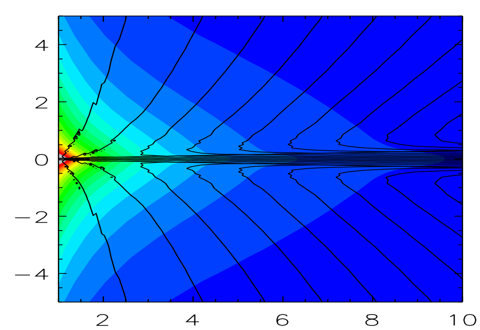

Fig. 2. The different magnetic geometry for accretion-disk dynamos with positive α-effect (in the upper disk plane) is quadrupolar. The figure is taken from Rekowski et al. (2000)

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the magnetic wind launching concept for accretion disks not only for unrealistic high turbulent magnetic Prandtl numbers.

On the other hand, an accretion disk can only exist if there is an instability which transports the angular momentum outwards, or, with other words, the 'viscosity-alpha' is positive. This is not a trivial constraint as we know from several hydrodynamical simulations (Ryu & Goodman 1992; Cabot & Pollack 1992; Kley et al. 1993; Goldman & Wandel 1995; Stone & Balbus 1996, see also Balbus et al. 1996)). The situation drastically changes for electrically conducting media, however, if (weak) magnetic fields are allowed to play there own role and, in particular, to feedback to the momentum transport via the Lorentz force (Balbus & Hawley 1991; Hawley et al. 1996, Brandenburg et al. 1995). On the other hand, Brandenburg (1998) proposes for magnetic shear flows an interesting argument that for positive viscosity-α the dynamo-α must be negative in the upper disk plane (and must be positive in the lower disk plane).

In the present paper for a magnetic-driven turbulence field subject to a large-scale shear flow the angular momentum transport, the dynamo-α-effect and the two helicities are simultaneously derived. Drastic differences of the results for rigid rotation and Kepler rotation are found.

2. Mean-field electrodynamics

The equations are close to those in Rüdiger et al. (2000). The momentum equation for non-rigid rotation in the inertial system with buoyancy included is

$$\frac{\partial \boldsymbol{u}'}{\partial t} + \boldsymbol{u}' \cdot \nabla \bar{\boldsymbol{u}} + \bar{\boldsymbol{u}} \cdot \nabla \boldsymbol{u}' = -\frac{1}{\bar{\rho}} \,\mathrm{grad}\left(p' + \frac{\boldsymbol{B}' \cdot \bar{\boldsymbol{B}}}{\mu_0}\right) + \\ + \frac{\rho'}{\bar{\rho}}\boldsymbol{g} + \frac{1}{\mu_0 \bar{\rho}}(\bar{\boldsymbol{B}} \cdot \nabla)\boldsymbol{B}' + \nu \Delta \boldsymbol{u}'. \quad (1)$$

Overbars indicate prescribed mean quantities, such as magnetic field, flow and density. $\boldsymbol{g}$ denotes the acceleration due to gravity. Mass conservation in Boussinesq approximation requires

$$\frac{\partial \rho'}{\partial t} + \bar{\rho}\,\mathrm{div}\,\boldsymbol{u}' = 0. \quad (2)$$

Note that we do *not* apply the anelastic approximation. As the energy equation for the turbulence we use the adiabacy relation

$$p' = c_{\mathrm{ac}}^2 \rho' \quad (3)$$

with c_{ac} as the isothermal speed of sound. Equations (1)...(3) lead to a turbulence field $\boldsymbol{u}'$, driven by the Lorentz force on the RHS of (1). The original prescribed magnetic field fluctuations may here be denoted by $\boldsymbol{B}^{(0)}$. Their correlation tensor is assumed as forming a homogeneous, isotropic and stationary field of magnetic turbulence. The resulting kinetic turbulence is subject to a basic rotation and subject to a shear, i.e. subject to a differential rotation.

