Astron. Nachr. / AN **328**, No. 10, 1037 – 1039 (2007) / **DOI** 10.1002/asna.200710876

The differential rotation of ε Eri from MOST data*

H.-E. Fröhlich**

Astrophysikalisches Institut Potsdam, An der Sternwarte 16, D-14482 Potsdam, Germany

Received 2007 Nov 2, accepted 2007 Nov 5
Published online 2007 Dec 15

Key words methods: data analysis – methods: statistical – stars: individual (ϵ Eri) – stars: rotation – starspots

From high-precision MOST photometry spanning 35 days the existence of two spots rotating with slightly differing periods is confirmed. From the marginal probability distribution of the derived differential rotation parameter k its *expectation* value as well as confidence limits are computed directly from the data. The result depends on the assumed range in inclination i, not on the shape of the prior distributions. Two cases have been considered: (a) The priors for angles, inclination i of the star and spot latitudes $\beta_{1,2}$, are assumed to be constant over i, β_1, and β_2; (b) the priors are assumed to be constant over $\cos i$, $\sin \beta_1$, and $\sin \beta_2$. In both cases the full range of inclination is considered: $0° \leq i \leq 90°$. Scale-free parameters, i. e. periods and spot areas (in case of small spots) are taken logarithmically. Irrespective of the shape of the prior, k is restricted to $0.03 \leq k \leq 0.10$ ($1\,\sigma$ limits). The inclination i of the star is photometrically ill-defined.

1 Introduction

In late-type stars with their deep convection zones non-uniform rotation is driven by the action of the Coriolis force on that convective turbulence (cf. Kitchatinov & Rüdiger 1993). Now quantitative models of differential rotation for the Sun and solar-like stars are available (Rüdiger & Kitchatinov 2005; Küker & Rüdiger 2005; Küker & Rüdiger 2007) which should be compared with real stars.

The outcome of these theoretical efforts is a lapping time which is for a given main-sequence star nearly independent of its rotation period (in the case of the Sun, a G2 dwarf, roughly 100 days).

Precision photometry of a spotted star with spots differing in latitude allows a direct measurement of the differential rotation parameter k. It parameterizes the surface rotation. With $P_{\rm eq}$ denoting the equatorial rotation period that at latitude β is $P_\beta = P_{\rm eq}/(1 - k\,\sin^2\beta)$.

Here the MOST data (Croll et al. 2006; Croll 2006) of the star ε Eri have been reanalyzed in a Bayesian framework. The motivation was to get realistic error estimates by computing k's marginal distribution and to find out, how it depends on the chosen prior. In the essence a Bayesian approach explores the whole likelihood mountain in the N-dimensional parameter space. It looks not only to the most probable set of parameter values, but provides *expectation* values. Moreover, from a marginal distribution reliable confidence limits can be easily derived.

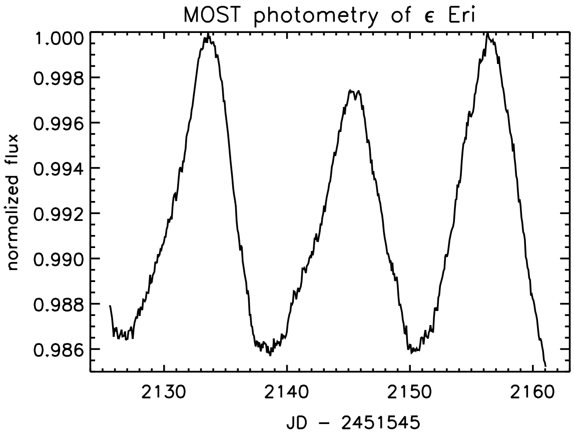

Fig. 1 The ε Eri light curve with an obvious trend removed.

The nearby K2 dwarf is young (≤ 1 Gyr), rotates twice as fast as the Sun, and shows strong chromospheric activity (cf. Biazzo et al. 2007).

2 The MOST data

The MOST photometric satellite (Walker et al. 2003) observed three consecutive rotations of ε Eri in 2005. The data consist of 492 data points, one point per orbit of the satellite. The point-to-point precision of the data is 50 ppm rms (i.e. $\pm\,0.000\,05$ mag). The rectified light curve (Fig. 1) spanning 35 days shows an overall variation of a hundredth of a magnitude.

