

Modelling the differential rotation of F stars

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Received 2007 Aug 31, accepted 2007 Oct 16

Published online 2007 Dec 15

Key words stars: interiors – stars: rotation

We model stellar differential rotation based on the mean-field theory of fluid dynamics. DR is mainly driven by Reynolds stress, which is anisotropic and has a non-diffusive component because the Coriolis force affects the convection pattern. Likewise, the convective heat transport is not strictly radial but slightly tilted towards the rotation axis, causing the polar caps to be slightly warmer than the equator. This drives a flow opposite to that caused by differential rotation and so allows the system to avoid the Taylor-Proudman state. Our model reproduces the rotation pattern in the solar convection zone and allows predictions for other stars with outer convection zones. The surface shear turns out to depend mainly on the spectral type and only weakly on the rotation rate. We present results for stars of spectral type F which show signs of very strong differential rotation in some cases. Stars just below the mass limit for outer convection zones have shallow convection zones with short convective turnover times. We find solar-type rotation and meridional flow patterns at much shorter rotation periods and horizontal shear much larger than on the solar surface, in agreement with recent observations.

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1 Introduction

While the differential rotation (DR) of the solar surface has been known for a long time, stellar DR has only recently become accessible to observation. It can be derived from stellar butterfly diagrams (Henry et al. 1995; Donahue et al. 1996; Messina & Guinan 2003), Doppler imaging (Strassmeier 2004), or the Fourier transform method (Reiners & Schmitt 2002). The latter has the advantage of allowing the measurement of the surface DR from a single exposure and does not depend on large spots or activity. Like Doppler imaging, it requires large values of $v \sin i$, however.

Surface rotation is usually assumed to follow a law of the form

$$\Omega = \Omega_{\text{eq}}(1 - \alpha \cos^2 \theta), \quad (1)$$

where Ω_{eq} is the angular velocity at the equator. The surface shear

$$\delta\Omega = \Omega_{\text{eq}} - \Omega_{\text{pole}}, \quad (2)$$

where Ω_{pole} is the rotation rate at the poles, can alternatively be expressed in term of the lapping time, $t_{\text{lap}} = 2\pi/\delta\Omega$. For the Sun, the lapping time is about 100 d.

Barnes et al. (2005) investigated the dependence of stellar differential rotation on rotation rate and effective temperature. Using the combined samples from previous work they found no significant dependence on the stellar rotation rate but a strong dependence on the effective temperature, namely

$$\delta\Omega \propto T_{\text{eff}}^{8.92 \pm 0.31}. \quad (3)$$

Reiners (2006) added observations of A- and F-type stars using the Fourier transform method and found the relation (3) confirmed. Though the scatter around the $T^{8.92}$ law is large, with the Sun lying way off the line, and the low temperature end is poorly covered (Fig. 5 in Reiners 2005 shows only three stars with temperatures below 5000 K), the data shows a clear trend of differential rotation increasing with increasing temperature, especially close to the limiting mass for outer convection zones.

Küker & Rüdiger (2005a) modelled the differential rotation of an F8 star and found a solar-type rotation law but stronger surface shear. In Küker & Rüdiger (2005b) the authors compared the DR resulting from the same model as in Küker & Rüdiger (2005a) for four types of star, namely the F8 star, a solar-type star, a K5 star and an M dwarf. The surface shear was found to depend weakly on the rotation rate and moderately on temperature, though the dependence is weaker than $T^{8.92}$.

In this paper we model the rotation of MS stars with geometrically thin outer convection zones. These objects lie just below the mass limit for outer convection zones and their spectral type is early F. Some of the stars studied by Reiners & Royer (2004) show surface shear up to an order of magnitude larger than we have so far found theoretically. These stars rotate faster and are of earlier spectral type than the stars we have modelled so far. The objective is therefore to find out whether our model can reproduce the observed strong surface shear if applied to the same spectral type and rotation period.

