Astron. Nachr. / AN **328**, No. 10, 1130 – 1132 (2007) / **DOI** 10.1002/asna.200710864

How can α^2-dynamos generate axisymmetric magnetic fields?

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Received 2007 Sep 25, accepted 2007 Oct 24
Published online 2007 Dec 15

Key words magnetohydrodynamics (MHD) – stars: magnetic fields

Inspired by a recently observed axisymmetric field in a fully convective star we investigate the influence of an anisotropic diffusivity on the dynamo. We find that with reasonable assumptions for the anisotropy of the diffusivity and the α-effect the preference of axisymmetric modes is achieved.

1 Introduction

Due to the anisotropy of the α-effect mean field dynamos for rigidly rotating systems are known to prefer nonaxisymmetric modes for the generated magnetic field (Rüdiger & Elstner 1994). Models for fully convective protostars presented by Chabrier & Küker (2006) lead to highly nonaxisymmetric magnetic solutions. Recently Donati et al. (2006) analyzed a rapidly rotating, very-low-mass, fully convective dwarf star through tomographic imaging from time series of spectropolarimetric data and found the magnetic field axis perfectly aligned with the rotation axis. This result suggests that fully convective stars are able to trigger axisymmetric large scale poloidal fields even without differential rotation. Rüdiger et al. (2003) found axisymmetric (oscillating) solutions for exceptional cases with *thin* convection zones and an concentration of the α-effect around the equator. Also the antisymmetric parts in the α-tensor could lead to preferred axisymmetric solutions. There exists a term, which can be interpreted as a differential rotation. Depending on the latitudinal and radial profiles (which are still uncertain) and the strength compared to the diagonal terms, an axisymmetric solution similar to the $\alpha\Omega$-dynamo would be possible (cf. Rüdiger et al. 2003).

Here we investigate the influence of the anisotropic turbulent *diffusion* with an enhanced component parallel to the rotation axis. This will be expected for rapid rotators. In direct simulations of rotating cylinders with a Roberts flow Tilgner (2004) found the occurrence of axisymmetric solutions under the influence of an increased diffusion in the direction parallel to the rotation axis. Chabrier & Küker (2006) also included the effects of the anisotropic diffusion, but they found predominantly nonaxisymmetric solutions. We shall demonstrate that this will be the case for too strong anisotropies in the α-tensor. Using data for the α-tensor similar to the results of simulations in the solar convection zone we find *axisymmetric* magnetic fields for a reasonable anisotropic diffusivity.

2 Basic equations and the model

The model consists of a turbulent fluid in a spherical shell of inner radius $r_{\rm in} = 0.1R$ and outer radius $r_{\rm out} = R$, which is embedded in a sphere with radius of $1.5R$. A magnetic field is generated in the shell by the α-effect. The turbulent magnetic diffusivity η_0 is constant in the whole sphere but anisotropic with respect to the rotation axis. At the outer radial boundary ($1.5R$) the tangential component of the magnetic field and the vertical component of the electric field are set to zero and a perfect conductor boundary was chosen at $r_{\rm in}$.

The induction equation is

$$\frac{\partial \boldsymbol{B}}{\partial t} = {\rm curl}\, \boldsymbol{\mathcal{E}} \quad (1)$$

with the electromotive force (EMF)

$$\mathcal{E}_i = \alpha_{ij} B_j + \eta_{ijk} \frac{\partial B_j}{\partial x_k}, \quad (2)$$

where $\boldsymbol{B}$ is the magnetic field, η is the turbulent magnetic diffusivity tensor and α is the α-tensor. It represents the interaction of an anisotropic turbulence with a global rotation and a uniform magnetic field. Here we are only interested in the structure of the solution of a pure α^2-dynamo, i.e. the rotation is *assumed* to be uniform. In Rüdiger & Kitchatinov (1993) one finds the overall structure of the α-tensor as

$$\alpha_{im} = -\alpha_1 (\boldsymbol{G}^0 \boldsymbol{\Omega}^0)\delta_{im} - \alpha_2 (G_i^0 \Omega_m^0 + \Omega_i^0 G_m^0) + \alpha_3 (G_m^0 \Omega_i^0 - G_i^0 \Omega_m^0) - \alpha_4 (\boldsymbol{G}^0 \boldsymbol{\Omega}^0) \Omega_i^0 \Omega_m^0 - \gamma \epsilon_{imk} G_k^0. \quad (3)$$

The unit vector $\boldsymbol{\Omega}^0$ denotes the direction of the axis of the global rotation of the turbulence and the radial unit vector $\boldsymbol{G}^0$ denotes its anisotropy.