One can find after some massive algebra the correlation tensor of the turbulence and, in particular, its covariance $\langle u_s' u_\phi' \rangle$, s here being the distance from the rotation axis. This quantity is part of the angular momentum transport. The total angular momentum transport is given by

$$T_{s\phi} = \langle u_s' u_\phi' \rangle - \frac{1}{\mu_0 \bar{\rho}} \langle B_s' B_\phi' \rangle, \quad (4)$$

taking into account also the magnetic effect. To this end one has to compute the magnetic fluctuations, $\boldsymbol{B}'$, driven by the turbulence field considered. The corresponding equation is the induction equation in its linearised version, i.e.

$$\frac{\partial \boldsymbol{B}'}{\partial t} - \mathrm{rot}(\bar{\boldsymbol{u}} \times \boldsymbol{B}') - \eta \Delta \boldsymbol{B}' = \mathrm{rot}(\boldsymbol{u}' \times \bar{\boldsymbol{B}}). \quad (5)$$

Here again both the influences of the global rotation (only on nonaxisymmetric field components) as well as differential rotation can be isolated.

The resulting (rather complex) magnetic fluctuations must be used to compute the Maxwell stress in (4), or, as the next interesting quantity, to compute the current helicity

$$\mathcal{H}_{\mathrm{curr}} = \langle \boldsymbol{j}' \cdot \boldsymbol{B}' \rangle = \frac{1}{\mu_0} \langle \mathrm{rot}\,\boldsymbol{B}' \cdot \boldsymbol{B}' \rangle, \quad (6)$$

which has the same kind of equatorial (anti-)symmetry like the dynamo-α. For *homogeneous* global magnetic fields the dynamo-α is related to the turbulent electromotive force (EMF) in accord to

$$\boldsymbol{\mathcal{E}} = \langle \boldsymbol{u}' \times \boldsymbol{B}' \rangle = \alpha \circ \bar{\boldsymbol{B}}, \quad (7)$$

so that

$$\alpha_{ij} \bar{B}_i \bar{B}_j = \boldsymbol{\mathcal{E}} \cdot \bar{\boldsymbol{B}}. \quad (8)$$

Rädler & Seehafer (1990) propose to read this equation in $\alpha\Omega$-dynamos as

$$\alpha_{\phi\phi} = \boldsymbol{\mathcal{E}} \cdot \bar{\boldsymbol{B}} / \bar{B}_\phi^2, \quad (9)$$

where $\alpha_{\phi\phi}$ is the dominant component of the α-tensor. We are, in particular, interested to check their and Keinigs' (1983) antiphase relation,

$$\alpha_{\phi\phi} \mathcal{H}_{\mathrm{curr}} < 0, \quad (10)$$

between α-effect and current helicity. There is an increasing number of papers presenting observations of the current helicity of the solar surface always with the result that it is *negative* at the northern hemisphere and positive at the southern hemisphere (Seehafer 1990; Pevtsov et al. 1995; Abramenko et al. 1996; Bao & Zhang 1998). If (10) is correct then there is a strong empirical indication for an α-effect which is *positive* in the northern hemisphere of the Sun. We start here to find the relation between α-effect and current helicity for shear-flow disks. We shall see that there are exceptions, indeed, from the simple relation (10). This is not a surprise. Blackman & Field (1999) argue that Keinigs' result,

$$\frac{\alpha_{\phi\phi} \bar{B}^2}{\mu_0 \mathcal{H}_{\mathrm{curr}}} = -\eta, \quad (11)$$

strongly depends on the assumed stationarity and homogeneity of the magnetic fields and flows which assumptions, however, are not realistic for dynamo problems.

