# SOLAR MESOGRANULATION CORRELATION OBSERVATIONS PROBING THE HELICITY

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**Abstract.** Motivated by new observations of solar surface flow patterns of mesogranulation, theoretical computations of the horizontal divergence-vorticity correlation are presented. Because of its close relation to the helicity in rotating turbulence such observations and discussions are of particular importance for the dynamo theory. For the northern hemisphere we find a small, but always *negative*, divergence-vorticity correlation. Both an analytical Second Order Correlation Approximation for slow rotation as well as a numerical simulation (originally done for accretion disks) for fast rotation yield very similar results.

**Key words:** Mesogranulation, turbulence, dynamo theory

## 1. Introduction

Modern stellar physics considers solar/stellar activity as being driven by a dynamo mechanism maintaining a large-scale magnetic field. The dynamo itself is thought to result from the interaction between turbulence and rotation, the latter modifying the turbulence structure. Non-local theories for the generation of differential rotation as well as large-scale magnetic fields are described by coefficients '$\Lambda$' and '$\alpha$' that capture the influence of the basic rotation on the turbulence.

In the present paper we ask the question whether the rotational influence on the turbulence can be observed at the solar surface. Granulation and mesogranulation are the main candidates due to their nonmagnetic character. Our particular question is whether the *horizontal* motions alone suffice to estimate the rotational influence.

The helicity $\langle \boldsymbol{u}' \cdot \mathrm{rot}\, \boldsymbol{u}' \rangle$ is a key ingredient in dynamo theory. Here we consider that part of the helicity which results from the vertical components of velocity and vorticity,

$$\mathcal{H} = \left\langle w \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \right\rangle , \tag{1}$$

where $(u, v, w) = \boldsymbol{u}'$ denote the deviations from the mean flow in a Cartesian coordinate system $(x, y, z)$. We adopt a right-handed coordinate system (e.g., if $x$ points east, $y$ north, then $z$ points radially outwards.) It is well known (e.g. Krause & Rädler 1980) that in a stratified convection zone rising material expands and rotates because of the action of the Coriolis force. On the northern hemisphere the results are *left-handed* helical motions, i.e. $\mathcal{H} < 0$. Expansion results in clockwise rotation and *vice versa*. Instead of (1) we consider now the scalar quantity

$$\mathcal{B} = \left\langle \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \right\rangle . \tag{2}$$

We adopt the anelastic approximation, $\mathrm{div}\rho\boldsymbol{u} = 0$, and neglect fluctuations in the density, so $\rho = \rho(z)$ and therefore $\mathrm{div}\rho\boldsymbol{u}' = 0$. Thus, we have

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} + w\frac{\partial \ln \rho}{\partial z} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + w\frac{\partial \ln |\rho w|}{\partial z} = 0. \tag{3}$$

We now define a scale height for the turbulence intensity,

$$H_{\rm turb} = -\left( \frac{\partial \ln |\rho w|}{\partial z} \right)^{-1}, \tag{4}$$

so the horizontal velocity divergence is then just $w/H_{\rm turb}$, and therefore

$$\mathcal{B} = \left\langle \frac{w}{H_{\rm turb}} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \right\rangle = \frac{\mathcal{H}}{H_{\rm turb}}. \tag{5}$$

Assuming furthermore that $w$ vanishes slightly above the location where the velocity correlation is measured, we have $H_{\rm turb} > 0$, i.e. the turbulence intensity, as measured by the magnitude of $\rho w$, increases with depth. Thus, from $\partial u/\partial x + \partial v/\partial y = w/H_{\rm turb}$ we see that a positive horizontal divergence corresponds to an updraft ($w > 0$) and a negative horizontal divergence corresponds to a downdraft ($w < 0$), which is in agreement with our expectation.

