Astronomy Letters, Vol. 27, No. 3, 2001, pp. 163–170. Translated from Pis'ma v Astronomicheskiĭ Zhurnal, Vol. 27, No. 3, 2001, pp. 194–202.

Expulsion of Magnetic Flux from the Core and Its Dissipation in the Crust of a Neutron Star

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Received March 13, 2000

Abstract—We construct a model for the magnetic-field evolution of an isolated neutron star by assuming that its core is a type II superconductor and that the field penetrates the core in the form of magnetic lines (fluxoids). We consider the fluxoid expulsion from the core and the field dissipation in a conducting crust. The magnetic-field evolution is calculated self-consistently by taking into account the inverse effect of crustal magnetic line bending on the fluxoid velocity in the core. We consider the evolution of two magnetic configurations, in which the bulk of the magnetic flux passes through the neutron-star core and crust. The buoyancy of fluxoids and the force from the neutron vortexes are mainly responsible for their expulsion from the core in the former and latter cases, respectively. *© 2001 MAIK "Nauka/Interperiodica".*

Key words: *pulsars, neutron stars, magnetic-field evolution*

INTRODUCTION

The magnetic-field evolution of neutron stars (NSs) has been the subject of much debate since the discovery of radio pulsars. It is primarily determined by the configuration of currents inside NSs and by the conductive properties of the layers, in which these currents are located. The magnetic-flux conservation during gravitational collapse and/or the effect of magnetic dynamo in a convective proto-neutron star (Thompson and Duncan 1993) result in a uniform distribution of the magnetic flux over the NS and in the passage of its bulk through the core. On the other hand, a magnetic field can be generated in the outer-crustal layers of a young NS after its birth under the effect of, for example, thermomagnetic instability (Urpin *et al.* 1986). In this case, the bulk of the magnetic flux is confined to the NS crust (see Fig. 1). Unfortunately, there is currently no consensus on the generation mechanism of NS magnetic fields.

Neutrons and protons in the NS core are believed to become superfluid at early cooling stages (Alpar 1991); the superfluid core of the NS is involved in its rotation, forming a lattice of neutron vortexes (see, e.g., Shapiro and Teukolsky 1983). The neutron vortexes are parallel to the spin axis. As was shown by Baym *et al.* (1969), protons form a type II superconductor, in which the magnetic field exists in the form of vortex lines, or fluxoids (Lifshitz and Pitaevskiĭ 1978). Each fluxoid carries a quantum of magnetic flux $\Phi_0 = hc/2e \approx 2 \times 10^7$ G cm^2. A fluxoid consists of a nonsuperconducting nucleus, with a characteristic diameter of the order of the proton coherence length ξ_p, surrounded by the shielding current of superconducting protons with sizes of the order of the London length of magnetic-field penetration into a superconductor, $\lambda_p \sim 10^{-12}$ cm. In a type II superconductor, $\xi_p/\lambda_p \le 1/\sqrt{2}$. The magnetic field is $B_p = \Phi_0/(4\pi\lambda_p^2)\ln(\lambda_p/\xi_p) \approx 1.9 \times 10^{16} x_p \rho_{15} \ln(\lambda_p/\xi_p)$ G inside the fluxoid and falls off exponentially outside the fluxoid, with a characteristic length λ_p (Ding *et al.* 1993). Here, x_p is the proton-to-neutron density ratio in the core, and ρ_{15} is the density in units of 10^{15} g cm^{-3}. By the mean core magnetic field, we mean $B_c = \Phi_0 n_p$, where $n_p = 5 \times 10^{18}(B_c/10^{12}$ G) cm^{-2}, is the number of fluxoids per unit area.

The magnetic-field evolution in the core is directly related to the motion of the fluxoids. The buoyancy force (Muslimov and Tsygan 1985a, 1985b), the force from neutron vortexes (Ding *et al.* 1993), and the drag force (Harvey *et al.* 1985) act on the fluxoids. The radial fluxoid velocity (and the magnetic-field evolution) in the core of an isolated NS under the action of these forces was first calculated by Ding *et al.* (1993). Jahan-Miri (1999) used the same model to calculate the magnetic evolution of NSs in binary systems. These authors determined the fluxoid velocity and the magnetic-field evolution in the core from the balance condition for the forces exerted on fluxoids. The surface field relaxed to the core field in the dissipation time of the crustal currents, which is a parameter of the problem. However, as the fluxoid roots move, the crustal magnetic lines bend, the magnetic energy outside the

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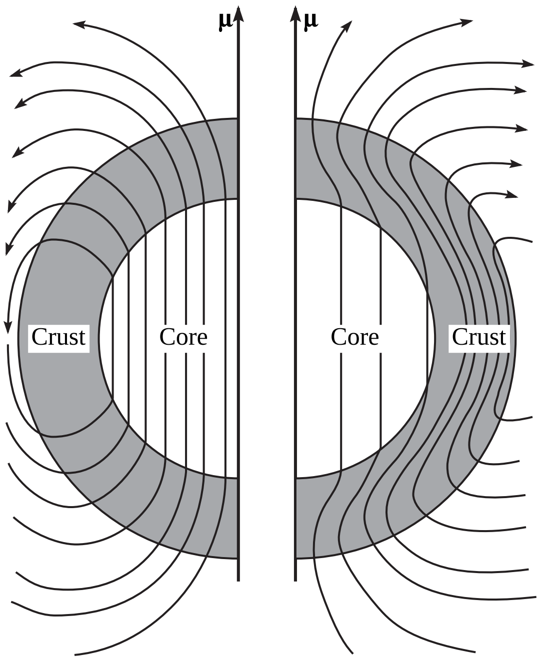