3. The current helicity and the α-effect

The complete relation for the current helicity is

$$\mathcal{H}_{\rm curr} = \epsilon_{inj} \left\{ g_j \bar{B}^2 \bar{u}_{n,i} + 2 g_j \bar{B}_n \bar{B}_p \bar{u}_{p,i} - \right. \\ \left. - g_n \bar{B}_j \bar{B}_p \bar{u}_{i,p} - (\boldsymbol{g}\cdot\bar{\boldsymbol{B}}) \bar{B}_j \bar{u}_{i,p} \right\} \frac{I_1}{\mu_0^2 \bar{\rho} c_{\rm ac}^2} \quad (12)$$

with

$$I_1 = \frac{2}{15} \iint_0^\infty \frac{\eta k^2(\nu^2 k^4 + \omega^2) - 2\omega^2 \nu k^2}{(\omega^2 + \nu^2 k^4)^2 (\omega^2 + \eta^2 k^4)} k^2 \mathcal{B}(k,\omega)\, dk d\omega \quad (13)$$

and the spectral function $\mathcal{B}$ in the definition $\langle B^{(0)2}\rangle = \iint \mathcal{B} dk d\omega$.

In disk geometry the deformation tensor is simply

$$\bar{u}_{i,j} = -\epsilon_{ijp}\Omega_p + e_i^\phi e_j^s \frac{\partial \Omega}{\partial \log s}, \quad (14)$$

with $\boldsymbol{e}^s$ and $\boldsymbol{e}^\phi$ as the unit vectors in radial and azimuthal directions. Hence,

$$\mathcal{H}_{\rm curr} = 3\bar{B}^2 (\boldsymbol{g}\cdot\boldsymbol{\Omega}) \left(1 + \frac{\partial \log \Omega}{\partial \log s}\right) \frac{I_1}{\mu_0^2 \bar{\rho} c_{\rm ac}^2}. \quad (15)$$

The sign of the I_1 determines the sign of the current helicity which we have to discuss for reliable turbulence models. The integral (13) does not prove as definite in sign. It is negative-definite for very large magnetic Prandtl numbers ($\eta = 0$) but it is positive for the more realistic case of moderate magnetic Prandtl number and spectral functions $\mathcal{B}$ decreasing for increasing frequency ω. In the sense of the 'τ-approximation' the spectrum of the given field of magnetic fluctuations has been approximated by $\mathcal{B} \propto \delta(k - \ell_{\rm corr}^{-1})\, \delta(\omega)$ with $\nu \simeq \ell_{\rm corr}^2/\tau_{\rm corr}$ (Kitchatinov 1991) and the I_1 becomes positive-definite. For the current helicity (6) of the shear flow we then find

$$\mathcal{H}_{\rm curr} = \frac{2}{5} \frac{\tau_{\rm corr}^3}{\ell_{\rm corr}^2} V_{\rm A}^2 \frac{\langle B^{(0)2}\rangle}{\mu_0 \bar{\rho} c_{\rm ac}^2} (\boldsymbol{g}\cdot\boldsymbol{\Omega}) \left(1 + \frac{\partial \log \Omega}{\partial \log s}\right). \quad (16)$$

$V_{\rm A}$, the Alfvén velocity is $V_{\rm A} = \bar{B}/\sqrt{\mu_0 \bar{\rho}}$. The current helicity is negative on the northern hemisphere for weak differential rotation but changes its sign for sufficiently large shear. For a Kepler disk with its vertical gravity, $g_z = -\Omega^2 z$, the current helicity becomes

$$\mathcal{H}_{\rm curr} = \frac{z\Omega^3}{5} \frac{\tau_{\rm corr}^3}{\ell_{\rm corr}^2} \frac{\bar{B}^2}{\mu_0} \frac{\langle B^{(0)2}\rangle}{\mu_0 \bar{\rho} c_{\rm ac}^2}, \quad (17)$$

which is *positive* in the upper disk plane and negative in the lower disk plane. This is exactly the numerical result of Brandenburg (1999) for the current helicity of the magnetic field fluctuations.

