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Global vortex systems on standard-accretion disk surfaces

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ABSTRACT

The hydrodynamical global stability of turbulent density-stratified and differentially rotating disks is studied. In the context of the mean-field approach the small-scale turbulence produces the Anisotropic Kinetic Alpha-effect which is shown here to generate global vortices. Different rotation laws and aspect ratios, in particular those of standard-accretion disks and galaxies, are considered. Linear stability analysis yields the AKA-effect leading to the self-excitation of large-scale flow patterns. The preferred excitations are nonaxisymmetric with an azimuthal wave number $m = 1$ and antisymmetric with respect to the disk mid-plane. Therefore, there are both hot and cold spots on each surface of the disk. The vertical flow always crosses the equator so that there is no concentration of dense material.

Key words: instability – vortices – accretion disks – flickering – planetary systems

1 MOTIVATION

The present study is motivated by a paper of Abramowicz et al. (1992) reporting the existence of X-ray active coherent structures at the surface of an accretion disk. In particular, large-scale vortices have been proposed as the source of short-term flickering of cataclysmic variables and the short-term variability of X-ray binaries (Bath et al. 1974).

Vortices, indeed, would produce time-dependent effects via rotational modulation if the observation is neither edge-on nor pole-on. The reason for this is that they are basically nonaxisymmetric patterns. It is known that the nonaxisymmetric phenomena always need a particular excitation mechanism in order to be stationary; e.g., a nonaxisymmetric magnetic field cannot survive against the smoothing action of differential rotation (Krause & Rädler 1980, Rädler 1986). Also, the arms of spiral galaxies would quickly be sheared by the differential rotation if there were no physical excitation like density waves.

The same could hold for maintaining large-scale vortices. Abramowicz et al. (1992) have suggested that nonaxisymmetric structures can exist in disks in spite of their strong differential rotation.

The long-living vortices may also be important for planets formation. The balance between the centrifugal and pressure forces in the vortical flow does not apply to planetesimals due to their relatively large density compared to the surrounding gas. The resulting planetesimals concentration in the central regions of vortices accelerates the planets formation (Barge & Sommeria 1995, Tanga et al. 1996, Klahr & Henning 1997).

The generation of self-gravitating vortices in circumstellar disks in a local approximation was already studied by Adams & Watkins (1995) considering an infinitely thin disk in a laminar regime. The present paper studies the alternative possibility, i.e. that a coherent global vortex system in an accretion disk is supported by *turbulence.* As known the turbulent Anisotropic Kinetic Alpha (AKA) effect is suspected to cause an instability of rotating, stratified turbulent fluids to global seed flows (cf. Frisch, She & Sulem 1987; Kitchatinov, Rüdiger & Khomenko 1994), similar to the hydromagnetic dynamo instability for large-scale magnetic seed fields. The expectation of a general vortex generation by the AKA-effect is suggested by recent simulations of v. Rekowski & Kitchatinov (1997) who considered spheres in uniform rotation. However, these results are not directly applicable to accretion disks with Kepler rotation. We have to check whether the shearing by the differential rotation will preclude the excitation of nonaxisymmetric vortices in the disks.

We shall see in the following that in spite of the shearing effect, nonaxisymmetric flow patterns are always preferred in excitation. This conclusion follows from a series of accretion disk models for various values of the viscosity alpha. All of them favor patterns with the zonal wave number $m = 1$, which are antisymmetric with respect to the disk mid-plane. Each of the patterns will produce one hot and one cool spot on the disk surface due to the vertical mixing by their meridional flow.

