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*The kink-type instability of toroidal stellar magnetic fields with thermal diffusion**

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The stability of toroidal magnetic fields in rotating radiative stellar zones is studied for realistic values of both the Prandtl numbers. The two considered models for the magnetic geometry represent fields with odd and even symmetry with respect to the equator. In the linear theory in Boussinesq approximation the resulting complex eigenfrequency (including growth rate and drift rate) are calculated for a given radial wave number of a nonaxisymmetric perturbation with $m = 1$. The ratio of the Alfvén frequency, $\Omega_{\rm A}$, to the rate of the basic rotation, Ω, controls the eigenfrequency of the solution. For strong fields with $\Omega_{\rm A} > \Omega$ the solutions do not feel the thermal diffusion. The growth rate runs with $\Omega_{\rm A}$ and the drift rate is close to $-\Omega$ so that the magnetic pattern will rest in the laboratory system. For weaker fields with $\Omega_{\rm A} < \Omega$ the growth rate strongly depends on the thermal conductivity. For fields with dipolar parity and for typical values of the heat conductivity the resulting very small growth rates are almost identical with those for vanishing gravity. For fields with dipolar symmetry the differential rotation of any stellar radiative zone (like the solar tachocline) is shown as basically stabilizing the instability independent of the sign of the shear.

Finally, the current-driven kink-type instability of a toroidal background field is proposed as a model for the magnetism of Ap stars. The recent observation of a lower magnetic field treshold of about 300 Gauss for Ap stars is understood as corresponding to the minimum magnetic field producing the instability.

Keywords: stellar magnetic fields; stellar rotation; differential rotation; solar tachocline

1 Introduction

This paper considers the stability of toroidal magnetic fields in rotating radiation zones of stars and focuses on the destabilizing effect of finite thermal diffusion and the stabilizing effect of differential rotation. The equations for the linear stability of toroidal magnetic fields under the influences of basic rotation and gravitational buoyancy are solved for two different latitudinal profiles of the toroidal field, i.e. with the two possible symmetries with respect to the equator. Our equations are global in both horizontal dimensions but they are local in radius, i.e. the short-wave approximation $kr \gg 1$ is used (k is the radial wave number).

We know that a toroidal magnetic field B_ϕ which fulfills the condition

$$\frac{\mathrm{d}}{\mathrm{d}R}\left(RB_\phi^2\right) > 0 \tag{1}$$

is unstable against nonaxisymmetric disturbances in an ideal and incompressible medium (Tayler 1957, 1973, Vandakurov 1972, Acheson 1978). Here R is the distance from the axis where $B_\phi \equiv 0$. This 'Tayler instability' (TI) is suppressed by rigid rotation unless the magnetic field is strong enough to fulfill the condition

$$\Omega_{\rm A} \geq \Omega \tag{2}$$

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(Pitts and Tayler 1985) with the magnetic frequency $\Omega_{\rm A} = B_\phi/\sqrt{\mu_0\rho}R$ and the rotational frequency $\Omega =$ const of the star. For a simple spherical model with $\Omega_{\rm A} =$ const (one magnetic belt with maximum in the equatorial plane) the dashed line in figure 1 demonstrates this situation where the solar value of the magnetic Prandtl number $\mathrm{Pm} = \nu/\eta$ of 5×10^{-3} was used. Here ν and η are the microscopic values of the viscosity and magnetic diffusivity. The growth rate γ at the vertical axis of the plot is normalized with the basic rotation so that the result of the calculation is

$$\gamma \propto \Omega_{\rm A} \qquad \text{for} \qquad \Omega_{\rm A} \gtrsim \Omega. \tag{3}$$

The instability thus exists only for subAlfvénic rotation and it is obviously very fast, i.e. $\gamma \simeq \Omega_{\rm A} > \Omega$.

Due to the assumed isothermal state of the medium no buoyancy-term exists in this calculation. Generally, in stably-stratified plasma the real buoyancy is 'negative' and should stabilize the magnetic instability. This is indeed the case. The solid line in figure 1 results from a model with adiabatic density fluctuations in Boussinesq approximation, i.e. for vanishing thermal diffusivity χ. The resulting field strength for onset of instability slightly exceeds its value without buoyancy. Surprisingly, the stabilization of the TI by 'negative buoyancy' is a small effect. The opposite case with $\chi \to \infty$ provides results identical with those for the incompressible model (dashed line). No temperature fluctuations can develop for $\chi \to \infty$ so that the stabilizing effect of buoyancy does not apply.