The next step is the evaluation of the α-effect. It results from the general expression

$$\alpha_{fm} = \frac{\epsilon_{fij}}{3} \left(2 g_m \bar{u}_{i,j} - 3 g_i \bar{u}_{m,j} + 2 g_l \delta_{mj} \bar{u}_{i,j}\right) \frac{I_2}{\mu_0 \bar{\rho} c_{\rm ac}^2} - \\ - \epsilon_{fij} \left(g_i (\bar{u}_{m,j} + \bar{u}_{j,m}) + \delta_{mj} g_l (\bar{u}_{i,l} + \bar{u}_{l,i}) \right) \frac{I_3}{\mu_0 \bar{\rho} c_{\rm ac}^2} \quad (18)$$

with

$$I_2 = \frac{1}{5} \iint_0^\infty \frac{\nu^2 k^4 - \omega^2}{(\omega^2 + \nu^2 k^4)^2} \mathcal{B}(k,\omega) dk d\omega, \quad (19)$$

$$I_3 = \frac{1}{15} \iint_0^\infty \frac{\nu^2 k^4 (\nu^2 k^4 - 3\omega^2)}{(\omega^2 + \nu^2 k^4)^3} \mathcal{B}(k,\omega) dk d\omega. \quad (20)$$

Only the most important component $\alpha_{\phi\phi}$ will be discussed. We obtain

$$\alpha_{\phi\phi} = -(\boldsymbol{g}\cdot\boldsymbol{\Omega}) \left(\left(1 + \frac{\partial \log \Omega}{\partial \log s}\right) \frac{I_2}{\mu_0 \bar{\rho} c_{ac}^2} - \frac{\partial \log \Omega}{\partial \log s} \frac{I_3}{\mu_0 \bar{\rho} c_{ac}^2} \right). \quad (21)$$

Our magnetic flux tube model yields

$$\alpha_{\phi\phi} = -\frac{4}{15} \frac{\tau_{\rm corr}^2}{c_{\rm ac}^2} (\boldsymbol{g}\cdot\boldsymbol{\Omega}) \left(\frac{3}{4} + \frac{\partial \log \Omega}{\partial \log s}\right) \frac{\langle B^{(0)2}\rangle}{\mu_0 \bar{\rho}}, \quad (22)$$

so that for Kepler disks

$$\alpha_{\phi\phi} = -\frac{z\Omega^3}{30} \frac{\tau_{\rm corr}^2}{c_{\rm ac}^2} \frac{\langle B^{(0)2}\rangle}{\mu_0 \bar{\rho}}. \quad (23)$$

results. For rigid rotation the α-effect proves to be positive in the northern hemisphere and negative in the southern hemisphere (Rüdiger et al. 2000). The opposite is true for Kepler flows. The dynamo-α becomes negative in the upper disk plane and positive in the lower disk plane. Again the results comply with the results of the numerical simulations by Brandenburg (1999). The ratio (11) of α-effect and current helicity here follows to

$$\frac{\alpha_{\phi\phi} \bar{B}^2}{\mu_0 \mathcal{H}_{\rm curr}} = -\frac{1}{6} \frac{\ell_{\rm corr}^2}{\tau_{\rm corr}}. \quad (24)$$

For rigid rotation the factor 1/6 grows to 1/2. The differences to Keinigs' result (his factor is unity) are certainly resulting from the fact that we are not using the anelastic approximation.

A similar question arises concerning the kinetic helicity

$$\mathcal{H}_{\rm kin} = \langle \boldsymbol{u}' \cdot {\rm rot}\, \boldsymbol{u}' \rangle, \quad (25)$$

which is often believed to be in antiphase to the α-effect (Moffatt 1978; Krause & Rädler 1980). Here we obtain

$$\mathcal{H}_{\rm kin} = -\frac{4}{15} \epsilon_{inj} g_j \bar{B}_n \bar{B}_p (\bar{u}_{p,i} - \bar{u}_{i,p}) \frac{I_4}{\mu_0^2 \bar{\rho}^2 c_{\rm ac}^2} + \\ + \frac{4}{15} \epsilon_{pij} g_i \bar{B}_n \bar{B}_j \,(\bar{u}_{p,n} + \bar{u}_{n,p}) \frac{I_7}{\mu_0^2 \bar{\rho}^2 c_{\rm ac}^2} \quad (26)$$