In almost all papers about α-dynamos the expression (3) is reduced to its first term of the tensorial expression. The inclusion of the remaining parts of the α-tensor reveals the variety of the solutions of the α^2-dynamo and even allows the

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excitation of axisymmetric fields for thin convection zones (Rüdiger et al. 2003). The influence of the large-scale flow pattern is ignored.

It is interesting to consider the antisymmetric parts in the tensor (3). γ_k in the last term plays the role of a radial advection ("pumping") of the magnetic field. On the other hand, if formally a basic rotation with $\boldsymbol{u} = \boldsymbol{\Omega} \times \boldsymbol{x}$ is used for the velocity field then $\mathcal{E}_i = (\Omega_m x_i - \Omega_i x_m) B_m$. It follows that the α_3-term in (3) plays the role of a global differential rotation. More exactly speaking, in cylindrical coordinates (s, ϕ, z) we find $(\alpha_{sz} - \alpha_{zs})/2s$ playing the role of an angular velocity, where the gradient induces magnetic fields.

For a uniform rotation the α-tensor in cylindrical coordinates (s, φ, z) is

$$\alpha = \begin{pmatrix} \alpha_{ss} & \alpha_{s\varphi} & \alpha_{sz} \\ \alpha_{\varphi s} & \alpha_{\varphi\varphi} & \alpha_{\varphi z} \\ \alpha_{zs} & \alpha_{z\varphi} & \alpha_{zz} \end{pmatrix}. \tag{4}$$

In spherical coordinates (r, θ, ϕ) we have

$$\alpha = \begin{pmatrix} \alpha_{rr} & \alpha_{r\theta} & \alpha_{r\phi} \\ \alpha_{\theta r} & \alpha_{\theta\theta} & \alpha_{\theta\phi} \\ \alpha_{\phi r} & \alpha_{\phi\theta} & \alpha_{\phi\phi} \end{pmatrix}. \tag{5}$$

The transformation rules are

$$\begin{aligned}
\alpha_{rr} &= \alpha_{ss} \sin^2\theta + \alpha_{zz} \cos^2\theta + (\alpha_{sz} + \alpha_{zs}) \sin\theta\cos\theta\,, \\
\alpha_{\theta\theta} &= \alpha_{ss} \cos^2\theta + \alpha_{zz} \sin^2\theta - (\alpha_{sz} + \alpha_{zs}) \sin\theta\cos\theta\,, \\
\alpha_{\phi\phi} &= \alpha_{\varphi\varphi}\,, \\
\alpha_{r\theta} &= (\alpha_{ss} - \alpha_{zz}) \sin\theta\cos\theta - \alpha_{sz} \sin^2\theta + \alpha_{zs} \cos^2\theta\,, \\
\alpha_{\theta r} &= (\alpha_{ss} - \alpha_{zz}) \sin\theta\cos\theta + \alpha_{sz} \cos^2\theta - \alpha_{zs} \sin^2\theta\,, \\
\alpha_{r\phi} &= \alpha_{s\varphi} \sin\theta + \alpha_{z\varphi} \cos\theta\,, \\
\alpha_{\phi r} &= \alpha_{\varphi s} \sin\theta + \alpha_{\varphi z} \cos\theta\,, \\
\alpha_{\theta\phi} &= \alpha_{s\varphi} \cos\theta - \alpha_{z\varphi} \sin\theta\,, \\
\alpha_{\phi\theta} &= \alpha_{\varphi s} \cos\theta - \alpha_{\varphi z} \sin\theta\,.
\end{aligned} \tag{6}$$