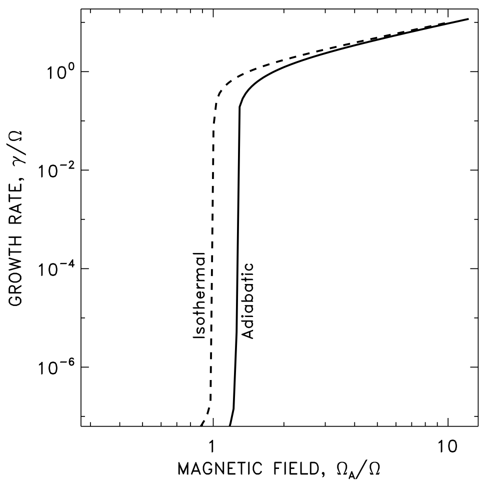

Figure 1. Normalized growth rates of the nonaxisymmetric ($m = 1$) kink-type instability of an equatorsymmetric toroidal field in a sphere with $\mathrm{Pm} = 5 \times 10^{-3}$. The solid line represents the adiabatic approximation with $\chi = 0$. The dashed line gives the limit of $\chi \to \infty$. The stabilizing effect of buoyancy is rather small.

After inspection of the extremes $\chi \to 0$ and $\chi \to \infty$ one could believe that the inclusion of the thermal equation into the instability theory is not significant. We know, however, that the heat conductivity in the stellar radiation zones is the most effective dissipation process, far from the adiabatic limit, i.e.

$$\chi \gg \eta \gg \nu. \tag{4}$$

The Prandtl number of the gas is of order

$$\mathrm{Pr} = \frac{\nu}{\chi} \simeq 2 \times 10^{-6} \tag{5}$$

which yields a Roberts number $\mathrm{q} = \chi/\eta$ of $\mathrm{q} \simeq 2500$. Note that liquid metals of MHD laboratory experiments have $\mathrm{q} \ll 1$.

The nonadiabatic models with small Prandtl number (see equation (5)) will lead to a surprising result. The buoyancy does *not* suppress the instability any longer but it creates another instability which exists for much weaker magnetic fields. Acheson (1978) found that the Roberts number q enters the stability equations in a rather complicated manner so that one term acts stabilizing while another one acts destabilizing. “It is natural, therefore, to expect quite complicated changes in the stability properties of the system ...”. All terms, however, vanish when $q \to \infty$. We shall find that indeed finite thermal conductivities lead to a destabilization of magnetic fields with $\Omega_{\rm A}$ much smaller than Ω; the growth rates of this doubly-diffusive instability, however, are small. Neither time stepping codes nor technical experiments can find such slow instabilities with growth rates of order $10^{-4}\Omega$. Of course, such timescales are still very short compared to stellar evolutionary times but the physical relevance of such instabilities is still in question (Cally 2003).

Critical for the astrophysical meaning of nonaxisymmetric magnetic instabilities is also the differential rotation. One expects that the nonuniform rotation acts against the excitation of nonaxisymmetric modes if they are already excited for rigid rotation. In the solar tachocline the rotation law is – per definition – nonuniform. The differential rotation itself is needed for the production of the strong toroidal fields leading to the tachocline phenomenon. A possible magnetic instability of these toroidal fields (“Tayler-Spruit dynamo”) should basically be stabilized by the nonuniformity of the rotation. One has thus to check how important the differential rotation is to stabilize/destabilize the toroidal magnetic fields. There are indications that the influence of the differential rotation strongly depends on the symmetry type of the toroidal field patterns with respect to the equator. In the present paper only fields with antisymmetry with respect to the equator are considered as subject to differential rotation.

2 The model

The equations for small disturbances ($'$) of background magnetic ($\bar{\mathbf{B}}$) and velocity ($\bar{\mathbf{u}}$) fields are

$$\frac{\partial \mathbf{u}'}{\partial t} + (\bar{\mathbf{u}}\cdot\nabla)\,\mathbf{u}' + (\mathbf{u}'\cdot\nabla)\,\bar{\mathbf{u}} + \frac{1}{\mu_0\rho}\left(\nabla\left(\bar{\mathbf{B}}\cdot\mathbf{B}'\right) - \right.$$
$$\left. - \left(\bar{\mathbf{B}}\cdot\nabla\right)\mathbf{B}' - \left(\mathbf{B}'\cdot\nabla\right)\bar{\mathbf{B}}\right) = -\left(\frac{1}{\rho}\nabla P\right)' + \nu\Delta\mathbf{u}' \tag{6}$$