resulting in

$$\mathcal{H}_{\rm kin} = -\frac{4}{15} \frac{\bar{B}^2}{\mu_0^2 \bar{\rho}^2 c_{\rm ac}^2} (\boldsymbol{g}\cdot\boldsymbol{\Omega}) \left(2 + \frac{\partial \log \Omega}{\partial \log s}\right) I_4 + \\ + \frac{\partial \log \Omega}{\partial \log s} I_7 \quad (27)$$

with the positive quantity

$$I_4 = \iint_0^\infty \frac{\nu k^4 \mathcal{B}(k,\omega)}{(\omega^2 + \nu^2 k^4)^2} dk d\omega, \quad (28)$$

and with

$$I_7 = \iint \frac{\nu k^4(\nu^2 k^4 - \omega^2)}{(\omega^2 + \nu^2 k^4)^3} \mathcal{B}(k,\omega) dk d\omega. \quad (29)$$

Hence, for rigid rotation the kinetic helicity is positive in the upper disk plane and negative in the lower disk plane. For the one-mode flux tube model we find for the amplitude the value

$$\mathcal{H}_{\rm kin} = \frac{8}{15} \frac{\tau_{\rm corr}^3 \Omega g}{\ell_{\rm corr}^2} \left(1 + \frac{\partial \log \Omega}{\partial \log s}\right) V_{\rm A}^2 \frac{\langle B^{(0)2} \rangle}{\mu_0 \bar\rho c_{\rm ac}^2}. \quad (30)$$

Obviously, again the sign of the (pseudo-)scalar changes with increasing shear in the same way as it happens for the current helicity, see Eq. (16). For Kepler rotation the kinetic helicity in the upper hemisphere proves to be negative – as it does in the simulations by Brandenburg (1999).

The amplitude ratio of the dynamo-α and the kinetic helicity is

$$\frac{\alpha_{\phi\phi}}{\mathcal{H}_{\rm kin}} \simeq \tau_{\rm corr} \frac{\rm Ma^2}{\rm Mm^2} \quad (31)$$

with the turnover velocity $u_{\rm T} = \ell_{\rm corr}/\tau_{\rm corr}$, the turbulence Mach number ${\rm Ma} = u_{\rm T}/c_{\rm ac}$ and the magnetic Mach number ${\rm Mm} = V_{\rm A}/c_{\rm ac}$. For equipartition of the magnetic field with the thermal pressure (Mm$\simeq 1$) we find the α-effect to be much smaller than the traditional value ('helicity times the correlation time') if the turbulence is subsonic. Even the sign is opposite.

4. Angular momentum transport

Our turbulence model can only model the situation in accretion disks if it transports the angular momentum outwards, i.e. if the stress $T_{s\phi}$ is positive. On the other hand, we know from observations the value of the normalized angular momentum transport,

$$\alpha_{\rm SS} = \frac{T_{s\phi}}{c_{\rm ac}^2}, \quad (32)$$

being of order $10^{-3} - 1$ so that – if we find a relation between both the alphas the dynamo-α can be estimated. From historical reasons the quantity (32) is called the viscosity-α. For the correlation tensor of the magnetic-forced turbulence the expression

$$\begin{aligned}\langle u_i' u_j' \rangle = & -\frac{23}{105} \frac{\bar B^2}{\mu_0^2 \bar\rho^2} \left(\bar u_{i,j} + \bar u_{j,i}\right) I_4 + \\ & + \Big\{ -\left(\bar B_i g_j + \bar B_j g_i\right)\left(\bar u_{m,n} + \bar u_{n,m}\right) \bar B_n g_m - \\ & -5 \left(\bar B_i g_j + \bar B_j g_i\right) \bar B_m g_n \bar u_{m,n} + \\ & +3g^2 \left(\bar B_i \bar u_{j,n} + \bar B_j \bar u_{i,n}\right) \bar B_n - \frac{\bar B^2}{9} \left(g_i \bar u_{j,n} + g_j \bar u_{i,n}\right) g_n \\ & -\frac{5}{3} \bar B^2 g_n \left(\bar u_{n,i} g_j + g_i \bar u_{n,j}\right) - \\ & -\frac{13}{3} g^2 \bar B^2 \left(\bar u_{i,j} + \bar u_{j,i}\right) \Big\} \frac{I_5}{\mu_0^2 \bar\rho^2 c_{\rm ac}^4} - \\ & -\Big\{ 4 \left(g_i \bar B_j + g_j \bar B_i\right)\left(\bar u_{m,n} + \bar u_{n,m}\right) \bar B_n g_m - \\ & -\frac{76}{9} \bar B^2 \left(g_i \left(\bar u_{j,n} + \bar u_{n,j}\right) + g_j \left(u_{i,n} + u_{n,i}\right)\right) g_n + \\ & + \frac{16}{9} \bar B^2 g^2 \left(\bar u_{i,j} + \bar u_{j,i}\right) \Big\} \frac{I_6}{\mu_0^2 \bar\rho^2 c_{\rm ac}^4} \end{aligned} \quad (33)$$