The inverse transformation is

$$\begin{aligned}
\alpha_{ss} &= \alpha_{rr} \sin^2\theta + \alpha_{\theta\theta} \cos^2\theta + (\alpha_{r\theta} + \alpha_{\theta r}) \sin\theta\cos\theta\,, \\
\alpha_{\varphi\varphi} &= \alpha_{\phi\phi}\,, \\
\alpha_{zz} &= \alpha_{rr} \cos^2\theta + \alpha_{\theta\theta} \sin^2\theta - (\alpha_{r\theta} + \alpha_{\theta r}) \sin\theta\cos\theta\,, \\
\alpha_{sz} &= (\alpha_{rr} - \alpha_{\theta\theta}) \sin\theta\cos\theta + \alpha_{\theta r} \cos^2\theta - \alpha_{r\theta} \sin^2\theta\,, \\
\alpha_{zs} &= (\alpha_{rr} - \alpha_{\theta\theta}) \sin\theta\cos\theta + \alpha_{r\theta} \cos^2\theta - \alpha_{\theta r} \sin^2\theta\,, \\
\alpha_{s\varphi} &= \alpha_{r\phi} \sin\theta + \alpha_{\theta\phi} \cos\theta\,, \\
\alpha_{\varphi s} &= \alpha_{\phi r} \sin\theta + \alpha_{\phi\theta} \cos\theta\,, \\
\alpha_{\varphi z} &= \alpha_{\phi r} \cos\theta - \alpha_{\phi\theta} \sin\theta\,, \\
\alpha_{z\varphi} &= \alpha_{r\phi} \cos\theta - \alpha_{\theta\phi} \sin\theta\,.
\end{aligned} \tag{7}$$

The ratio

$$\hat{\alpha}_z = \frac{\alpha_{zz}}{\alpha_{\varphi\varphi}} \tag{8}$$

will be of particular relevance for the resulting solutions. For rapidly rotating objects its value will be much smaller than unity. For a uniform diffusivity these small values of $\hat{\alpha}_z$ lead to nonaxisymmetric fields as preferred modes.

Ossendrijver et al. (2001) presented simulations for all the components of the α-tensor. As an example in Fig. 1 the radial behaviour of the main diagonal components of the α-tensor in Cartesian coordinates is given corresponding to the cylindrical components $\alpha_{\varphi\varphi} = \alpha_{yy}$ and $\alpha_{zz} = \alpha_{zz}$. Note the negative sign of α_{yy} (in the southern hemisphere!) throughout the convection zone. Only in the overshoot region the sign changes.

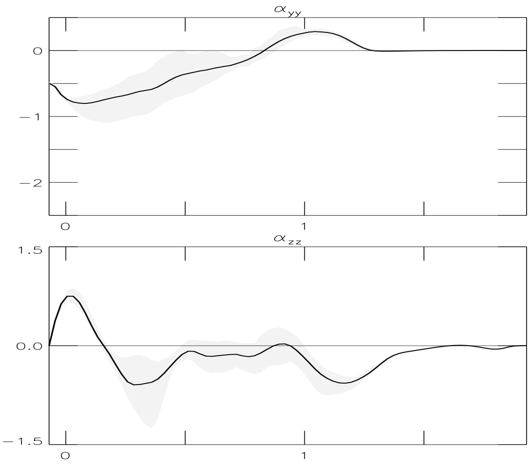

Fig. 1 Numerical results for the Cartesian α-tensor components $\alpha_{yy} = \alpha_{\phi\phi}$ and $\alpha_{zz} = \alpha_{rr}$, measured in units of $0.01\sqrt{dg}$, as a function of depth in units of d from Ossendrijver et al. (2001) for the case of a box located at the south pole (run A00). The simulation domain consists of a thin cooling layer ($z < 0$), a convectively unstable layer ($0 < z < d = 1$), and a stably stratified layer with overshooting convection ($z > d$). The Coriolis number of the run is about 2.4, which is in the appropriate range for the bottom of the solar convection zone. For the other parameters and for a detailed description of the model we refer to Ossendrijver et al. (2001). The black curves are spatial and temporal averages; the shaded areas provide an error indication. Note, that the $\alpha_{\phi\phi}$ does not vanish at the pole.