for the velocity fluctuations $\mathbf{u}'$,

$$\frac{\partial \mathbf{B}'}{\partial t} = \mathrm{curl}\left(\bar{\mathbf{u}}\times\mathbf{B}' + \mathbf{u}'\times\bar{\mathbf{B}} - \eta\ \mathrm{curl}\mathbf{B}'\right), \tag{7}$$

for the magnetic fluctuations $\mathbf{B}'$ and

$$\frac{\partial s'}{\partial t} + \bar{\mathbf{u}}\cdot\nabla s' + \mathbf{u}'\cdot\nabla\bar{s} = \frac{C_{\rm p}\chi}{T}\Delta T' \tag{8}$$

for the entropy fluctuations s' related to the density fluctuations by $s' = -C_{\rm p}\rho'/\rho$.

The basic flow is a rotation with uniform angular velocity Ω and the mean magnetic field $\bar{\mathbf{B}}$ has only a toroidal component B_ϕ. The mathematical method for solving the equation system (6)–(8) has been described earlier (Kitchatinov and Rüdiger 2008). In this previous paper the instability has been considered for the case of rigid rotation and for given heat conductivity. In the present paper the *destabilizing* role of the heat conductivity is demonstrated and first results for the *stabilizing* role of differential rotation are given.

The equations are global in horizontal dimensions but local in radius. The radial scale of disturbances is assumed as short and their dependence on radius r is taken in the form of Fourier modes $\exp(\mathrm{i}(m\phi - \omega t + kr))$. There are only symmetry conditions along the polar axes which will be fulfilled by the series expansions after Legendre polynomials. Only the modes with $m = 1$ are considered. Then at the axes the radial components of flow and field must vanish and also the θ-derivatives of the horizontal components (see Elstner *et al.* 1990, Gilman and Fox 1997). For given radial wave number and for given field amplitude (in units of the basic rotation velocity) the resulting eigenfrequency is computed including growth rate (imaginary part of ω) of the instability and drift rate of the eigensolutions (real part of ω).

The key parameter for the effect of the stable stratification is

$$\hat{\lambda} = \frac{N}{\Omega k r}, \tag{9}$$

where N is the buoyancy frequency

$$N^2 = \frac{g}{C_\mathrm{p}} \frac{\partial s}{\partial r}. \tag{10}$$

In stellar radiation zones it is $N \gg \Omega$. For the most unstable modes we find $\hat{\lambda} < 1$ so that the radial scale of the modes is indeed small, i.e. $kr \gg 1$. Our equations include finite diffusion via

$$\epsilon_\eta = \frac{\eta N^2}{\Omega^3 r^2}, \qquad \epsilon_\nu = \frac{\nu N^2}{\Omega^3 r^2}, \tag{11}$$

with η and ν as the magnetic resistivity and the kinematic viscosity. The thermal conductivity χ enters the equations in the normalized form

$$\epsilon_\chi = \frac{\chi N^2}{\Omega^3 r^2}, \tag{12}$$

which is the free parameter in the following discussion.

The present article focuses on the effect of thermal diffusivity from the following reason. The current-driven Tayler instability requires radial displacements. The instability does not exist in the 2D case of purely horizontal disturbances (see Dicke 1979). In a stably stratified radiation zone the radial displacements are opposed by buoyancy. Finite thermal diffusivity reduces the buoyancy and thus supports the instability. The relevant parameter is the ratio C_χ

$$C_\chi = \frac{\chi k^3 r}{N} = \frac{\epsilon_\chi}{\hat{\lambda}^3} \tag{13}$$

of the frequency χk^2, with which the thermal diffusion destroys the buoyancy, to the characteristic frequency $N/(kr)$ of gravity waves. When ϵ_χ and $\hat{\lambda}$ are simultaneously varied for constant C_χ-parameter (13) the results change little.

The toroidal field profile of the model is parameterized as

$$B_\phi = \sqrt{\mu_0 \rho}\; r \sin\theta \cos^n\theta \; \Omega_\mathrm{A}, \tag{14}$$

where now Ω_A is the *amplitude* of the magnetic Alfvén frequency. Computations were made for $n = 0$ so that the toroidal field is symmetric (or quadrupolar) with respect to the equator. The other model with $n = 1$ represents a (dipolar) background field antisymmetric relative to the equator or – in other words – with two belts of opposite signs.

The diffusion parameters $\epsilon_\eta = 4 \times 10^{-8}$ and $\epsilon_\nu = 2 \times 10^{-10}$ characteristic for the upper radiation zone of the Sun are kept fixed and ϵ_χ is varied to study the effect of thermal diffusion.