2 BASIC EQUATIONS

We study the turbulent flow perturbations of a differentially rotating disk. For the turbulent flow we use the mean-field approach, $\boldsymbol{u} = \bar{\boldsymbol{u}} + \boldsymbol{u}'$, where $\bar{\boldsymbol{u}}$ is the mean velocity and $\boldsymbol{u}'$ describes the fluctuations. We then write the equation for the mean flow which is assumed to be small compared to the unperturbed background differential rotation. Thus, we start from the linearized mean-field momentum equation for a turbulent stratified rotating fluid,

$$\begin{aligned} \frac{\partial \bar{m}_i}{\partial t} \quad &+ \quad \nabla\left(\boldsymbol{V}\bar{m}_i + \bar{\boldsymbol{m}}V_i\right) \; + \; \frac{\partial \bar{p}}{\partial x_i} \; - \\ &- \quad \frac{\partial}{\partial x_j}\left(\nu_{\mathrm{T}}\left(\frac{\partial \bar{m}_i}{\partial x_j} + \frac{\partial \bar{m}_j}{\partial x_i} + \frac{2}{3}\delta_{ij}\left(\boldsymbol{G}\cdot\bar{\boldsymbol{m}}\right) - \right.\right. \\ &- \quad \left.\left. G_i\bar{m}_j - G_j\bar{m}_i\right)\right) \; = -\frac{\partial}{\partial x_j}\left(\Gamma_{ijk}\bar{m}_k\right), \qquad (1) \end{aligned}$$

(Pipin, Rüdiger & Kitchatinov, 1996), where $\boldsymbol{V} = \boldsymbol{\Omega} \times \boldsymbol{r}$ is the mean velocity for the undisturbed state which represents the nonuniform rotation, $\bar{\boldsymbol{m}} = \rho\bar{\boldsymbol{u}}$ is the mean momentum density, $\bar{p}$ is the pressure, ν_{T} is the eddy viscosity, $\boldsymbol{G} = \nabla \log \rho$ is the stratification vector, and the right-hand side of the equation involves the AKA-effect with the symmetric Γ-tensor,

$$\begin{aligned} \Gamma_{ijk} \quad = \quad \Gamma_1 &\left(\Omega^\circ_j\epsilon_{ikp} + \; \Omega^\circ_i\epsilon_{jkp} + \right. \\ &\left. + \; \delta_{kj}\epsilon_{ilp}\Omega^\circ_l + \; \delta_{ki}\epsilon_{jlp}\Omega^\circ_l\right)G_p. \qquad (2) \end{aligned}$$

$\boldsymbol{\Omega}^\circ = \boldsymbol{\Omega}/\Omega_0$ is the vertical unit vector. The Γ-term of Eq. (1) comes from the correlation tensor for fluctuating velocities (Krause & Rüdiger 1974; Roberts & Soward 1975; Moffatt & Tsinober 1992),

$$\langle u'_i \; u'_j\rangle = -\nu_{\mathrm{T}}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\,\mathrm{div}\,\bar{\boldsymbol{u}}\right) \; + \; \Gamma_{ijk}\bar{u}_k. \qquad (3)$$

It is linear in the mean velocity similar to the hydromagnetic alpha-effect which is also linear but in the mean magnetic field. With the AKA-effect included, the fluid may be unstable to generate large-scale flow patterns (Schüssler 1984; Frisch, She & Sulem 1987).

We perform the (linear) stability analysis by solving the eigenvalue problem for Eq. (1) for 3D flow perturbations.

The flow is assumed to be inelastic,

$$\mathrm{div}\,\bar{\boldsymbol{m}} \; = \; 0, \qquad (4)$$

hence it can be described in terms of two scalar potentials,

$$\bar{\boldsymbol{m}} \; = \; \boldsymbol{\Omega}^\circ \times \nabla U \; + \; \mathrm{rot}\left(\boldsymbol{\Omega}^\circ \times \nabla\Psi\right), \qquad (5)$$

where U defines the toroidal part of the flow with its velocity vector normal to $\boldsymbol{\Omega}^\circ$, and Ψ is the stream function for the poloidal flow.