* Based on data from the MOST satellite, a Canadian Space Agency mission, jointly operated by Dynacon Inc., the University of Toronto Institute for Aerospace Studies and the University of British Columbia, with the assistance of the University of Vienna.

** Corresponding author: HEFroehlich@aip.de

3 A Bayesian data analysis

In the case of two circular spots the light curve is determined by nine parameters: two periods, two epochs, two latitudes, two areas, and the inclination of the star. The parameters specifying limb darkening and spot rest intensity are assumed to be given (cf. Croll et al. 2006).

There are nuisance parameters: an offset in the photometric zero point and a long term trend in the data. With these two parameters introduced shifting the light curve vertically and even adding a trend does not alter the results. The flux error is considered a Gaussian with unknown variance. Integrating away that error σ, assuming Jeffreys $1/\sigma$ prior, sounds strange, but perhaps it is not so bad an assumption and it leads to quite a simple formula for the likelihood. The likelihood function used is therefore a *mean* likelihood with respect to these three parameters. One can say the method itself determines offset, slope and measurement error.

The result, especially the star's inclination i, depends on the prior distribution functions. In the following it is assumed that rotational frequencies as well as spot areas (if small compared with the star), i. e. parameters missing a characteristic scale, have constant priors if taken logarithmically. Two cases are considered: In Case A the prior is assumed constant over the inclination i and the spot latitudes β_1, and β_2 as in the Croll (2006) paper. In Case B it is assumed constant over $\cos i$, $\sin \beta_1$, and $\sin \beta_2$, respectively. An inclination prior constant over $\cos i$ means that nothing is a priori known about the orientation of the rotational axis. A latitude prior constant over $\sin \beta$ takes into account that there are more possibilities to locate by chance a spot near the equator than near the poles.

In order to explore the likelihood mountain over a nine-dimensional parameter space the Markov chain Monte Carlo (MCMC) method (cf. Press et al., 2007) has been applied. The results should be therefore best compared with the analysis of the MOST data by Croll (2006), who has employed that MCMC technique.

A set of 64 Markov chains was generated. Each chain has performed 10^7 steps. After a burn-in period of 1000 steps every 1000th successful step has been recorded to suppress the correlation between successive steps. For modeling light curves Budding's star-spot model (1977) has been used.

4 Results

Expectation values and modes as well as one-σ confidence limits are presented in Table 1, augmented by the solution of Croll (2006; his "Wide Prior" Case). The Case-B probability distributions for the parameters k and i are shown in Figs. 2 and 3, respectively. Especially the inclination parameter i proves ill-defined by photometry alone. There is no strong correlation between i and k (Fig. 4).

The differences between the two cases A and B are marginal, i. e. the outcome is rather insensitive to the *shape* of the prior. Most important is the restriction of the inclination i to values between 15° and 40° in the "Wide Prior" Case of the Croll MCMC analysis[1]. Here the whole range, $0^\circ \le i \le 90^\circ$, is taken into account. It includes a second peak in the likelihood mountain: $i \approx 72^\circ, \beta_1 \approx 61^\circ, \beta_2 \approx 73^\circ$. This high-$i$ solution is even more probable, by a factor of 4.6, then the *best* low-i solution ($i \approx 24^\circ, \beta_1 \approx 14^\circ, \beta_2 \approx 25^\circ$). Contrary to the Croll solution the second spot is always visible.

By fitting spectroscopic measurements (2000/2001) to a spot/plage model Biazzo et al. (2007) find two spots but larger than ours. By the way, this could be due to the lower temperature contrast between spot and photosphere these

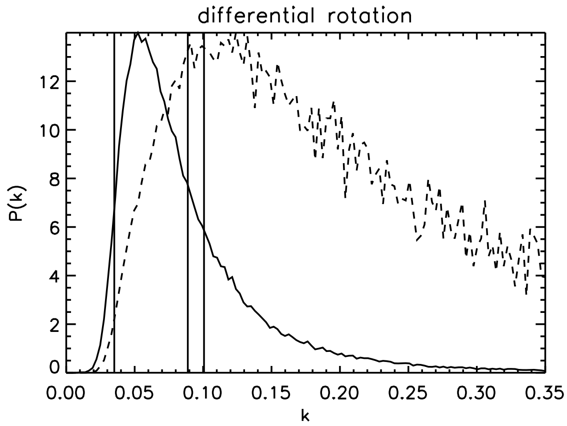

Fig. 2 The marginal distribution for the differential rotation parameter k (Case B). Vertical lines denote expectation value and the 68 per cent confidence region. Additionally to the marginalized distribution the run of *mean* likelihood (dashed) is given, providing an impression how the goodness of fit varies with k. The mismatch hints at the non-Gaussianity of the likelihood mountain (cf. Lewis & Bridle 2002).