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2 Model

Our model is a variant of the one used for the Sun in Küker & Stix (2001). Its present form is described in more detail in Bonanno et al. (2006). To determine the differential rotation we solve the Reynolds equation,

$$\rho \left[\frac{\partial \bar{\mathbf{u}}}{\partial t} + (\bar{\mathbf{u}} \cdot \nabla) \bar{\mathbf{u}} \right] = -\nabla \cdot \rho \mathbf{Q} - \nabla P + \rho \mathbf{g}, \quad (4)$$

for the stellar convection zone assuming an axisymmetric gas flow. In Eq. (4) $\bar{\mathbf{u}}$ is the mean velocity, P the gas pressure, ρ the gas density, and \mathbf{g} the gravity. $Q_{ij} = \langle u'_i u'_j \rangle$ is the correlation tensor of the velocity fluctuations. The heat transport is described by the equation

$$\rho T \frac{\partial \delta s}{\partial t} = -\nabla \cdot (\mathbf{F}^{\text{conv}} + \mathbf{F}^{\text{rad}} + \rho T \bar{\mathbf{u}} \delta s), \quad (5)$$

where δs is defined by

$$\delta s = \bar{s} - s_0, \quad \delta s \ll \bar{s}, s_0 \quad (6)$$

with the constant s_0 . The convective heat transport is given by

$$F_i^{\text{conv}} = \rho c_p \langle u'_i T' \rangle = \rho c_p \chi_{ij} \beta_j, \quad (7)$$

with

$$\beta = \frac{\mathbf{g}}{c_p} - \nabla T = -\nabla \delta T. \quad (8)$$

The quantity δs describes the deviation from a state of constant specific entropy, i.e. from adiabatic stratification. The relation between the temperature and entropy perturbations is

$$\delta T = \frac{T}{c_p} \delta s. \quad (9)$$

The stratifications of density and temperature are taken from stellar models computed with a stellar evolution code (Sienkiewicz et al. 1988). As input parameters for the latter we chose $X = 0.715$, $Z = 0.0185$, and $\alpha_{\text{MLT}} = 1.6$. This reproduces the solar luminosity at the age of 4.6 Gyrs with a sufficient accuracy for our purpose. The output from the stellar evolution code defines the background state for modelling differential rotation, meridional flow, and convective heat transport using mean-field hydrodynamics. As we treat the convection zone only, we assume that the ground state has constant specific entropy. For thin convection zones, we can also assume that the total heat flux is the same at the lower and upper boundaries. As the unperturbed state already includes convection, we split δs into its horizontally-averaged part δs_1 (which does the whole net radial transport) and the remainder, δs_2 , and replace δs with δs_2 in the advection term in Eq. (5) in order to ensure that the perturbation δs_2 does not take over the convective heat transport.

3 Results

We have carried out computations for a series of stellar masses ranging from one to 1.5 solar masses. As mass increases, so does the stellar radius but the relative depth

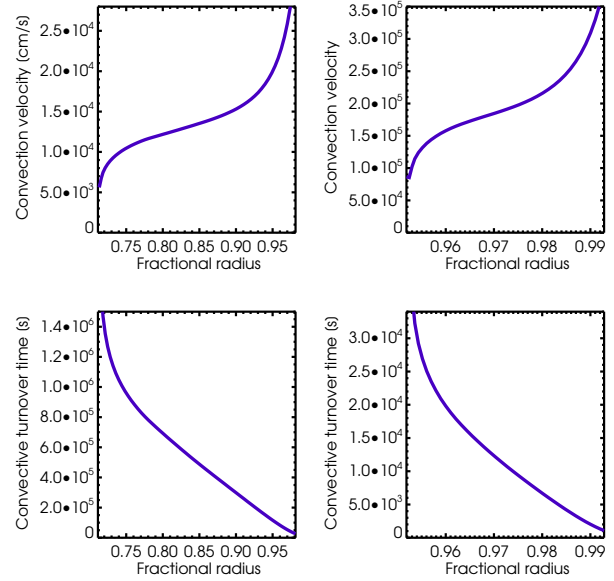


Fig. 1 (online colour at: www.an-journal.org) Convection velocity (*top*) and convective turnover time (*bottom*) vs. the fractional radius in the convection zone. *Left*: Sun, *right*: a $1.4 M_{\odot}$ main sequence star.

of the convection zone decreases. As the latter effect is stronger the total depth of the convection zone decreases. At a mass of 1.5 solar masses it has become as shallow as one percent of the total stellar radius.