is obtained with

$$I_5 = \frac{1}{105} \iint\limits_0^\infty \frac{\nu k^2 \mathcal{B}(k,\omega)}{(\omega^2 + \nu^2 k^4)^2} dk d\omega \quad (34)$$

and

$$I_6 = \frac{1}{105} \iint\limits_0^\infty \frac{\nu k^2 (\nu^2 k^4 - \omega^2) \mathcal{B}(k,\omega)}{(\omega^2 + \nu^2 k^4)^3} dk d\omega. \quad (35)$$

It follows with $g = (0, 0, -g)$

$$\langle u_s' u_\phi' \rangle = -\frac{\bar B^2 \Omega}{\mu_0^2 \bar\rho^2} \left(\frac{3 g^2 I_5}{c_{\rm ac}^4} + \left(\frac{23}{105} I_4 + \frac{13}{3} \frac{g^2 I_5}{c_{\rm ac}^4} + \frac{16}{9} \frac{g^2 I_6}{c_{\rm ac}^4} \right) \frac{\partial \log \Omega}{\partial \log s} \right). \quad (36)$$

Our turbulence model then yields

$$\langle u_s' u_\phi' \rangle = -\frac{\tau_{\rm corr}^3 \Omega}{105} \left(3 \frac{g^2}{c_{\rm ac}^4} + \frac{1}{\ell_{\rm corr}^2} \left(23 + \frac{55}{9} \frac{g^2 \ell_{\rm corr}^2}{c_{\rm ac}^4} \right) \right) \frac{\partial \log \Omega}{\partial \log s} V_{\rm A}^2 \frac{\langle B^{(0)2} \rangle}{\mu_0 \bar\rho} \quad (37)$$

for the Reynolds stress at the equator. It does not vanish for rigid rotation so that simply here the term 'viscosity' makes no sense. Moreover, the rigid-rotation term in (37) reflects the Λ-effect of rotating turbulence fields which is responsible for the maintenance of differential rotation in stellar convection zones. It is here negative. The total angular momentum transport, however, for a Kepler flow is positive as then

$$\langle u_s' u_\phi' \rangle \approx \frac{1}{105} \frac{\tau_{\rm corr}^3}{\ell_{\rm corr}^2} \left(\frac{69}{2} + \frac{37}{6} \left(\frac{\ell_{\rm corr}}{H_p} \right)^2 \right) \Omega V_{\rm A}^2 \frac{\langle B^{(0)2} \rangle}{\mu_0 \bar\rho} \quad (38)$$

results if $B_\phi \gg B_s, B_z$ and $g H_p \simeq c_{\rm ac}^2, H_p$ as the radial pressure scale. In Kepler flows the Reynolds stress is positive, hence the angular momentum is always transported outwards.