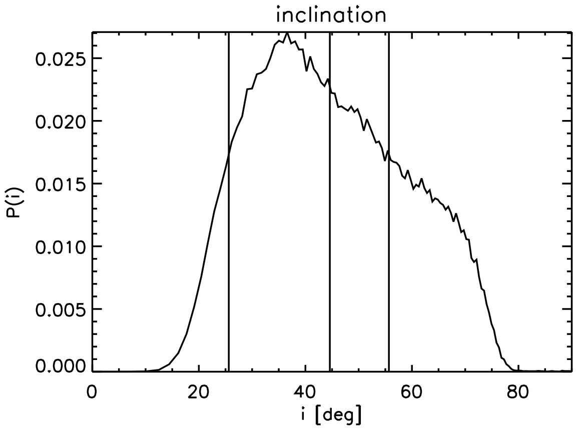

Fig. 3 The marginal distribution for the inclination i (Case B). Vertical lines denote expectation value and the 68 per cent confidence region. The high-i solution is suppressed by the chosen prior.

[1] The marginalized likelihood of i, the last plot of his Fig. 3, indicates that inclinations beyond 40° are not ruled out.

Table 1 Results: Given are the *expectation* values, the modes (Case B) and 1σ errors. Periods P are in days, epochs are with regard to E = HJD – 2451545. Note that for Case A i and β_1 are both bi-modal. To give only one interval, for once the 90% confidence region is chosen.

Parameter		Case A	Case B		Croll 2006
		Mean	Mean	Mode	Mode
		0°	$\leq\ i\ \leq$	90°	$15^{\circ} \leq i \leq 40^{\circ}$
Differential rotation	k	$0.088^{+0.011}_{-0.055}$	$0.089^{+0.012}_{-0.054}$	0.053	$0.058^{+0.084}_{-0.018}$
Inclination	i	$46\overset{\circ}{.}6^{+26}_{-23}$	$44\overset{\circ}{.}6^{+11}_{-19}$	36°	$25\overset{\circ}{.}9^{+12.6}_{-6.7}$
1st latitude	β_1	$35\overset{\circ}{.}1^{+25}_{-23}$	$33\overset{\circ}{.}0^{+9.1}_{-19}$	25°	$16\overset{\circ}{.}5^{+7.0}_{-5.0}$
2nd latitude	β_2	$51\overset{\circ}{.}2^{+21}_{-12}$	$48\overset{\circ}{.}8^{+18}_{-14}$	48°	$24\overset{\circ}{.}8^{+15.1}_{-3.5}$
1st radius	γ_1	$5\overset{\circ}{.}71^{+0.25}_{-1.1}$	$5\overset{\circ}{.}62^{+0.20}_{-0.98}$	4.9°	$5\overset{\circ}{.}3^{+2.1}_{-0.2}$
2nd radius	γ_2	$7\overset{\circ}{.}78^{+0.15}_{-1.6}$	$7\overset{\circ}{.}49^{+0.07}_{-1.3}$	6.6°	$6\overset{\circ}{.}8^{+1.4}_{-0.3}$
1st period	P_1	$11.348^{+0.037}_{-0.036}$	$11.349^{+0.037}_{-0.034}$	11.35	$11.35^{+0.03}_{-0.03}$
2nd period	P_2	$11.553^{+0.020}_{-0.020}$	$11.554^{+0.019}_{-0.020}$	11.555	$11.55^{+0.02}_{-0.02}$
1st epoch	E_1	$2130.43^{+0.20}_{-0.21}$	$2130.41^{+0.19}_{-0.22}$	2130.37	$2130.43^{+0.20}_{-0.21}$
2nd epoch	E_2	$2126.47^{+0.11}_{-0.11}$	$2126.46^{+0.11}_{-0.12}$	2126.46	$2126.47^{+0.11}_{-0.12}$