With decreasing depth of the convection zone the time and length scales of the convective motions become shorter. Figure 1 shows the convection velocity and the convective turnover time vs. the radius for the Sun and star with 1.4 solar masses. With almost 30 percent of the outer radius the solar convection zone is much deeper than that of the $1.4 M_{\odot}$ star, which covers only five percent of the stellar radius. Taking into account the larger radius of the latter (1.09×10^{11} cm) the convection zone of the $1.4 M_{\odot}$ star amounts to 54 500 km, about 1/4 of the corresponding solar value. The convection velocity of the $1.4 M_{\odot}$ star is about one order of magnitude larger than that of the Sun. This is in part because the depth of the convection zone is so small, making it roughly the equivalent of the solar supergranulation layer, and the greater luminosity of the star ($4.4 L_{\odot}$.) The convective turnover times of the two stars differ by a similar factor, that of the Sun being larger by more than an order of magnitude. The mixing lengths are roughly the same at the same (fractional) depth. The Sun reaches larger values in the lower part of its convection zone because the latter is much deeper than the CZ of the $1.4 M_{\odot}$ star.

The short convective turnover time in the $1.4 M_{\odot}$ star has drastic consequences for its differential rotation and meridional flow. Convection is strongly affected by the Coriolis force if the Coriolis number is greater than or equal one:

$$\Omega^* = 2\tau\Omega \geq 1, \quad (10)$$

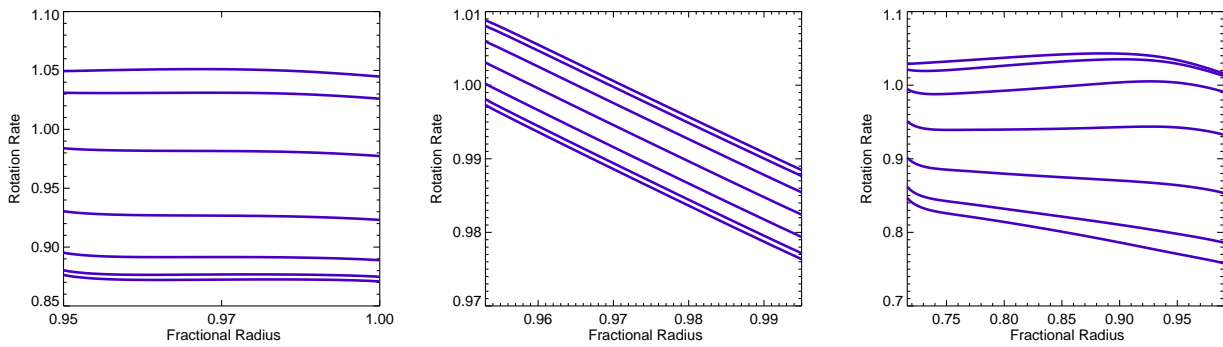


Fig. 2 (online colour at: www.an-journal.org) The normalised rotation rate vs. radius for 0, 15, 30, 45, 60, 75, and 90° latitude, from top to Bottom. *Left*: the 1.4 M_{\odot} star, $P = 1$ d. *Center*: the 1.4 M_{\odot} star, $P = 27$ d. *Right*: solar-type star, $P = 27$ d.

where τ is the convective turnover time. The Coriolis number is closely related to the Rossby number, $Ro = P_{rot}/\tau$, and sometimes also referred to as the inverse Rossby number but strictly speaking, the relation between Coriolis and Rossby number is $\Omega^* = 4\pi/Ro$.