Fig. 1. Two possible configurations of the NS poloidal magnetic field. On the left, the bulk of the magnetic flux passes through the NS core; if the core is a type II superconductor, then the magnetic field penetrates it in the form of fluxoids. On the right, the bulk of the magnetic flux passes through the NS crust.

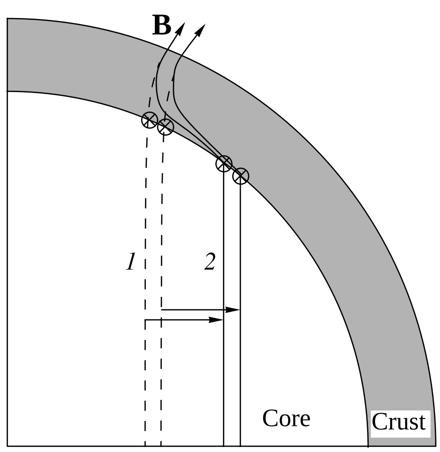

Fig. 2. Motion of fluxoids from position *1* to position *2*. The fluxoid roots are marked by ⊗.

core changes, and currents are generated in the crust. Consequently, additional work needs to be done to move the fluxoid root (Fig. 2). This factor was disregarded by Ding *et al.* (1993) and Jahan-Miri (1999). Here, we calculate the expulsion of magnetic flux from the NS core, in a self-consistent way, by taking into account this effect. We consider the evolution of the two possible magnetic configurations shown in Fig. 1.

STATEMENT OF THE PROBLEM

Alpar *et al.* (1984) showed that a magnetic field comparable in magnitude to the magnetic field inside a fluxoid is generated inside neutron vortexes. As a result, fluxoids and neutron vortexes interact with each other as they draw closer together, with the interaction energy being $E_p \sim 10$ MeV per intersection. The radial velocity of neutron vortexes is determined by the spindown rate of an isolated NS; in turn, spindown is attributable to the losses of rotational kinetic energy of an isolated NS through the magnetodipole radiation and the ejection of relativistic particles:

$$v_n(t) = \frac{rk(t)\Omega_s}{2}, \quad (1)$$

where

$$k(t) = K\frac{8B_e^2(t)R^6}{3Ic^3}.$$

Here, r is the radial coordinate, Ω_s is the angular velocity of the superfluid NS core, R is the NR radius, I is the moment of inertia, B_e is the NS surface magnetic field on the magnetic equator, and c is the speed of light. In general, the coefficient $K \le 1$ depends on the inclination of the spin axis to the magnetic axis, on the spin period, and on the magnetic field. For simplicity, we assume that $K = 1$. We emphasize that Ω_s is not equal to the observed angular velocity Ω_c of the crust. There are three modes of relative motion of fluxoids and neutron vortexes: fluxoids can move either faster than neutron vortexes (forward creeping), or the velocities of both types of vortex lines can be the same (comoving), or neutron vortexes can move faster than fluxoids (reverse creeping). The force exerted per unit fluxoid length by neutron vortexes is given by (Ding *et al.* 1993)

$$f_n = \frac{n_v}{n_p}F_M \approx \frac{2\Phi_0\rho r\Omega_s(t)\omega(t)}{B_c(t)}, \quad (2)$$

where n_v is the number of neutron vortexes per unit area, $F_M = \rho\kappa r\omega$ is the Magnus force per unit vortex length, ρ is the core matter density, $\kappa = h/2m_n$ is the velocity circulation quantum, and $\omega = \Omega_s - \Omega_c$ is the difference between the angular velocities of a superfluid core and a conducting solid crust. Depending on the sign of ω, the force from neutron vortexes can be positive (directed to the crust, i.e., expels a fluxoid outward) or negative (directed into the NS, i.e., prevents fluxoid expulsion).

The maximum magnitude of the force exerted by a neutron vortex on a fluxoid per intersection can be estimated by using the formula $f_p \approx E_p/\lambda_p$. The Magnus force acting on neutron vortexes is balanced by the force from fluxoids. It thus follows that $|\omega| \le \omega_{cr}$ (Ding *et al.* 1993; Jahan-Miri 1999). In the forward creeping, comoving, and reverse creeping modes, $\omega = \omega_{cr}$, $|\omega| <$

ω_{cr}, and $\omega = -\omega_{cr}$, respectively. Ding *et al.* (1993) derived the following expression for ω_{cr}:

$$\omega_{cr} = 8.7\times10^{-2}x_p\alpha_g r_6^{-1}\left(\frac{\delta m_p^*}{m_p}\right)\left(\frac{m_p^*}{m_p}\right)^{-1/2}$$
$$\times(B_c/10^{12}\ \mathrm{G})^{1/2}\ln(\lambda_p/\xi_p)\sin(2\chi)\ \mathrm{rad\ s^{-1}}.$$

Here, α_g is a geometric factor of the order of unity, r_6 is the distance from the neutron vortex to the spin axis (in units of 10^6 cm), m_p^* is the effective proton mass, $\delta m_p^* = m_p - m_p^*$, and χ is the angle between the spin axis and the magnetic dipole axis. In our calculations, we assume that $x_p = 0.025$, $m_p^* = 0.8m_p$, $\lambda_p/\xi_p = 1/\sqrt{2}$, and $\sin(2\chi) = 1$.