The procedure to derive the dynamical equations for given flow potentials is very similar to derivations for spheres (cf. Krause & Rädler 1980). We substitute (5) into (1), calculate the curl of the equation to exclude pressure and take the scalar product with the vector $\boldsymbol{e}$. This yields the dynamical equation for the quantity $\hat{R}U$, where $\hat{R}$ is the 'horizontal part' of the Laplacian delta,

$$\hat{R} \; = \; \frac{1}{\varpi}\frac{\partial}{\partial\varpi}\varpi\frac{\partial}{\partial\varpi} \; + \; \frac{1}{\varpi^2}\frac{\partial^2}{\partial\phi^2}. \qquad (6)$$

Then, we perform the curl of (1) twice and take again the scalar product with $\boldsymbol{\Omega}^\circ$. This yields the dynamical equation for $\hat{R}W$, where W is the potential for the vorticity $\boldsymbol{w}$,

$$\boldsymbol{w} \; = \; \mathrm{rot}\;\bar{\boldsymbol{m}} \; = \; \boldsymbol{\Omega}^\circ \times \nabla W \; + \; \mathrm{rot}\left(\boldsymbol{\Omega}^\circ \times \nabla U\right). \qquad (7)$$

The Poisson equation resulting from the comparison of (5) and (7),

$$\Delta\Psi \; = -W, \qquad (8)$$

closes the equation system. The complete equation system is too bulky to reproduce here.

We apply the equations to disks bounded by parallel planes crossing the z-axis of rotation at the positions $z = \pm H$ with H being the disk's half-thickness. No boundaries in the radial directions are imposed. A new variable,

$$\mu \; = \; \cos\theta \; = \; \frac{\left(\frac{\varpi}{\varpi_0}\right)^2 - 1}{\left(\frac{\varpi}{\varpi_0}\right)^2 + 1}, \qquad (9)$$

is introduced to compensate for the infinite range of ϖ and the finite range of μ. Eq. (6) in terms of the new variable is

$$\begin{aligned} \hat{R} \quad &= \quad \frac{1}{\varpi_0}(1-\mu)^2\hat{\mathcal{L}}, \\ \hat{\mathcal{L}} \quad &= \quad \frac{\partial}{\partial\mu}\left(1-\mu^2\right)\frac{\partial}{\partial\mu} \; + \frac{1}{1-\mu^2}\frac{\partial^2}{\partial\phi^2}, \qquad (10) \end{aligned}$$

where $\hat{\mathcal{L}}$ has the Legendre polynomials, P^m_n, as the eigenfunctions. Hence, the polynomial expansion is convenient. In fact, expansions in appropriate functions over all three dimensions were applied to solve the eigenvalue problem by the fully-spectral method,

$$\begin{aligned} \Psi \quad = \quad &\sum_{n,m,l}\Psi^A_{nml}\cos\left(\pi(l-\frac{1}{2})\frac{z}{H}\right)\mathrm{e}^{im\phi}P^m_n(\mu) \; + \\ + \quad &\sum_{n,m,l}\Psi^S_{nml}\sin\left(\frac{\pi l z}{H}\right)\mathrm{e}^{im\phi}P^m_n(\mu), \qquad (11) \end{aligned}$$

and the same expansion applies to W. The representation for U, however, is slightly different,

$$\begin{aligned} U \quad = \quad &\sum_{n,m,l}U^A_{nml}\sin\left(\pi(l-\frac{1}{2})\frac{z}{H}\right)\mathrm{e}^{im\phi}P^m_n(\mu) \; + \\ + \quad &\sum_{n,m,l}U^S_{nml}\cos\left(\frac{\pi l z}{H}\right)\mathrm{e}^{im\phi}P^m_n(\mu). \qquad (12) \end{aligned}$$

The expansions (11) and (12) automatically satisfy the stress-free and no-vertical-flow boundary conditions, the symmetry conditions on the rotation axis and zero-flow condition for $\varpi \to \infty$ (cf. Kitchatinov & Mazur 1997). The first and second sums in (11) and (12) describe the flows with different types of symmetry about the disk mid-plane. The superscripts A and S denote the antisymmetric and symmetric flow patterns relative to the mirror-reflection about the mid-plane, respectively. It may also be shown that the complete equation system for the linear excitations splits into two independent subsystems describing the S- and A-modes, when the eddy viscosity and Γ_1-coefficient of (2) are symmetric and the stratification vector $\boldsymbol{G}$ is antisymmetric about the mid-plane.