While their magnitudes are determined by stratification and luminosity, the structure of the Reynolds stress and heat diffusivity tensor depend on the Coriolis number only. The λ -effect will only drive a noticeable surface shear if $\Omega^* \approx 1$ or larger. With τ being much smaller than in the bulk of the solar CZ, the 1.4 M_{\odot} star must rotate much faster than the Sun to reach the same Coriolis number, as the following comparison shows. Halfway down from the top to the bottom of the solar convection zone, at $x = 0.86$, the convective turnover time takes a value of 4.4×10^5 s. With a rotation period of 27 d the Coriolis number at that radius is 2.4. The 1.4 M_{\odot} star has a convective turnover time of 10^4 s at $x = 0.975$. With a rotation period of 1 d this yields a value of 1.5 for Ω^* .

The left part of Figure 2 shows the rotation pattern of the 1.4 M_{\odot} star rotating with a period of 1 d. The rotation rate is mainly a function of latitude with hardly any dependence on radius. With a value of 0.167, the horizontal surface shear, $\delta\Omega/\Omega_0$, where Ω_0 is the average angular velocity, is of the same order as the corresponding value for the Sun and the general pattern is very similar, too, namely the equator rotating faster than the poles. The absolute value of the surface DR, however, is much larger for the 1.4 M_{\odot} star than for the Sun. With $\Omega_0 = 7.27 \times 10^{-5} \text{ s}^{-1}$, we have $\delta\Omega = 1.05 \text{ rad/d}$, about 18 times the solar value of 0.06. The center part of the figure shows the rotation profile for the 1.4 M_{\odot} star rotating with a period of 27 d. The variation of the rotation rate with latitude is weak, but there is a pronounced vertical shear. The rotation rate increases with depth at all latitudes. The equator rotates faster than the poles, but only by one percent. This case resembles the solar supergranulation layer. The latter shows much stronger horizontal shear, but that is because its lower boundary is not stress-free.

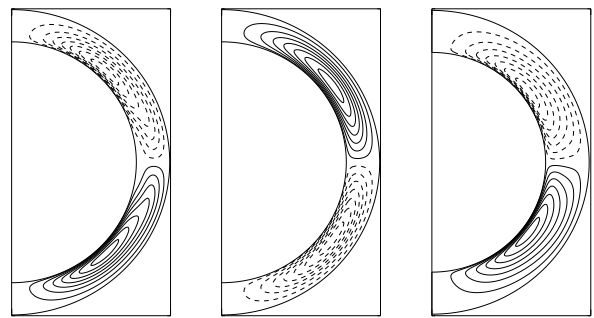


Fig. 3 Streamlines of the meridional flow for the 1.4 M_{\odot} star and the Sun. Solid lines refer to clockwise, dashed lines to counterclockwise flow. *Left*: $M = 1.4 M_{\odot}$, $P = 1$ d. *Middle*: $M = 1.4 M_{\odot}$, $P = 27$ d. *Right*: solar-type star, $P = 27$ d. In the left and middle diagrams the radial length scale has been stretched by a factor of 5 to show more detail. The right diagram is to scale.

Figure 3 shows streamlines of the meridional flow. Like the rotation pattern, the flow pattern for $P = 1$ d is similar to that on the Sun (at $P = 27$ d). The flow is directed towards the poles at the top and towards the equator at the bottom of the convection zone. The maximum flow speed at the surface is 144 m/s, one order of magnitude larger than the solar value. The flow at the bottom of the convection zone reaches an amplitude of 103 m/s. At the “solar” rotation period of 27 d, however, the direction of the flow has the opposite direction. The flow is directed towards the equator and the surface and towards the poles at the bottom, as shown in the right part of the figure.

Figure 4 summarizes the results for a variety of main sequence stars. The two first diamonds from the left indicate the maximum values obtained for the M- and K-type stars in Küker & Rüdiger (2005b). The others denote models for the solar analogues κ^1 Ceti and ϵ Eri, the Sun, and a series of models computed with the Paczynski code for masses of 1.1–1.4 M_{\odot} and various rotation periods. The solid line indicates the $T^{8.92}$ law found by Barnes et al. (2005). Our model stars lie far above the line in the left part of the dia-