For the α_{zz}-component the sign changes already in the convection zone. We reflect this property in our inhomogeneous model, where

$$\begin{aligned}
\alpha_{\varphi\varphi} &= \alpha_{\varphi 0} \sin(\pi(r - r_{in})/d) \cos\theta\,, \\
\alpha_{zz} &= \alpha_{z0} \sin(2\pi(r - r_{in})/d) \cos\theta\,,
\end{aligned} \tag{9}$$

with d as the thickness of the convection zone (Fig. 2). We also calculated critical dynamo numbers for the basic modes with radial independent α-profiles.

The magnetic diffusivity tensor for a rotating fluid is

$$\eta_{ijk} = \epsilon_{ijm} \left(\eta_{\mathrm{T}} \delta_{km} + \eta_{||} \frac{\Omega_k \Omega_m}{\Omega^2} \right) \tag{10}$$

(Kitchatinov et al. 1994). For slow rotators $\eta_{||}$ vanishes and one has an isotropic turbulent diffusion. For rapid rotation one finds $\eta_{||} = \eta_{\mathrm{T}}$ and reduced non-diffusive terms. E.g. a Coriolis number $\Omega^* = 2\tau_{\mathrm{corr}}\Omega = 10$ leads to

$$\eta_{\mathrm{T}} = \eta_{||} = 0.1\eta_0, \tag{11}$$

with $\eta_0 = \tau_{\mathrm{corr}}\langle u'^2\rangle/3$, while $\Omega^* = 1$ gives

$$\eta_{\mathrm{T}} = 0.8\eta_0 \qquad \text{and} \qquad \eta_{||} = 0.1\eta_0. \tag{12}$$

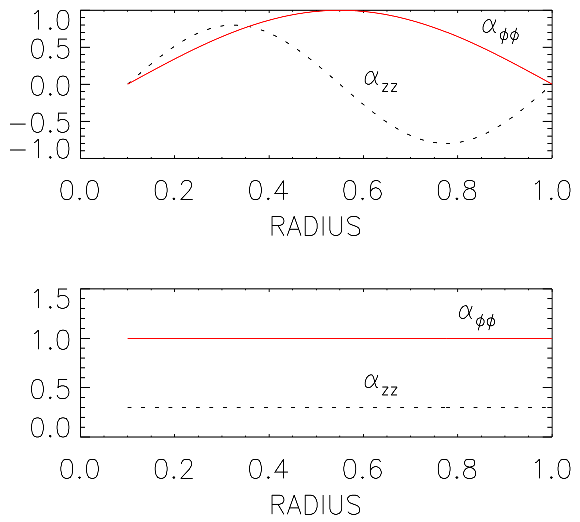

Fig. 2 (online colour at: www.an-journal.org) Normalized α-profiles for the inhomogeneous (*top*) and homogenous (*bottom*) case.

Inspecting the EMF for cylindrical coordinates, i.e.

$$\mathcal{E}_s = \alpha_{ss} B_s + \eta_{||} \frac{\partial}{\partial z} B_\varphi - \eta_\mathrm{T} \left(\frac{\partial}{s \partial \varphi} B_z - \frac{\partial}{\partial z} B_\varphi \right),$$
$$\mathcal{E}_\varphi = \alpha_{\varphi\varphi} B_\varphi - \eta_{||} \frac{\partial}{\partial z} B_s - \eta_\mathrm{T} \left(\frac{\partial}{\partial z} B_s - \frac{\partial}{\partial s} B_z \right), \quad (13)$$
$$\mathcal{E}_z = \alpha_{zz} B_z - \eta_\mathrm{T} \frac{1}{s} \left(\frac{\partial}{\partial s} s B_\varphi - \frac{\partial}{\partial \varphi} B_s \right),$$

we see that $\eta_{||}$ enhances the diffusivity in the radial and azimuthal components only. Therefore the effect of a reduced α_{zz} is weakened and the system becomes more similar to the isotropic case for α and η. We expect a preferred axisymmetric solution for sufficiently strong $\eta_{||}$. For a total vanishing α_{zz} the system remains anisotropic in α. Here we have no influence of the anisotropic diffusion regarding the excitation of axisymmetric modes. This will be also the case for negative $\hat{\alpha}_z$. We define the dimensionless dynamo number $C_\alpha = \alpha_{\varphi 0} R / \eta_\mathrm{T}$.