Apart from the force of neutron vortexes, the buoyancy force acts per unit fluxoid length (Muslimov and Tsygan 1985a, 1985b):

$$f_b = \left(\frac{\Phi_0}{4\pi\lambda_p}\right)^2\frac{1}{R_c}\ln\left(\frac{\lambda_p}{\xi_p}\right), \quad (3)$$

where R_c is the core radius. This force is always positive, i.e., it tends to expel a fluxoid from the core.

Finally, the drag force attributable to electron scattering by the fluxoid magnetic field acts per unit length of a fluxoid moving at velocity v_p. This force is proportional to the fluxoid velocity and is given by (Harvey *et al.* 1985)

$$f_v = -\frac{3\pi}{64}\frac{n_e e^2\Phi_0^2}{E_F\lambda_p}\frac{v_p}{c}, \quad (4)$$

where n_e is the electron density in the core (we assume it to be equal to the proton density), and E_F is the electron Fermi energy. This equation for the drag force remains valid when the collective effects during fluxoid motion are ignored, which is justifiable for $B_c \ll B_p$.

According to Ding *et al.* (1993), the equation for the fluxoid velocity can be written as

$$f_n + f_b + f_v(v_p) = 0. \quad (5)$$

These authors also attempted to take into account the force that arises as fluxoids bend. Allowance for these forces gave rise to a coefficient of the order of unity near f_b in Eq. (5). On the other hand, Eq. (33) from Ding *et al.* (1993), which relates the field evolution in the core to the fluxoid velocity, explicitly implies that the core field is uniform (i.e., the fluxoids are straight, and their density is constant throughout the entire core). We assume, for simplicity, that the fluxoids remain straight as they move. Whether this simplification is acceptable is discussed below. The magnetic field concentrated in the core of the fluxoids also passes through the nonconducting crust. The crustal-magnetic lines, bend as the fluxoid roots move (Fig. 2); consequently, the forces exerted on a fluxoid do work as the fluxoid root moves, which was not included in Eq. (5). A more consistent allowance for this effect requires that Eq. (5) be replaced by

$$\sum_{\text{fluxoids}}\int(\mathbf{f}_n + \mathbf{f}_b + \mathbf{f}_v)\mathbf{v}_p dl = \int_{V_{crust}}\frac{j^2}{\sigma}dV + \frac{d}{dt}\int_V\frac{B^2}{8\pi}dV. \quad (6)$$

The left part of this equation represents the total power of the forces exerted on fluxoids, the integration is performed along the fluxoid length, and the summation is carried out over all fluxoids. In the right part, the first integral is taken over the crust volume, while the second integral is taken over the crust volume and the entire space outside the NS. Assuming, for simplicity, that the mean core magnetic field is uniform and substituting for all quantities their values at the crust-core boundary, we can write the left part as $(f_n + f_b + f_v)v_p(R_c)N_p\langle l_p\rangle$, where $N_p = 4\pi R_c^2 B_c/\Phi_0$ is the total number of fluxoids, and $\langle l_p\rangle = 4R_c/3$ is the mean fluxoid length. Thus, instead of the forces per unit length, we introduce the total forces exerted on the fluxoids in the core:

$$F_{n,b,v} = f_{n,b,v}\times 4R_c/3\times N_p. \quad (7)$$

We can also introduce a quantity that has the meaning of the force acting on the fluxoid roots as they move:

$$F_{crust}(v_p) = -\frac{1}{v_p}\left(\int_{V_{crust}}\frac{j^2}{\sigma}dV + \frac{d}{dt}\int_V\frac{B^2}{8\pi}dV\right). \quad (8)$$

If the fluxoids move outward (the velocity is positive) and if the magnetic energy in the crust increases, then this force is negative; i.e., it prevents the motion of the fluxoid roots toward the magnetic equator. Equation (6) can now be rewritten as

$$F_n + F_b + F_v(v_p) + F_{crust}(v_p) = 0. \quad (9)$$

The magnetic-field evolution in a solid conducting crust is described by the induction equation without a convective term:

$$\frac{\partial\mathbf{B}}{\partial t} = -\frac{c^2}{4\pi}\nabla\times\left(\frac{1}{\sigma}\nabla\times\mathbf{B}\right), \quad (10)$$

where σ is the crust conductivity. We study the magnetic evolution of NSs with classical magnetic fields (10^{12}–10^{13} G). In this case, the crustal electrons are nonmagnetized, and the conductivity is a scalar.