For the Maxwell stress we obtain

$$\langle B_i' B_j' \rangle = \frac{23}{14} V_{\rm A}^2 (\bar u_{i,j} + \bar u_{j,i}) I_1. \quad (39)$$

which immediately leads to the magnetic-induced angular momentum transport

$$\langle B_s' B_\phi' \rangle = \frac{23}{105} \frac{\tau_{\rm corr}^3}{\ell_{\rm corr}^2} V_{\rm A}^2 \langle B^{(0)2} \rangle \frac{\partial \log \Omega}{\partial \log s} \Omega. \quad (40)$$

Then we arrive at the positive value

$$\alpha_{\rm SS} = \left(\frac{23}{35} + \frac{2}{35} \frac{\ell_{\rm corr}^2}{H_p^2} \right) \frac{\tau_{\rm corr}^3}{\ell_{\rm corr}^2} \frac{V_{\rm A}^2}{c_{\rm ac}^2} \frac{\langle B^{(0)2} \rangle}{\mu_0 \bar\rho} \Omega \quad (41)$$

with the amplitude

$$\alpha_{\rm SS} \simeq \frac{\Omega^*}{2}\frac{V_{\rm A}^2}{u_{\rm T}^2}\frac{\langle B^{(0)2}\rangle/\mu_0\bar\rho}{u_{\rm T}^2}\frac{u_{\rm T}^2}{c_{\rm ac}^2}, \quad (42)$$

where the Coriolis number $\Omega^* = 2\tau_{\rm corr}\Omega$ has been used. If the magnetic fields are in equipartition with the turbulence then we find the simple estimation

$$\alpha_{\rm SS} \lesssim {\rm Ma}\ \Omega^*. \quad (43)$$

The maximal value of (43) might be Ω^*, which itself does hardly exceed the order of unity. The viscosity-alpha, therefore, proves to be *smaller* than unity for subsonic turbulence.

5. Discussion: dynamo-α and viscosity-α

In Table 1 a summary is given for the signs of the resulting MHD-mean-field coefficients obtained by the presented turbulence model for the two cases of rigid rotation and Kepler rotation. The first line with positive α-effect, with positive kinetic helicity and negative current helicity (all in the upper disk plane) is just the same as given by Brandenburg & Schmitt (1998) for a simulation of the solar northern pole. For a MHD shear flow simulated by Brandenburg et al. (1999) the case 'Kepler' in Table 1 valids and there is also not even one exception from the agreement. The kinetic helicity in the upper disk plane for Brandenburg's simulation is negative and the same is true in our flux tube model. It is interesting to formulate for Kepler disks the relation between both the alphas. With (23) and (41) follows

Table 1. The signs of the MHD coefficients for rigid rotation and Kepler rotation.

rotation	location	$\alpha_{\rm SS}$	$\alpha_{\phi\phi}$	$\mathcal{H}_{\rm curr}$	$\mathcal{H}_{\rm kin}$
rigid	north	$-$	$+$	$-$	$+$
	south	$-$	$-$	$+$	$-$
Kepler	north	$+$	$-$	$+$	$-$
	south	$+$	$+$	$-$	$+$

$$\frac{\alpha_{\phi\phi}}{\alpha_{\rm SS}} \lesssim -\frac{1}{30}\ \frac{z\Omega^2\tau_{\rm corr}u_{\rm T}^2}{V_{\rm A}^2}, \quad (44)$$

so that the amplitude of the dynamo-α becomes

$${\rm MAX}(\alpha_{\phi\phi}) \lesssim \frac{\alpha_{\rm SS}}{10}\ \frac{\ell_{\rm corr}}{H}\ \frac{u_{\rm T}}{{\rm Mm}^2} \quad (45)$$

where the magnetic Mach number Mm can be assumed of order unity. We have also used the relation $H\Omega \simeq c_{\rm ac}$ between the disk thickness and the temperature of a thin accretion disk.

We find the dynamo-α as a rather small fraction of the turbulent velocity $u_{\rm T}$. The dynamo-α proves thus to be very small. It is negative in the upper disk plane and positive in the lower one. We can thus expect a dipolar symmetry with respect to the equator for the dynamo-maintained large-scale magnetic fields. In order to ensure self-excitation for the magnetic fields with such a small α-effect the eddy diffusivity of the turbulence must be sufficiently small. We can not compute this effect with our model working with uniform magnetic fields. The same also holds for almost all numerical simulations so that it must remain open whether a magnetic dynamo really works.

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