The normalized wavelengths (9) were also kept constant. The constant values were $\hat{\lambda} = 0.6$ for quadrupolar background field ($n = 0$)and $\hat{\lambda} = 0.1$ for dipolar fields ($n = 1$). The choice is motivated by the finding that for this values of $\hat{\lambda}$ the marginal field strengths for onset of the instability are minimized. The values of $\hat{\lambda}$ corresponding to maximum growth rates of supercritical excitations change with the external parameters but only slightly. Some results of this paper were obtained for the instability modes of symmetry type S1, the equatorially symmetric excitations with azimuthal wave number $m = 1$. For $\hat{\lambda} = 0.1$ and for the given

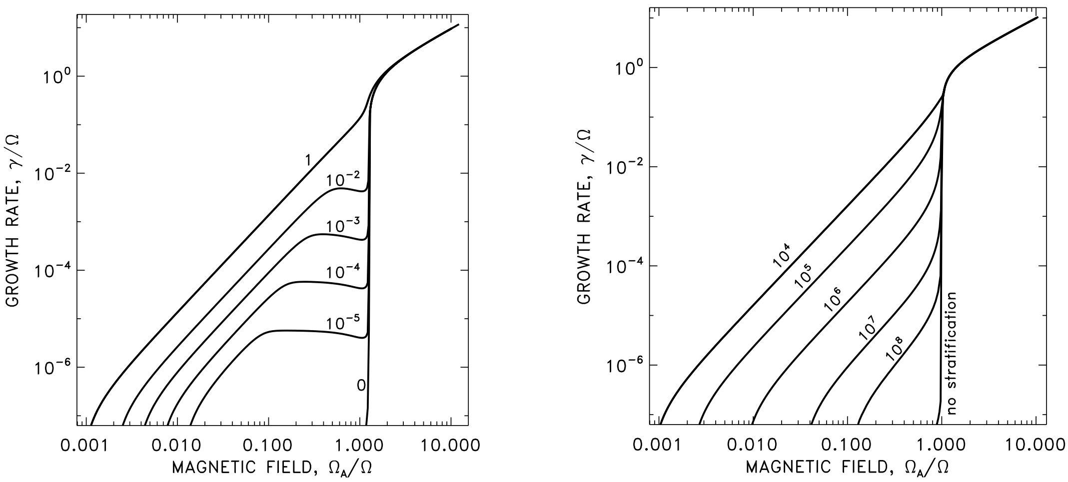

Figure 2. Growth rates of nonaxisymmetric disturbances of a background field with quadrupolar symmetry for small (left) and large (right) thermal diffusivity. The lines are marked by C_χ after equation (13). The growth rates for weak fields with $\Omega_{\rm A} < \Omega$ initially increase with C_χ but then decrease for very large thermal diffusivities.

Prandtl number (5) one finds $C_\chi = 0.1$ as the solar value. Computing the critical magnetic frequencies under the presence of differential rotation the wave numbers have been varied as long as the minimum eigenvalues have been found (see below).

The linear code is able to calculate the eigenfrequencies for the realistic small magnetic Prandtl number

$$\mathrm{Pm} = 5 \times 10^{-3}. \tag{15}$$

So far the best nonlinear MHD codes reach values down to 0.001 (Schekochihin et al. 2005, Brandenburg 2009).

3 Results

In the following the stability of the magnetic background field (14) is probed for nonaxisymmetric disturbances with the azimuthal wave number $m = 1$ and with a fixed radial scale. By use of the method described by Kitchatinov and Rüdiger (2008) the complex equation system (6)–(8) is numerically solved to find the growth rate of a possible instability. Generally, the results do not change if m is replaced by $-m$. Indeed, for purely toroidal magnetic fields there is no handedness in the system. The addition of even a weak poloidal field would break the symmetry between m and $-m$.

3.1 *Quadrupolar magnetic geometry*

We start with $n = 0$ as the most simple case. As the fields are symmetric with respect to the equator this model resembles stellar models with toroidal fields of quadrupolar parity.

The growth rates of the instability for various values of the thermal diffusivity parameter C_χ are shown in figure. 2. All the lines converge for sufficiently strong fields, $\Omega_{\rm A} > \Omega$, showing that then the instability is insensitive to diffusion. The growth rates γ for strong fields can be estimated as $\gamma \simeq \Omega_{\rm A}$ (Spruit 1999).