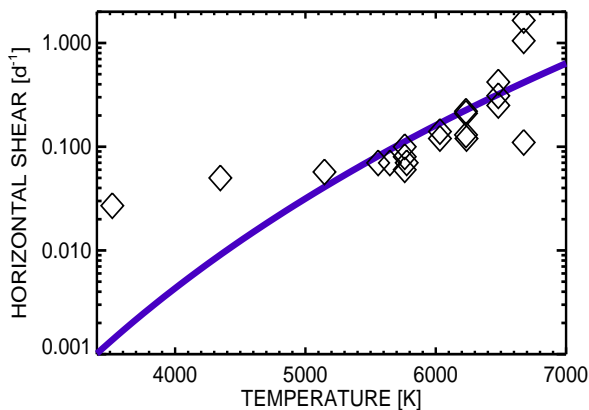


Fig. 4 (online colour at: www.an-journal.org) Differential rotation for stars on the lower main sequence. The diamonds indicate model stars, the solid line the $T^{8.92}$ law of Barnes et al. (2005).

gram but close to it in the right. The surface DR of the $1.4 M_{\odot}$ star varies strongly (as indicated by the three diamonds at the right margin of the diagram), depending on the rotation rate. While it is rather weak for rotation periods similar to that of the Sun, it reaches values above 1 rad d^{-1} .

4 Discussion

In shallow convection zones the convective time scales are shorter than in solar-type stars. As Reynolds stress and anisotropic heat diffusion will only produce substantial surface shear for Coriolis numbers of order unity and larger, the stars have to rotate faster for a solar-type pattern to occur. The rotation pattern of a $1.4 M_{\odot}$ star rotating with a period of 1 d is very similar to that of the Sun rotating with a period of 27 d, mainly because the Coriolis numbers are similar.

Slow rotation causes negative radial shear, $\partial\Omega/\partial r < 0$, little horizontal shear (and thus little surface DR), and a meridional flow that is directed towards the equator at the surface and towards the poles at the bottom of the convection zone. Fast rotation causes a rotation pattern similar to that found in the solar convection zone. The rotation rate varies with latitude but not radius and the equator rotates faster than the poles.

The relative shear α found for a rotation period of 1 d is about half the value found for the Sun at a rotation period of 27 d. The meridional flow pattern is similar to solar meridional flow, namely a surface flow towards the poles and a flow towards the equator at the bottom of the convection zone. However, both the total shear $\delta\Omega$ and the flow speed are substantially larger than the corresponding solar values.

Both the low detection rate of DR and the large shear values can be explained with our model. On the one hand the Fourier transform method requires fast rotation, i.e. large values of $v \sin i$. As the quantity determined is α and values below 0.1 can not be distinguished from rigid rotation, the

stars with detected DR all show very large values of $\delta\Omega$. On the other hand, our model predicts substantial surface differential rotation for short rotation periods only because the convective turnover times are short and therefore the Coriolis number reaches values of the order one only for rotation periods of the order 1 d.

For the stars with $M > 1 M_{\odot}$ in Fig. 4, the surface shear has been computed for a variety of rotation periods. This causes a scatter similar to that in Fig. 5 of Reiners (2006). The values for $T \leq 5000 \text{ K}$ are the largest values found for these stars so far. None of these models has been explored for rotation periods shorter than 5 d yet. Inclusion of longer rotation periods would produce scatter for these stars, too, with the additional data points lying below the ones shown in the figure.

The results from our model suggest that the direct dependence on the (effective) temperature is rather weak but there is a strong dependence on the depth of the convection zone for early F stars. Our findings are consistent with the $T^{8.92}$ law for $T_{\text{eff}} \geq 5000 \text{ K}$ but for lower temperatures we find larger shear and a weaker dependence on temperature. It should be noted that the Barnes sample has only two stars with temperatures below 4500 K and that the $T^{8.92}$ law predicts twice the observed surface shear for the Sun. It is therefore possible that the high exponent is the result of the steep increase in the temperature range between 6500 and 7000 K. On the other hand, Zeeman-Doppler imaging of the low mass star V 374 Peg by Donati et al. (2006) indicates weak differential or rigid rotation. More effort is needed both in observation and theory in the temperature range below 5000 K to clarify whether there is a significant discrepancy.

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