As usual, the assumption of the exponential time-dependence, $\Psi, U, W \; \sim \; \mathrm{e}^{i\omega t}$, reduces the linear stability analysis to an eigenvalue problem. The eigenvalues were determined numerically for the series of models formulated in the following section.

3 THE MODELS

The density distribution in astrophysical disks is usually much more non-uniform over the vertical direction than over the horizontal one. Due to this reason, we neglect the radial component of the stratification vector, $\boldsymbol{G}$. For the vertical stratification a parameterization is applied, i.e.

$$\boldsymbol{G} \;=\; \boldsymbol{\Omega}^{\circ} G(z), \qquad G(z) \;=\; \frac{1}{\rho}\frac{\partial \rho}{\partial z} \;=\; \frac{G_0}{H}\sin\left(\frac{\pi z}{H}\right). \qquad (13)$$

The opposite is, probably, true about the angular velocity distribution: it is much more variable with axial distance than with vertical position. The differential rotation profile is parameterized by the Brandt function,

$$\Omega(\varpi) \;=\; \frac{\Omega_0}{\left(1+\left(\frac{\varpi}{\varpi_0}\right)^{nq}\right)^{\frac{1}{n}}}, \qquad (14)$$

with $n = 2$ (Donner & Brandenburg 1990). The rotation profile (14) approaches a power law, $\Omega \simeq \Omega_0(\varpi/\varpi_0)^{-q}$, with $\varpi > \varpi_0$. A value of $q = 3/2$ corresponds to Keplerian rotation in accretion disks, while $q = 1$ is appropriate for galaxies.

The eddy viscosity $\nu_{\rm T}$ is assumed to be constant. $\Gamma_1(\varpi)$ is a function which is linear in $\nu_{\rm T}$. It is known that the AKA-effect comes from the involvement of rotation, therefore, we assume Γ_1 to be proportional to the normalized rotation profile,

$$\Gamma_1(\varpi) = \Gamma^* \nu_{\rm T} \frac{\Omega(\varpi)}{\Omega_0}. \qquad (15)$$

Apart from the q-parameter and the aspect ratio,

$$\tilde{r}_\Omega \;=\; \frac{\varpi_0}{H}, \qquad (16)$$

the models have two key governing parameters. The first is the Taylor number

$$\mathrm{Ta} \;=\; \frac{4\Omega_0^2 H^4}{\nu_{\rm T}^2}. \qquad (17)$$

With the viscosity ansatz $\nu_{\rm T} = \alpha_{\rm SS} H^2 \Omega_0$, one arrives at the estimation

$$\mathrm{Ta} = \left(\frac{2}{\alpha_{\rm SS}}\right)^2 \qquad (18)$$

in terms of the Shakura-Sunyaev viscosity parameter $\alpha_{\rm SS}$. The other parameter

$$C_\Gamma \;= \Gamma^* |G_0| \qquad (19)$$

measures the power of the AKA-effect.

Our problem is to define the critical values of C_Γ for the onset of the mean-flow instability and the shape of the preferred, i.e., having the lowest critical C_Γ, flow patterns.

4 RESULTS AND DISCUSSION

Figs 1 and 2 present the neutral stability lines for a thin standard-accretion disk and for a galaxy disk in the Ta-C_Γ-plane ($\alpha_{\rm SS}$-C_Γ-plane). Above the lines, the real parts of the eigenvalues are positive implying that the system is unstable to the flow excitations.

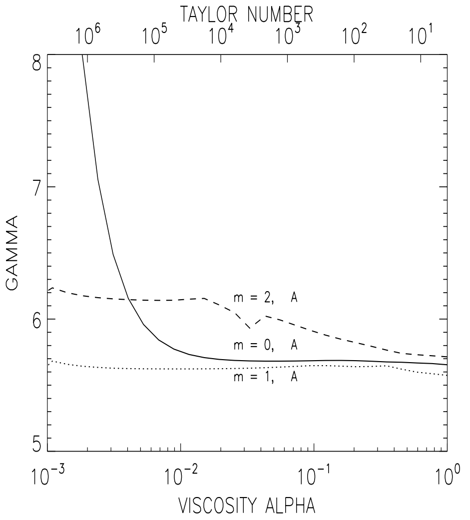

Figure 1. The diagram for marginal instability for a thin standard-accretion disk with Keplerian rotation ($q = 1.5$) and $\tilde{r}_\Omega = 30$. Only the equatorially antisymmetric modes are plotted. The critical values of AKA for the symmetric ones are always larger.