The magnetic instability in adiabatic fluids, i.e. with $\chi = 0$, is suppressed by the rotation, (Pitts and Tayler 1985, Cally 2003). But for finite thermal diffusion also weak fields under the presence of superAlfvénic rotation, $\Omega > \Omega_{\rm A}$, are unstable though with small growth rates. The growth rates initially increase with increasing C_χ to saturate for about $C_\chi = 1$ (figure 2, left). Due to its very small growth rates the instability of weak fields ($\Omega_{\rm A} < \Omega$) for both small and very large C_χ cannot be detected with time stepping codes (see Braithwaite 2006).

In the wide range

$$0.1 < C_\chi < 10^4 \tag{16}$$

also a weak-field instability is faintly sensitive to thermal diffusivity. The growth rate runs as

$$\gamma \propto (\frac{\Omega_{\rm A}}{\Omega})\ \Omega_{\rm A}. \tag{17}$$

Only for very large C_χ when the buoyancy is totally suppressed the results for unstratified fluids with their strong rotational suppression of the instability are reproduced (figure 2, right).

The case of one magnetic belt, $B_\phi \sim \sin\theta$, is exceptional (Pitts and Tayler 1985). When the ratio of rotational velocity to Alfvén velocity is uniform then there is no preferred latitude for the instability to start. Finite thermal diffusion promotes the instability as the effect of diffusion depends on the latitudinal scale of the disturbances. The isothermal limit $\chi \to \infty$ represents also the case of unstratified fluid and the instability is totally suppressed for fast rotation.

3.2 *Dipolar magnetic geometry*

We find a different situation for dipolar background fields ($n = 1$) with two belts of opposite polarity as resulting from the interaction of differential rotation and dipolar poloidal fields. Figure 3 shows the results. In the above model of quadrupolar fields, the vertical lines of figure 1 for adiabatic ($\chi = 0$) and isothermal ($\chi \to \infty$) disturbances are rather parallel and close together. Now the (dashed) line for $\chi \to \infty$ (or, what is the same, for unstratified fluids) and the vertical line for $\chi = 0$ differ completely. The dashed line is close below the line for $C_\chi = 1000$ for which the maximum growth rates appear. It also shows the typical behavior (17) for subequipartition fields ($\Omega_{\rm A} < \Omega$). Consequently, one finds the relation (17) true for all nonadiabatic disturbances in fluids with $C_\chi > 0.1$. Hence, if the very small growth rates are acceptable, then already toroidal fields with $\Omega_{\rm A} \simeq 0.01\ \Omega$ become unstable if the C_χ is not too small. For the Sun the maximum strength of stable fields is about 500 Gauss (cf. Spruit 1999). Note that the growth rates for the solar value of $C_\chi = 0.1$ differ slightly from the growth rates for isothermal case. For the stellar magnetic fields with dipolar parity, $C_\chi \simeq 0.1$ already represents the situation for $\chi \to \infty$. The inclusion of the buoyancy is thus not even necessary. This statement does not hold for the above model of quadrupolar field geometry.

A very new feature of the considered two-belts geometry are resonances. The lines with fixed but small C_χ show two peaks at magnetic field amplitudes slightly below the equipartition level of $\Omega_{\rm A} = \Omega$. The resonant eigenmodes do not appear in the one-belt model and they show a more detailed fine-structure than the nonresonant ones (figure 4). As a doubling of the resolution does not change the results the numerically detected resonances seem to be real. Note that all the unstable modes are global in horizontal dimensions.

The resonant nature of the peaks in figure 3 is illustrated by figure 5 showing the drift velocities for a small $C_\chi = 10^{-4}$. Generally, the unstable modes drift against the direction of the global rotation. Note that the drift rates of the resonant modes are zero. These modes corotate with the fluid.

For strong fields with $\Omega_{\rm A} > \Omega$ the instability pattern does not follow the basic rotation. The normalized drift rates approach the value of -1 which means that the modes are resting in the inertial frame of reference. Test calculations showed that this result also holds for Pm $= 1$ which for the interior of hot stars is not unrealistic due to the large radiative viscosity. The nonaxisymmetric field pattern produced by instability of a strong field seems to rest in the laboratory system. If only such a magnetic pattern is observed on a star then it seems to rest or to exhibit an extremely slow rotation. The steepness of the drift rate profile of figure 5 suggests that the transition between the nondrifting solution (where the difference between the rotation of the star and the rotation of the magnetic field is very small) and the drifting solution (where the magnetic pattern rotation disappear) is very sharp. If Ap stars are assumed as stars with unstable toroidal background fields (which themselves are invisible) then two groups among them should exist depending on the ratio of $\Omega_{\rm A}$ to Ω. The magnetic field pattern of the group with $\Omega_{\rm A} < \Omega$