We consider the evolution of a poloidal magnetic field with only a dipole component outside the NS. In spherical r, θ, ϕ coordinates, in which the vertical axis coincides with the magnetic dipole axis, it is convenient to introduce a vector potential $A = (0, 0, A_\phi)$, where $A_\phi = S(r, t)\sin\theta/r^2 = B_{e0}R^2 s(r, t)\sin\theta/r$, B_{e0} is the NS field on the magnetic equator at the initial time. Equation (10) can be rewritten as

$$\frac{4\pi\sigma}{c^2}\frac{\partial s}{\partial t} = \frac{\partial^2 s}{\partial r^2} - \frac{2s}{r^2}. \quad (11)$$

The magnetic-field components are written as a function of s as

$$B_r = \frac{2S}{r^2}\cos\theta, \quad B_\theta = -\frac{\sin\theta}{r}\frac{\partial S}{\partial r}. \quad (12)$$

There is no magnetic-field dissipation in a superconducting core, and its evolution is described by the equation

$$\frac{\partial \mathbf{B}_c}{\partial t} = \nabla \times (\mathbf{v}_p \times \mathbf{B}_c), \quad (13)$$

which can be rewritten for $s(r, t)$ as

$$\frac{\partial s}{\partial t} = -v_p\frac{\partial s}{\partial r}. \quad (14)$$

The integrals in the right part of Eq. (6) can be rewritten as

$$\int_{V_{\rm crust}} \frac{j^2}{\sigma}dV = \frac{c^2 B_{e0}^2 R^4}{6\pi}\int_{R_c}^{R}\frac{1}{\sigma}\left(\frac{\partial^2 s}{\partial r^2} - \frac{2s}{r^2}\right)^2 dr, \quad (15)$$

$$\frac{d}{dt}\int_{V_{\rm crust}}\frac{B^2}{8\pi}dV = B_{e0}^2 R^4 \frac{d}{dt}\left(\frac{2}{3}\int_{R_c}^{R}\frac{s^2}{r^2}dr + \frac{1}{3}\int_{R_c}^{R}\left(\frac{\partial s}{\partial r}\right)^2 dr\right), \quad (16)$$

$$\frac{d}{dt}\int_{V_{\rm vacuum}}\frac{B^2}{8\pi}dV = \frac{2B_{e0}^2 R^3}{3}s(R, t)\frac{ds(R, t)}{dt}. \quad (17)$$

The conductivity in a solid crust is mainly determined by the scattering of electrons by impurities and phonons. The scattering by phonons gives a major contribution to the conductivity at low densities and high temperatures, while the scattering by impurities dominates at high densities and low temperatures. We use expressions for the conductivity from Itoh *et al.* (1993) and Yakovlev and Urpin (1980). The frequency of electron scattering by phonons depends on the crustal temperature; we take the time dependence of the temperature from standard NS cooling calculations (Van Riper 1991). The frequency of scattering by impurities does not depend on temperature, but depends on impurity density. The latter is characterized by the impurity parameter Q, which has the meaning of rms deviation of the nuclear charge from the mean. Unfortunately, theory currently gives no definite impurity density in the NS crust. We therefore calculate the magnetic-field evolution for various values of this parameter in the range 0.01–1 and assume that Q does not depend on depth and time.

For a uniform core magnetic field ($r < R_c$), we can write

$$v_p = \alpha(t)r, \quad (18)$$

$$B_e(t) = B_{e0}\exp\left(-\int_0^t \alpha(t')dt'\right), \quad (19)$$

$$s(r, t) = \frac{B_c(t)}{B_{c0}}\frac{r^2}{R_c^2}, \quad (20)$$

where B_{c0} is the initial core field and $\alpha(t)$ can be determined from Eqs. (6) and (10). The following boundary condition must be satisfied at the NS surface ($r = R$):

$$\partial s/\partial r = -s/R. \quad (21)$$

The function s/r^2 must remain finite at the NS center ($r \longrightarrow 0$), which is automatically satisfied by Eq. (20). Equations (7)–(9) simultaneously give the fluxoid velocity and the inner boundary condition (at $r = R_c$) for Eq. (11).

We calculate the magnetic-field evolution of a NS as follows. Having chosen the initial core magnetic field, the initial $s(r, t = 0)$ profile in the crust, and the initial NS spin period P_0, we specify the initial condition. In addition, it is necessary to specify the density profile in the crust, the impurity parameter Q, the crust thickness, and the NR radius. For our calculations, we use the model of a $1.4M_\odot$ neutron star constructed for the hard equation of state by Pandharipande and Smith (1975), the NS radius R = 16.4 km, the crust thickness δR = 4200 m, and the moment of inertia $I = 2.12 \times 10^{45}$ g cm^2.