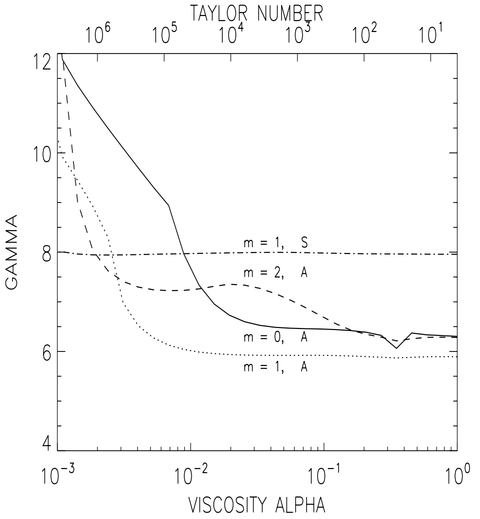

Figure 2. The same as in Fig. 1 but for a galaxy disk with uniform azimuthal velocity ($q = 1$) and $\tilde{r}_\Omega = 10$.

The accretion disk is modeled by assuming a Keplerian profile at large distances from the rotation axis. An aspect ratio of $\tilde{r}_\Omega = 30$ defines the disk to be thin compared to its rigidly rotating core. The galaxy disk is described by a uniform azimuthal velocity and a smaller aspect ratio of $\tilde{r}_\Omega = 10$.

As one can see from Fig. 1, in accretion disks the nonaxisymmetric modes with an azimuthal wave number $m = 1$ are the first which are excited by the action of the AKA-effect. This is true over the whole range of studied values of the viscosity parameter $\alpha_{\rm SS}$, which is commonly assumed to be smaller than unity. With respect to the mirror-reflection about the mid-plane, the antisymmetry is clearly preferred.

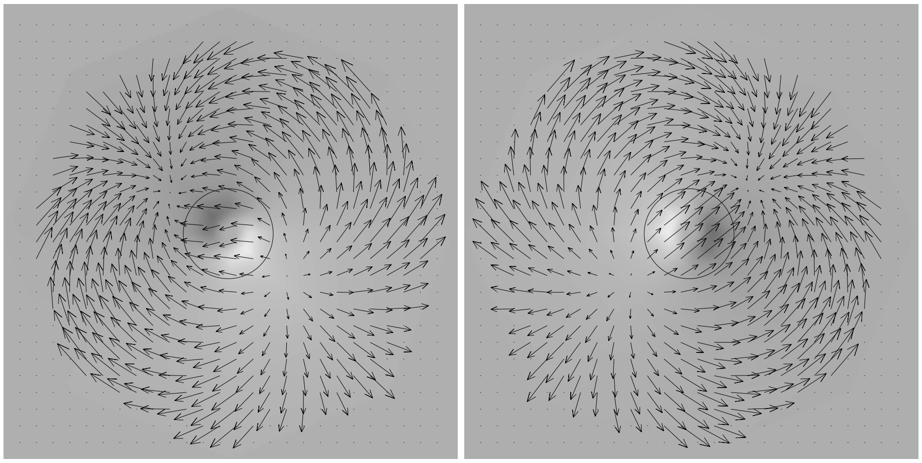

Figure 3. The flow patterns of the marginally-stable A1 excitation at the surface of an accretion disk at $\alpha_{\rm SS} \simeq 0.1$ (LEFT) and $\alpha_{\rm SS} = 0.001$ (RIGHT). The vectorplots show the horizontal velocity at the top of the disk while the greyscale images give the intensity and direction of the vertical flow. Black means downwards, white means upwards direction. The solid circle indicates the boundary of the rigidly rotating core.