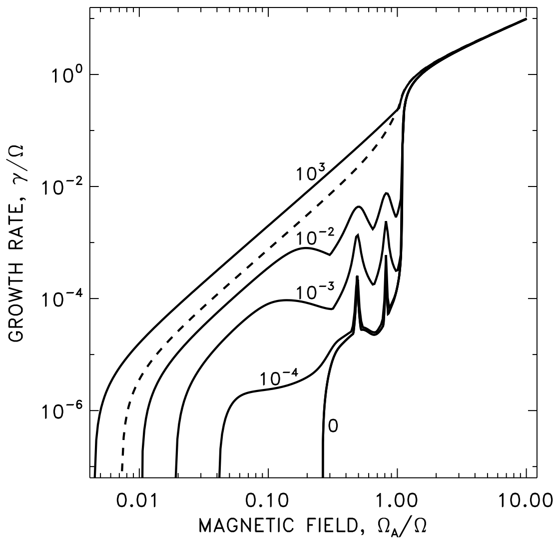

Figure 3. Growth rates of the instability for two magnetic belts of opposite signs ($n = 1$). The lines are marked with the corresponding C_χ-parameter (13). The dashed line shows the results for $g = 0$ and/or $\chi \to \infty$.

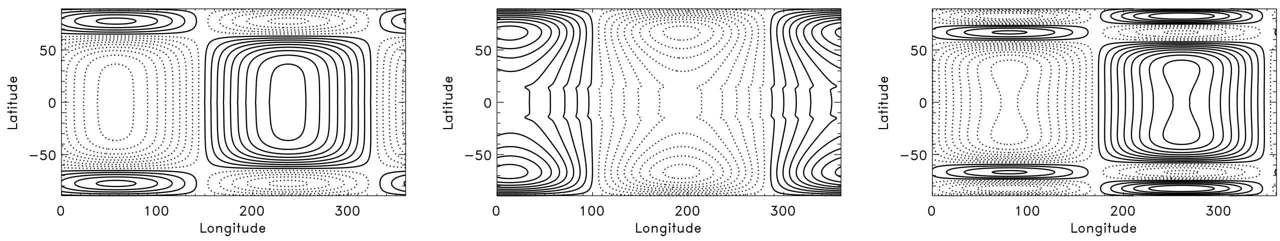

Figure 4. Streamlines of the toroidal flow for the most rapidly growing eigenmodes for the small value $C_\chi = 10^{-4}$. The left panel corresponds to the left peak in the growth rates of figure 3. The right panel is for the right peak and the middle panel is for a nonresonant mode between the peaks. Full and dotted lines show opposite senses of circulation.

rotates slightly slower than the star but the magnetic field pattern of the group with $\Omega_A > \Omega$ should rotate extremely slow.

The instability for strong fields is very fast. The growth times for this case after the equation (3) are shorter than the rotation period. On the other hand, after (17) the instability of so weak fields that $\Omega_A < \Omega$ is much slower. It is, however, hardly controlled by the thermal diffusion. The instability of weak fields in stellar radiation zones should thus not be too sensitive to the chemical details.

4 Differential rotation

Sofar we have assumed the stellar rotation as rigid. This is only true if the star is old enough. A possible differential rotation produces a strong toroidal field from the original fossil poloidal field. The resulting Maxwell stress suppresses the differential rotation producing an almost rigid rotation after the Alfvén travel time estimated for the poloidal field which for hot stars with fields of order mGauss lasts longer than 10 Myr.

Hence, for young stars the instability of the field pattern must be considered under the presence of differential rotation. As the current-driven instability is basically nonaxisymmetric one must expect the action of the differential rotation as stabilizing so that a possible instability might occur only after the Alfvén travel time.

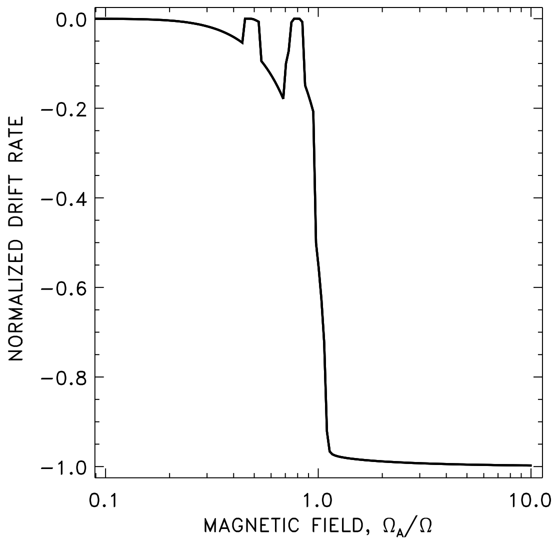

Figure 5. Drift rates of the most rapidly growing modes in the corotating reference frame. The rates are normalized with Ω. Negative values mean the counter-rotation drift. Resonant modes corotate with the fluid. The eigenmodes for strong fields, $\Omega_{\rm A} > \Omega$, are resting in non-rotating frame. The plot is for $C_\chi = 10^{-4}$.