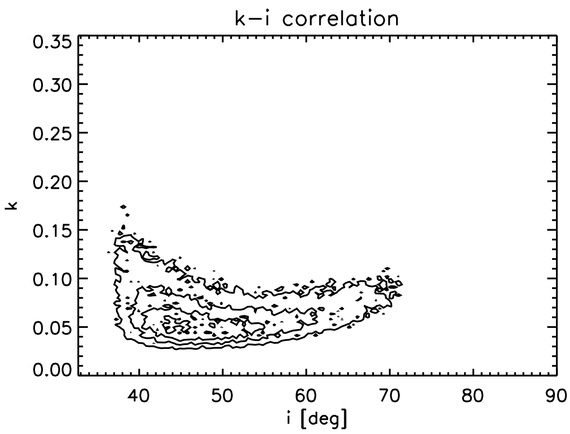

Fig. 4 The parameters of differential rotation k and inclination i are somewhat correlated (Case B).

authors have found. Perhaps the spots are persistent. The stated spot latitudes, $\beta_1 \approx 21^{\circ}$ and $\beta_2 \approx 48^{\circ}$, are within the uncertainties of our estimate.

Equatorial rotational period ($P_{\rm eq} = 11.2$ d) and radius ($0.72\,{\rm R}_{\odot}$) of ε Eri are well known. So in principle spectroscopic determinations of the projected rotational velocity may restrict the inclination i. Unfortunately, ε Eri rotates slowly. Saar & Osten (1997) find $v \sin i \approx 1.7 \pm 0.3$ km/s. This leads to a $\sin i = 0.5 \pm 0.3$ hinting at the low-i solution, but is too uncertain as to constrain the range of i values very much. A recent compilation by Valenti & Fischer (2005) gives $v \sin i \approx 2.4 \pm 0.5$ km/s. This favours an inclination close to 50°.

One should note that the planetary companion (Hatzes et al. 2000) as well as a 130 AU dust ring show both an inclination distinctly below 30°, namely $26\overset{\circ}{.}2$ (Benedict et al. 2006) and $\approx 25^{\circ}$ (Greaves et al. 2005), respectively.

5 Conclusions

As expected, with $0.03 \leq k \leq 0.10$, the estimated differential rotation parameter k proves smaller than that of the Sun ($k_{\odot} \approx 0.2$). The horizontal shear is $0.017 \leq \delta\Omega \leq 0.056$ rad/d, the lapping time 130 d. An independent estimate of the star's inclination would exclude either the low-i solution or the high-i one and would help to constrain k even more.

Acknowledgements. The author thanks the MOST team for the ε Eri data and W.W. Weiss from Vienna for valuable discussions.

References

Benedict, G.F., McArthur, B., Gatewood, G., et al.: 2006, AJ 132, 2206
Biazzo, K., Frasca, A., Henry, G.W., Catalano, S., Marilli, E.: 2007, ApJ 656, 474
Budding, E.: 1977, Ap&SS 48, 207
Croll, B., Walker, G.A.H., Kuschnig, R., et al.: 2006, ApJ 648, 607
Croll, B.: 2006, PASP 118, 1351
Greaves, J.S., Holland, W.S., Wyatt, M.C., et al.: 2005, ApJ 619, L187
Hatzes, A.P., Cochran, W.D., McArthur, B., et al.: 2000 ApJ 544, L145
Kitchatinov, L.L., Rüdiger, G.: 1993, A&A 276, 96
Küker, M., Rüdiger, G.: 2005, AN 326, 265
Küker, M., Rüdiger, G.: 2007, AN 328, 1050
Lewis, A., Bridle, S.: 2002, PhRvD 66, 103511
Rüdiger, G., Kitchatinov, L.L.: 2005, AN 326, 379
Press, W.H., Teukolsky, S.A., Vetterling, W.T., Flannery, B.P.: 2007, *Numerical Recipes*, 3rd Edition, Cambridge University Press
Saar, S.H., Osten, R.A.: 1997, MNRAS 284, 803
Valenti, J.A., Fischer, D.A.: 2005, ApJS 159, 141
Walker, G., Matthews, J., Kuschnig, R., et al.: 2003, PASP 115, 1023