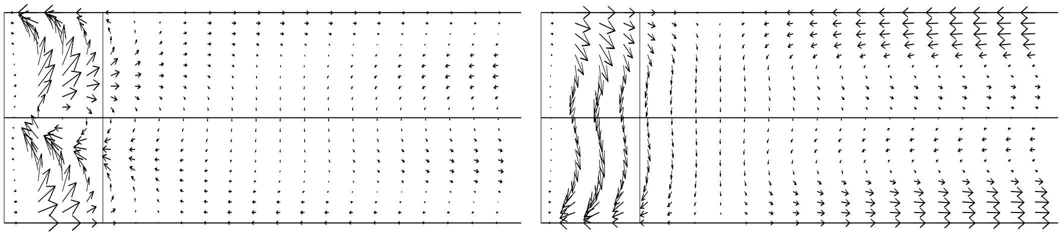

Figure 4. Corresponding vectorplots to Fig. 3 for the flow in the ϖ-z-plane.

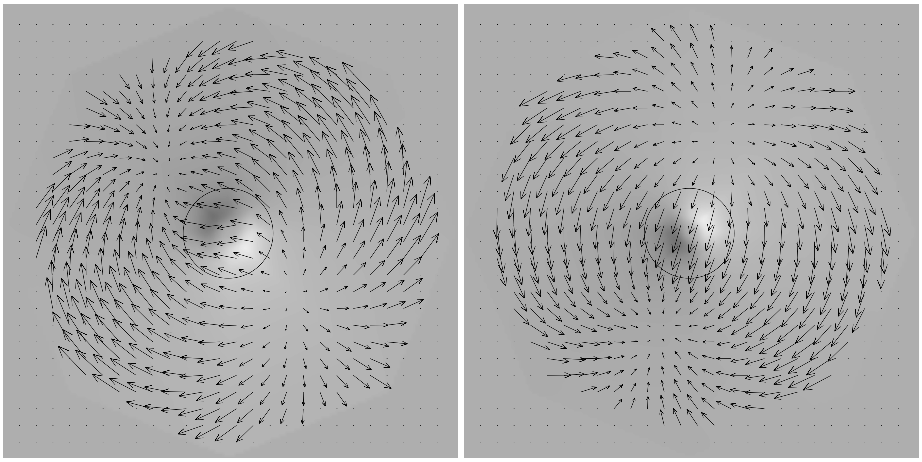

Figure 5. The same as in Fig. 3 but for a galaxy disk. At $\alpha_{\rm SS} \simeq 0.1$ (LEFT) the A1 mode is dominating, while at $\alpha_{\rm SS} = 0.001$ (RIGHT) the dominant mode is S1.

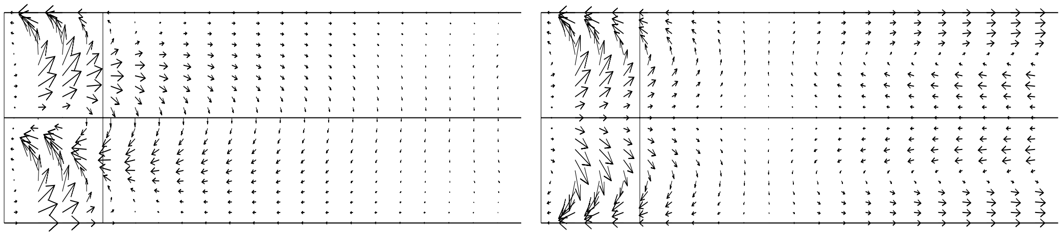

Figure 6. Corresponding vectorplots to Fig. 5 for the flow in the ϖ-z-plane.