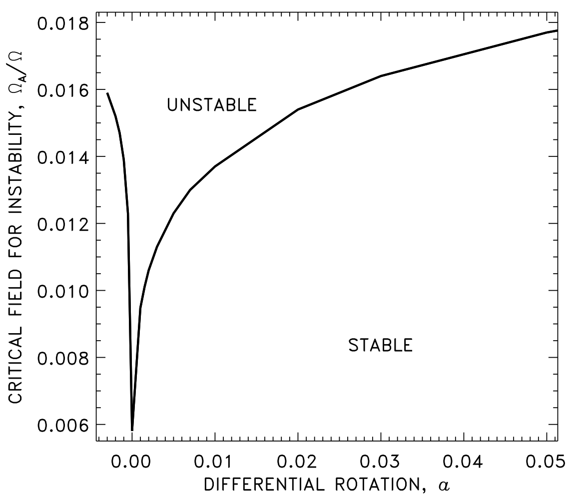

Figure 6. The stabilizing influence of differential rotation on toroidal fields with dipolar symmetry. The critical magnetic fields for onset of the instability optimized by choice of the wave number are shown. Note that the stabilization by differential rotation does only slightly depend on the sign of rotational shear. $C_\chi = 0.1$

We have worked with the simple rotation law

$$\Omega = \Omega_0(1 + a' r^2 \sin^2\theta), \tag{18}$$

which in its simplified local formulation reads as $\Omega \propto 1 - a\ \cos^2\theta$. Equation (18) describes a rotation law with cylindric isolines. For negative a the rotation rate decreases outwards and v.v. The stabilizing effect of the differential rotation should *not* depend on the sign of a.

Here we only consider the magnetic field geometry (14) for dipolar field structures, i.e. with antisymmetry of the fields with respect to the equator. Figure 6 gives the results for $C_\chi = 0.1$. The critical magnetic frequencies are optimized by choice of the wave numbers. We find the stabilization by differential rotation as highly effective. Already very small shear values lead to an increase of the critical magnetic field by a factor of (say) five. The influences of both the sign and the real value of a are small.

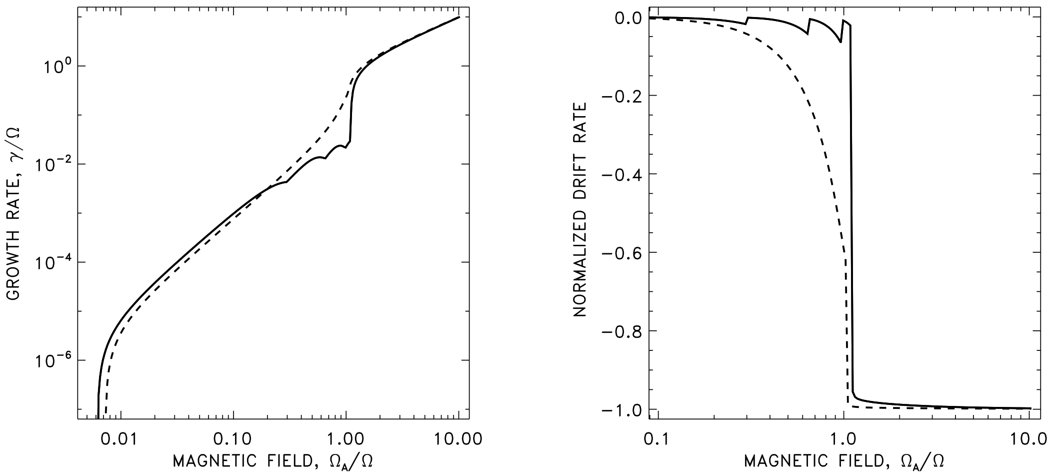

Figure 7. Dipolar field geometry: Normalized growth rate (left) and drift frequency (right) for the solar value of $C_\chi = 0.1$ (solid lines). The dashed lines are for $g = 0$ and/or $\chi \to \infty$. Note the small differences between solid and dashed lines and the abrupt changes of the eigenfrequencies at $\Omega_A = \Omega$. Test calculations have shown that the location of the footpoints of both the dashed and the solid lines do basically not depend on the magnetic Prandtl number ν/η.