We assume that $v_p = v_n$, where v_n is given by Eq. (1). Having specified a sufficiently small time step Δt, we calculate $s(R_c, \Delta t)$ from Eq. (20), i.e., obtain the inner boundary condition for Eq. (11), and calculate the evolution of the crustal magnetic field in time Δt using an implicit scheme. Thus, we obtain $s(r, \Delta t)$ in the crust. Next, we calculate the integrals (15)–(17) and $F_{\rm crust}$ and then F_b and F_v ($v_p = v_n$). Finally, we derive F_n from Eq. (9) and determine ω using Eqs. (7) and (2). If $-\omega_{cr} < \omega < \omega_{cr}$, then the fluxoids and neutron vortexes actually move at the same velocity (comoving). If, alternatively, $\omega > \omega_{cr}$ or $\omega < -\omega_{cr}$, then our assumption that the velocities of fluxoids and neutron vortexes are equal is wrong: in the former and latter cases, fluxoids move more slowly (forward creeping) and faster than neutron vortexes (reverse creeping), respectively. If fluxoids move more slowly than neutron vortexes, then we assume that $\omega \equiv \omega_{cr}$. Next, we calculate $F_n = F_n(\omega_{cr})$ from Eqs. (2) and (7) and then find the true value of v_p as the root of Eq. (9) in the interval $[0, v_n]$ by the bisection method. Otherwise ($v_n < v_p$), we assume that $\omega \equiv -\omega_{cr}$, calculate $F_n = F_n(-\omega_{cr})$, and again find the true value of v_p in the interval $[v_n, \infty]$ from Eq. (9).

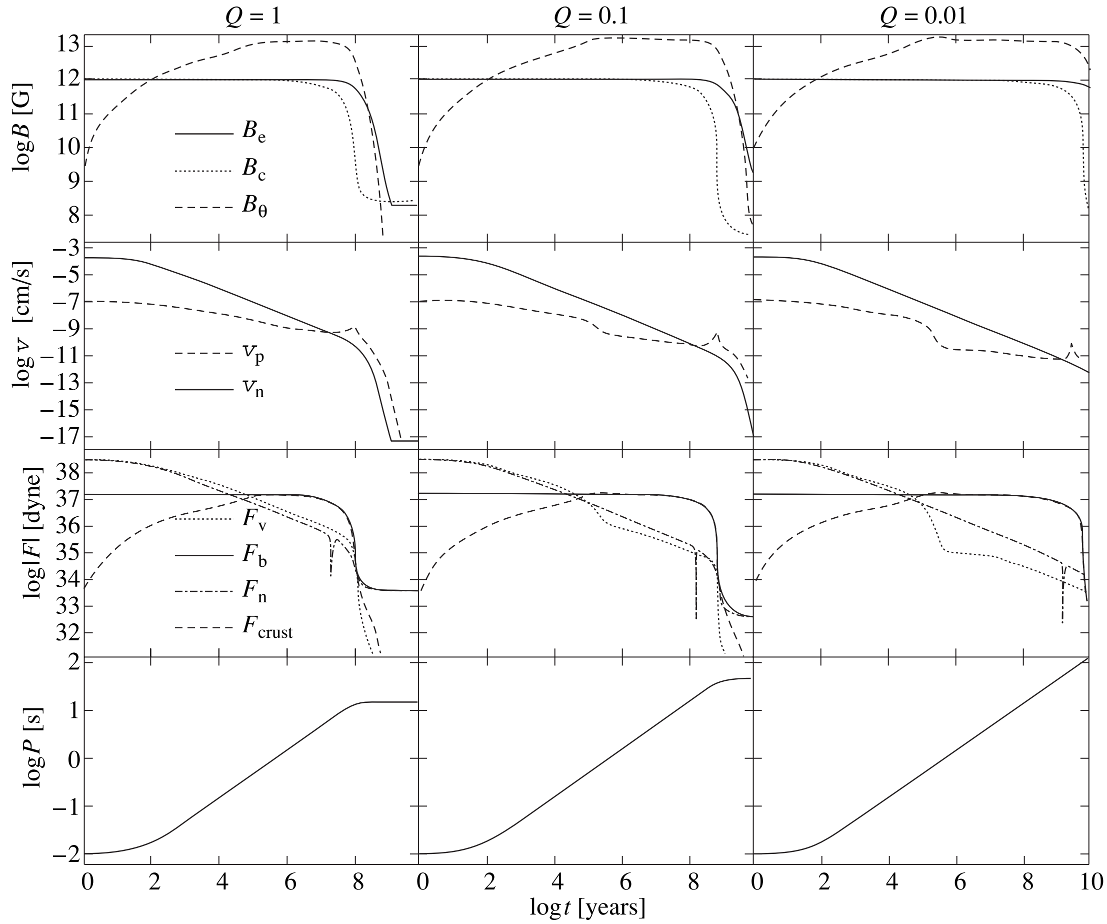

Fig. 3. Evolutionary curves for (from top to bottom) the crustal magnetic field, the NS surface magnetic field, and the θ field component at the core-crust boundary; the vortex and fluxoid velocities; the force exerted on fluxoids; and the NS spin period.

Because of the losses of rotational kinetic energy through the magnetodipole radiation and the ejection of a relativistic particle, the evolution of the NS spin period P can be calculated by using the formula

$$P\frac{dP}{dt} = \frac{32\pi^2}{3}\frac{B_e(t)^2R^6}{c^3I}.$$

We repeat these calculations at each time step.