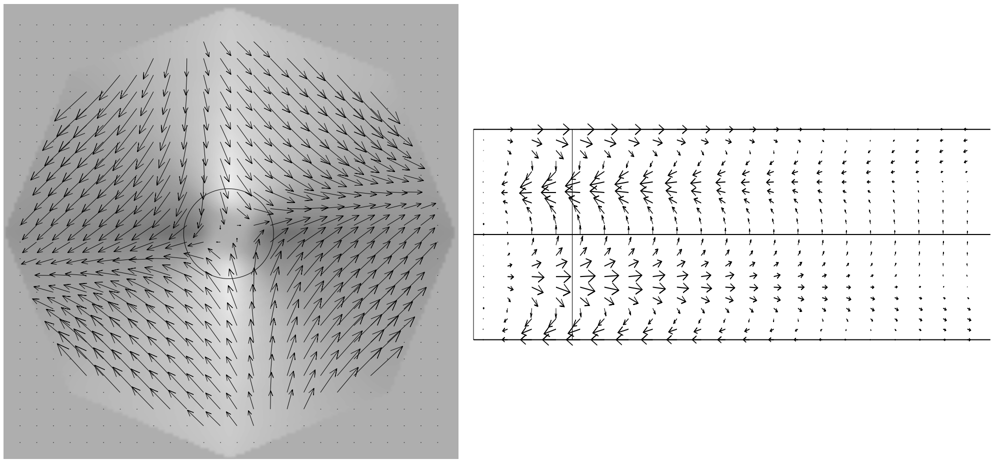

Figure 7. The same as in Figs 5 and 6 for the dominating mode A2 at $\alpha_{\rm SS} \simeq 0.0024$.

The symmetric modes require much higher values for C_Γ to be excited; they are therefore not plotted. Surprisingly, the C_Γ-value is nearly completely independent of $\alpha_{\rm SS}$, assuming values of $C_\Gamma \simeq 5.6$.

In the case of galactic disks, the situation is a slightly more complicated. Fig. 2 shows that the type of mode which is excited first depends on the viscosity alpha. For relatively high $\alpha_{\rm SS}$, A1 dominates and C_Γ remains nearly constant, $C_\Gamma \simeq 6$. With decreasing $\alpha_{\rm SS}$, C_Γ increases and the dominant mode changes from A1 to A2. For $\alpha_{\rm SS} < 2 \cdot 10^{-2}$ (Ta $> 10^6$), C_Γ remains constant at $C_\Gamma \simeq 8$, with the dominant mode now being S1. Therefore, nonaxisymmetric flow structures, e.g. vortices, are always preferred. But the boundary of the instability region is defined by different azimuthal wave numbers and by both symmetry types relative to the mid-plane.

The next figures (Figs 3-7) show the surface flow patterns for the primary A1 (accretion disks) or else A1, A2 and S1 (galaxy disks) modes for different $\alpha_{\rm SS}$ where they are dominant.

The radial and the azimuthal surface velocities describe a vortex structure which is extended over the whole disk (Figs 3, 5 and 7). In the case of $m = 1$ (Figs 3 and 5), there exists outside the rigidly rotating core two spots with almost no horizontal velocity and a vertical flow going in an upwards and downwards direction, respectively. The vertical flow attains its maximum at the mid-plane for the accretion disk (Fig. 4), and between the mid-plane and surface for the galactic disk (Fig. 6). In the small range of $\alpha_{\rm SS}$ between $2 \cdot 10^{-3}$ and $3 \cdot 10^{-3}$, where A2 is dominating, the vertical flow, as well as the horizontal flow, are concentrated outside the core.

All the eigenvalues have finite imaginary parts indicating the rotation of the flow pattern with time. The frequency of this rotation increases with an increasing basic rotation rate (or decreasing viscosity).

We note that the eigenvalue problem for a one-dimensional galaxy-model was solved by Kitchatinov, Rüdiger & Khomenko (1994). The critical value of AKA was found to be $C_\Gamma \simeq 5.98$ which is comparable to $C_\Gamma \simeq 6 - 8$, found here in the three-dimensional model. The flow evolution of the one-dimensional problem was studied in v. Rekowski, Kitchatinov & Rüdiger (1995). As for the present model, the flow patterns are antisymmetric about the mid-plane and rotating on the order of 10^6 years.

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