3 Results

Both the models with homogeneous $\alpha_{zz} = 0$ and $\alpha_{zz} = -\alpha_{\varphi\varphi}$ have the smallest critical dynamo number C_α for the nonaxisymmetric mode independent of the diffusion coefficient $\eta_{||}$. Only the value for C_α increases with the additional diffusivity parallel to the rotation axis. This strong anisotropy for the α-tensor was used in all former models for fully convective stars with the consequence of a nonaxisymmetric field geometry in rigidly rotating stars. For a homogeneous profile of α with a positive $\hat{\alpha}_z = 0.3$ we find the preferred axisymmetric mode for $\eta_{||} = 2\,\eta_\mathrm{T}$. The axisymmetric solution appears already for $\eta_{||} = 0.5\eta_\mathrm{T}$ for our inhomogeneous α-profile (Table 2).

Following Tilgner (2004) we have shown that for rapidly rotating stars the anisotropy of the diffusivity leads to axisymmetric solutions, provided that there is still a substantial part of a positive $\hat{\alpha}_z$. In this case the enhanced diffusion parallel to the rotation axis weakens the anisotropy of the α-tensor. This seems the most promising effect for an explanation of axisymmetric fields in rapidly rotating fully convective stars with negligible differential rotation. These solutions are stationary, at least if the α-effect has a dominating region, where it does not change its sign. Other solutions like the oscillating axisymmetric dynamos for thin convective shells are not applicable to fully convective stars. Also the effect of the antisymmetric parts of the α-tensor are probably not strong enough to bring the system in an $\alpha\Omega$-type dynamo (cf. Rüdiger et al. 2003). It still remains an open question, however, under which conditions mixed modes exist.

Table 1 Critical dynamo number for axisymmetric dipole (A0), quadrupole (S0) and nonaxisymmetric modes A1 and S1 for homogeneous $\alpha_{zz} = 0$ (*upper part*) and $\alpha_{zz} = -\alpha_{\varphi\varphi}$ (*lower part*) and with $R_0 = 0.5$, $\eta_{||} = h\eta_\mathrm{T}$. The lowest values are marked bold.

h	**A0**	**S0**	**A1**	**S1**
0	15.36	15.05	11.66	**10.72**
0.2	17.54	17.08	13.61	**12.35**
0.5	20.5	20.0	16.51	**14.74**
2	47.21	44.34	41.40	**36.48**
0	18.91	19.48	13.62	**12.56**
0.2	21.91	22.74	16.16	**14.71**
0.5	26.36	27.6	19.95	**17.93**
2	33.61	33.64	29.83	**27.20**

Table 2 Critical dynamo number for different basic dynamo modes for homogeneous $\alpha_{zz} = 0.3\alpha_{\varphi\varphi}$ (*upper part*) and inhomogeneous $\alpha_{z0} = 0.8\alpha_{\varphi 0}$ (*lower part*) and with $R_0 = 0.1$, $\eta_{||} = h\eta_\mathrm{T}$.

h	**A0**	**S0**	**A1**	**S1**
0	10.88	11.19	9.48	**8.61**
0.5	12.66	13.31	12.48	**11.26**
1	14.08	15.04	14.80	**13.66**
2	**16.39**	17.83	17.96	18.00
0	13.68	13.77	13.63	**13.40**
0.5	**16.51**	16.63	18.46	18.16
1	**18.92**	19.06	22.16	21.98
2	**22.97**	23.17	26.72	26.79

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