RESULTS

If the magnetic field of a star was enhanced during collapse, then the bulk of the magnetic flux will pass through its core (Fig. 1, left part). However, a magnetic field can be generated in the surface layers of the already-formed young hot NSs (Urpin *et al.* 1986). In this case, the bulk of the NS total magnetic flux can pass through the crust (Fig. 1, right part), and the magnetic field in the NS crust will be much stronger than in its core. The Vela pulsar may have such a magnetic configuration (Chau *et al.* 1992). Here, we study the evolution of both configurations of the NS magnetic field.

In the former case, we chose $s(r, 0) = 1$ as the initial condition in the crust. In this case, the crustal field is initially radial, and the core field is equal to the field at the magnetic equator. Figure 3 shows the evolution of the magnetic fields, the velocities of the fluxoids and neutron vortexes, the forces exerted on the fluxoids, the NS spin period for $B_{e0} = B_{c0} = 10^{12}$ G, $P_0 = 0.01$ s, and various values of Q. Irrespective of the specific value of Q, the fluxoid velocity is initially lower than the velocity of neutron vortexes (forward creeping). The duration of this stage depends on Q: the smaller the Q, the shorter this stage. Thus, for example, fluxoids move more slowly than neutron vortexes at $Q = 0.01$ during ~10^9 years, while the velocities of both types of vortex lines become equal at $Q = 1$ in ~10^7 years. At $t \le 10^4$ years, the fluxoid velocity is virtually independent of Q and is determined by the balance of F_n and F_v [in Eq. (9), $(F_n, F_v) \gg (F_b, F_{crust})$]. In this case, $F_n > 0$ (expels fluxoids from the core) and $F_v < 0$. At $t > 10^4$ years, Dependence of $\log(t_e)$ on B_{e0} and Q

Q \ B_{e0}, G	10^{12}	10^{13}
1	8.15	8.6
0.1	9.15	9.6
0.01	10.15	10.6

neutron vortexes affect the fluxoid dynamics only slightly, and the buoyancy force F_b becomes the main force responsible for the expulsion of fluxoids from the core. A θ magnetic-field component is generated in the crust near the boundary with the core, which exceeds the surface field by a factor of ~10. F_{crust} becomes the main force that prevents the outward motion of fluxoids; $(F_b, F_{crust}) \gg (F_n, F_v)$, and the fluxoid velocity depends on crust conductivity: the higher the crust conductivity (the smaller the Q), the lower the fluxoid velocity and the longer the time of magnetic-flux expulsion from the core. Thus, for example, the core field at $Q = 0.01$ begins to decrease only after ~10^9 years of evolution, while the NS surface field is essentially constant over the lifetime of the Universe. At $Q = 1$, the time of magnetic-field expulsion from the core is ~10^7 years. In this time, the fluxoid velocity becomes equal to the velocity of neutron vortexes and subsequently exceeds it. At this instant of time, F_n changes sign and begins to hinder the expulsion of fluxoids from the core. The fluxoid expulsion from the core ceases when the buoyancy force F_b is balanced by the force F_n from neutron vortexes. In this case, the fluxoid and vortex velocities again become equal (comoving). For the model of a NS constructed with the hard Pandharipande–Smith equation of state (PS model) with the initial magnetic field $B_{e0} = 10^{12}$ G, this occurs only for $Q = 1$. At lower values of Q, this stage is not reached on the Hubble time scale. The time of field expulsion from the core for $Q = 1$ is about 10^7 years, but the buoyancy force is balanced by the force from neutron vortexes as the core field decreases to 1.5×10^8 G. In this case, the fluxoid and vortex velocities fall to 3×10^{-18} cm s^{-1}, and the magnetic-flux expulsion from the core virtually ceases. Note that the NS surface magnetic field follows the core field with a delay, because the time of field diffusion through the crust in the NS model under consideration is ~$10^8/Q$ years (Urpin and Konenkov 1997).

In applications (for example, when modeling the evolution of a population of neutron stars in the Galaxy), the following analytic formula describing the evolution of the NS surface magnetic field can be of use:

$$B_e = B_{e0}\exp(-t/t_e) + B_{res}, \quad (22)$$

where the characteristic decay time t_e of the surface field and the residual magnetic field B_{res} depend on B_{e0}, Q, and NS model. We calculated the magnetic and spin evolution of a NS for $B_{e0} = B_{c0} = 10^{13}$ G as well; the values of t_e for various B_{e0} and Q are given in the table. The residual magnetic field (~10^8 G for $B_{e0} = 10^{12}$ G and ~10^7 G for $B_{e0} = 10^{13}$ G) is reached only for $Q = 1$. In the remaining cases, we may set $B_{res} = 0$ in Eq. (22).

We see from the table that the time of reduction in a NS surface magnetic field depends not only on conductivity, but also on the magnetic-field strength itself. This is because the buoyancy force is $F_b \propto B_c$ and because the force preventing the expulsion is $F_{crust} \propto B_c^2$. Therefore, as the magnetic field increases, the fluxoid velocity decreases, and the time of magnetic-flux expulsion from the core increases.