The figure 6 suggests a very efficient stabilization of the toroidal fields by any kind of differential rotation if the field is antisymmetric with respect to the equator. The same might *not* be true for other field geometries (see Rüdiger and Schultz 2010). Hence, we find the toroidal magnetic fields in stellar tachoclines with their strong differential rotation (if due to a fossil poloidal dipolar field) much more stable than they are in the rigidly rotating cores of stars.

The stabilizing action of differential rotation does hardly depend on the form of the rotation law. If the star rotates nonhomogeneously then higher amplitudes of the induced toroidal fields remain stable. The results of this section suggest the importance of further studies of the interaction of magnetic fields and differential rotation. If the fields are produced by a dynamo mechanism then the magnetic field geometry can easily differ (like in galaxies) from that considered in the present paper.

5 Stellar magnetism

We have shown that for realistic values of the heat diffusivity the growth rates of the kink-type instability in stellar radiation zones do hardly differ from the growth rates obtained for fully incompressible models without buoyancy. There are, however, strong differences for other types of the magnetic geometry. Cylindric models with uniform Alfvén frequency do not completely cover the situation for spherical models with toroidal fields of dipolar parity. For stellar applications the main results for the instability of such fields are given in figure 7. Note how well the approximation without gravity $g = 0$ and/or $\chi \to \infty$ (dashed line) works in comparison to the 'exact' profile for $C_\chi = 0.1$ (solid line).

After the figure 7 (left) three groups of hot stars can be distinguished in dependence on the amplitude of their toroidal fields. The toroidal magnetic field of the first group fulfills the relation $\Omega_A < 0.01\ \Omega$ so that it remains stable. If the field amplitude exceeds the lower limit (or the rotation is slow enough) then there are two possibilities. If it is not too strong, i.e. $\Omega_A < \Omega$, then it becomes unstable with very small rates of growth and azimuthal drift.

If strong enough, the poloidal component of the resulting nonaxisymmetric field should be observable. The critical Alfvén velocity for a typical hot star is about 10 km/s corresponding to a magnetic field of order 10^6 Gauss. If only 1% of the magnetic energy move to the poloidal perturbation, the amplitude of poloidal field is about 10^5 Gauss (see Gellert *et al.* 2007). This value is even larger than the observed fields of Ap stars.

The third group of stars fulfills the condition $\Omega_A > \Omega$. They are unstable with very short growth times. Their drift rates, however, approach the value of -1 so that the magnetic patterns may only show a very slow global rotation. There are indeed examples among the group of the Ap stars with rotation periods of several years. The bright star γ Equ has a rotation period longer than 70 yr.

In the light of the presented theory the basic fact that the Ap stars are slow rotators compared with the normal A stars mainly means that slow rotation is less stabilizing for the toroidal magnetic fields. With other words, the condition $\Omega_{\rm A} > 0.01\ \Omega$ for instability is more easily fulfilled for slow rotators. It is thus understandable with our results that for slow rotation weaker toroidal fields become unstable and also the resulting amplitude of the $m = 1$ mode is smaller for slow rotation than for fast rotation – which indeed is observed (Hubrig et al. 2007).

The condition $\Omega_{\rm A} > 0.01\ \Omega$ for instability could easily be the counterpart of the lower limit of about 300 Gauss found by Auriére *et al.* (2007) for magnetic fields of Ap stars. The existence of rather strong toroidal magnetic fields within stellar radiation zones can be thought of as the outcome of the differential rotation and a weak fossil poloidal field. For the above calculations we have assumed that the differential rotation only exists before the magnetic instability develops. The reason is that (any form of) differential rotation stabilizes the instability of fields with equatorial antisymmetry. From this point of view all the Ap stars are considered as (slow) rigid rotators.

A complete explanation of the Ap star magnetism still meets open questions. So the axis of the magnetic field pattern is obviously *not* orthogonal to the axis of rotation (Oetken 1977). The obliquity of the field, i.e. the ratio of nonaxisymmetric and axisymmetric field parts, depends on the rotation rate: it is maximum for large Ω (Landstreet and Mathys 2000). The instability of a single $m = 1$ mode cannot explain this finding. It is also known that the magnetic Ap stars do *not* exist close to the ZAMS, they are concentrated toward the center of the main-sequence. The earliest observed evolution time across the main-sequence of a magnetic Ap and Bp star is about 20 Myr, no one younger magnetic star has been observed (Hubrig *et al.* 2000). There are also possibilities to explain these empirical findings but those are beyond the scope of the present paper.

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