In Eq. (6), we disregard the force attributable to fluxoid curvature. If the radius of fluxoid curvature is comparable to the core radius, then this force will give rise to a coefficient of the order of unity near f_b in Eq. (6), and the time of flux expulsion from the core will also change by a coefficient of the order of unity; our results will not change qualitatively.

The magnetic field of a NS may be generated in its surface layer after its birth (Urpin *et al.* 1986). To model the evolution of such a magnetic configuration, we chose the following initial condition in the crust:

$$s(r, 0) = \begin{cases} B_{c0}/B_{e0}, & \text{if} \quad r < r_0 \\ (1 - r^2/r_0^2)(1 - R^2/r_0^2), & \text{if} \quad r_0 < r < R. \end{cases} \quad (23)$$

The $s(r, 0)$ profile corresponds to the following magnetic configuration: the core field is $B_{c0} < B_{e0}$, the crustal field is radial at $R_c < r < r_0$, and the currents that produce the observed NS surface magnetic field B_{e0} flow in the $r_0 < r < R$ layer. Thus, the initial condition we chose models the magnetic configuration shown in the right part of Fig. 1. The parameters are the initial surface magnetic field B_{e0}, the initial core magnetic field B_{c0}, the thickness $z_0 = R - r_0$ of the surface layer where currents initially flow, and the impurity density described by parameter Q. Figure 4 shows the evolution of s. Specific values of the parameters are given in the caption to the figure. On the one hand, the field initially localized in the crustal surface layer (left part in Fig. 1) dissipates. In this case, it diffuses deep into the crust, just as in the case of evolution of the field localized only in the NS crust (Urpin and Muslimov 1992; Urpin and Konenkov 1997). On the other hand, the magnetic flux is expelled from the superconducting core into the crust; a θ magnetic-field component is generated near the crust-core boundary (or $\partial s/\partial r$), while the field in the core $s(R_c, t)$ decreases. Thus, for example, at the specified model parameters, the surface and core fields decrease by a factor of ~5 in 10^7 years (curve *2*) and diffuse to a depth of ~2000 m. The magnetic-flux expulsion from the core virtually ceases by 10^8 years, and s relaxes (curves *3*, *4*) to the state (curve *5*) specified by

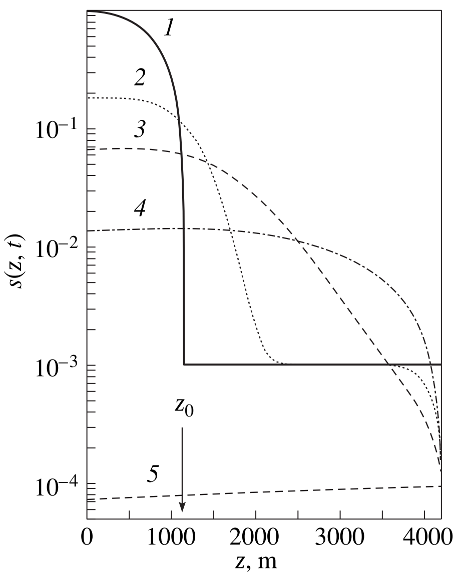

Fig. 4. Initial $s(z, 0)$ profile (*1*), where $z = R - r$ is the depth, and its evolution after (*2*) 10^7, (*3*) 10^8, (*4*) 10^9, and (*5*) 10^{10} years for the magnetic configuration in the right part of Fig. 1. The parameters are $B_{e0} = 10^{12}$ G, $B_{c0} = 10^9$ G, $Q = 1$, and $z_0 = 1146$ m. Here, B_{e0} is the initial surface magnetic field, B_{c0} is the initial core magnetic field, and z_0 is the thickness of the layer where currents flow at $t = 0$.

the boundary conditions $s(R_c, t)$ = const and Eq. (21) and by Eq. (11) for $\partial s/\partial t = 0$.

Figure 5 shows the evolution of the NS surface and core magnetic fields, the NS spin period, and the vortex and fluxoid velocities for $B_{e0} = 10^{12}$ G, $B_{c0} = 10^9$ G and for various impurity parameters Q and $z_0 = 1146$ m (the matter density at this depth is 10^{13} g cm^{-3}). Irrespective of Q, the fluxoid velocity is lower than the velocity of neutron vortexes for $t < 10^4$ years, while for $t > 10^4$ years, both types of vortex lines move at the same Q-dependent velocities. Thus, the effect of neutron vortexes on the fluxoid dynamics in this magnetic configuration is much stronger than that in the previous configuration. This directly follows from Eq. (2): the smaller the B_c (fluxoid density), the larger the f_n (the force exerted per unit fluxoid length by vortexes). The evolution of the NS surface magnetic field is determined by dissipation of the currents initially localized in a surface layer of thickness z_0 and coincides with the evolution of the magnetic field localized only in the NS crust (Urpin and Konenkov 1997) until all crustal currents dissipate. This occurs in $\sim 7 \times 10^8$ years for $Q = 1$ (with $B_c \approx 10^8$ G) and in $\sim 7 \times 10^9$ years for $Q = 0.1$ (with $B_c \approx 3 \times 10^7$ G). Subsequently, the characteristic evolution time of the NS surface magnetic field coincides with the time of magnetic-flux expulsion from the NS core, which exceeds the age of the Universe.

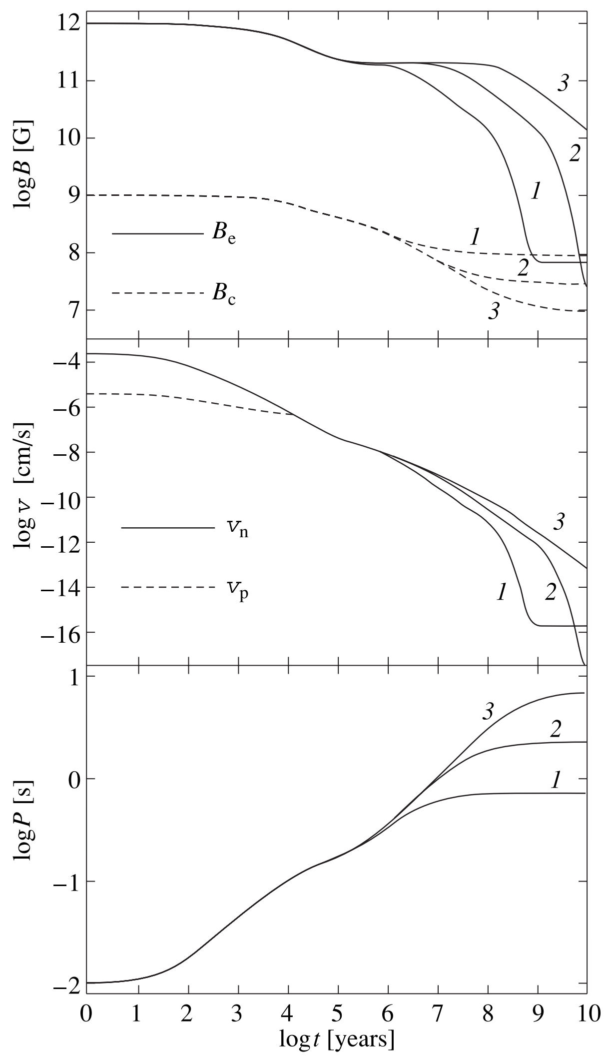

Fig. 5. Evolution of the NS surface and core magnetic fields, the vertex and fluxoid velocities, and the NS spin period for $B_{e0} = 10^{12}$ G, $B_{c0} = 10^9$ G, and $z_0 = 1146$ m. Q = (*1*) 1, (*2*) 0.1, and (*3*) 0.01.

DISCUSSION

We have investigated the expulsion of magnetic flux from a superconducting NS core and its dissipation in a conducting crust. In contrast to previous studies (Ding *et al.* 1993; Jahan-Miri 1999), we performed self-consistent calculations by taking into account the inverse effect of a crustal magnetic line bending on the fluxoid velocity in the core. We showed that, if the bulk of the magnetic flux passes through the NS core, then the buoyancy of fluxoids (Muslimov and Tsygan 1985a, 1985b), rather than their interaction with outwardly moving neutron vortexes, is mainly responsible for the flux expulsion into the crust. The flux expulsion time can be determined from the balance of the buoyancy force and the drag force exerted on the fluxoid roots by the NS crust. The higher the crust conductivity is and the stronger the NS magnetic field is, the longer the time of magnetic-flux expulsion from the core is. Konar and Bhattacharya (1999) calculated the expulsion of magnetic flux from the core by using the hypothesis of the so-called spindown-induced magnetic-field decay; in this hypothesis, neutron vortexes are assumed to be rigidly bound to fluxoids, and both types of vortex lines move at the same velocities throughout the entire NS evolution. We showed that if the bulk of the magnetic flux passes through the NS core, then this hypothesis is untenable.

It follows from the synthesis of populations of single radio pulsars (Bhattacharya *et al.* 1992; Hartmann *et al.* 1996) that the pulsar magnetic fields do not decay in their lifetimes. Our calculations are consistent with this conclusion: for all the values of B_{e0} and Q considered, the surface field decay time exceeds the lifetime of radio pulsars (10^7–10^8 years). For the model parameters to be determined more accurately, the results of calculations must be compared with the fields of NSs that passed the accretion stage in close binary systems. To perform such calculations requires that the effects associated with the crust heating by hot accreting material and with the emerging flow of accreting matter through the crust be included in the model. Both these factors can reduce the dissipation time of the crustal currents and the force F_{crust}.

The effect of neutron vortexes on the fluxoid dynamics is much stronger for the other possible magnetic configuration, when the bulk of the magnetic flux after the birth of a NS passes through its crust. In this case, however, the evolution of the observed NS surface magnetic field during the entire lifetime of the radio pulsar is entirely determined by crustal current dissipation. It coincides with the evolution of the field maintained by the currents that flow only in the NS crust. The evolution of such magnetic configurations was studied, for example, by Urpin and Muslimov (1992) and Urpin and Konenkov (1997).

ACKNOWLEDGMENTS

The work of D. Konenkov was supported by the A. Humboldt Foundation and by the Russian Foundation for Basic Research (project no. 00-02-04011) and the INTAS (grant no. 96-0154).

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Translated by V. Astakhov