Given that $\mathcal{B}$ is closely related to $\mathcal{H}$, it should be possible to estimate $\mathcal{H}$ from observations of *horizontal* flow components alone. At least the sign of the helicity should be observable with (5). In the Second Order Correlation Approximation (SOCA) the sign of the helicity is related to the sign of the $\alpha$-effect. For isotropic $\alpha$-effect negative helicity leads to positive $\alpha$ and *vice versa* – both for low and high conductivity (Krause & Rädler 1980). Positive values of $\alpha$ are thus expected for the northern solar hemisphere.* However, with a positive $\alpha$-effect and a rotation law taken from helioseismology inversions the solar dynamo produces the wrong butterfly diagram

* The situation is more complicated if the tensor character of the $\alpha$-effect is taken into account but also in this case the important azimuthal component of the $\alpha$-tensor, $\alpha_{\phi\phi}$, results as positive (Kitchatinov & Rüdiger 1993).

(see Parker 1987). It would, therefore, be useful, to *observe* the helicity in the solar atmosphere in order to compare the results with constraints of dynamo theory.

The helicity $\mathcal{H}$ is a pseudoscalar and its sign depends on the coordinate system. The only pseudoscalar, which can be constructed in anisotropic turbulence with the anisotropy direction $\boldsymbol{G}$, is the scalar product $\boldsymbol{G} \cdot \boldsymbol{\Omega}$, where $\boldsymbol{\Omega}$ is the angular velocity vector. Hence, we expect a nonvanishing correlation $\mathcal{B}$ only for rotating turbulence. The influence of the basic rotation is often represented by the Coriolis number (or inverse Rossby number)

$$\Omega^* = 2\tau_{\rm corr}\ \Omega, \tag{6}$$

i.e. the ratio between correlation time $\tau_{\rm corr}$ and the rotation period.

As the typical mesogranulation pattern only lives for a few hours $\Omega^*$ is estimated to be of order of $10^{-1}$. Hence, the correlation effect $\mathcal{B}$ can only be very small and one will need a sophisticated observational strategy to measure it. In Brandt et al. (1988) and in Simon et al. (1994) first results of an overall inspection of horizontal flow patterns on mesoscales are presented. The maximum velocities are $\sim$ 750 m/s, maximum vertical divergence is $\sim 4 \cdot 10^{-4}$ s$^{-1}$ and the maximum vertical vorticity is $\sim 2 \cdot 10^{-4}$ s$^{-1}$ (see also Simon et al. 1988). Brandt et al. (1988) even report a vorticity of an order of magnitude larger than those values. There are indications for a negative divergence-vorticity correlation (considered in our coordinate system, $\mathcal{B} < 0$) but a more detailed analysis must bring the final finding about the rather small effect.

## 2. Influence of the basic rotation

A radial unit vector $\boldsymbol{g}^\circ$ may be introduced. The desired correlation (2) can generally be written as the $z$-component of the vector

$$\boldsymbol{\mathcal{B}} = \langle {\rm div}\, \tilde{\boldsymbol{u}}\, {\rm rot}\, \boldsymbol{u}' \rangle \tag{7}$$

with $\tilde{\boldsymbol{u}} = \boldsymbol{u}' - (\boldsymbol{g}^\circ \cdot \boldsymbol{u}')\boldsymbol{g}^\circ$ as the horizontal component of the random flow field. The $z$-component of (7) forms the desired divergence-vorticity correlation (2).

It makes sense to transform the latter to the random momentum field with $\boldsymbol{m}' = \rho \boldsymbol{u}'$, i.e.

$$\rho\, {\rm rot}\, \boldsymbol{u}' = {\rm rot}\, \boldsymbol{m}' + \boldsymbol{m}' \times \boldsymbol{G}, \tag{8}$$

$$\rho\, {\rm div}\, \boldsymbol{u}' = {\rm div}\, \boldsymbol{m}' - \boldsymbol{G} \cdot \boldsymbol{m}', \tag{9}$$

with $\boldsymbol{G} = \nabla \log\rho$. It follows $\rho\, \boldsymbol{g}^\circ \,\mathrm{rot}\, \mathbf{u}' = \mathbf{g}^\circ \,\mathrm{rot}\, \boldsymbol{m}'$ and $\rho\, \mathrm{div}\, \tilde{\boldsymbol{u}} = \mathrm{div}\, \tilde{\boldsymbol{m}}$ so that

$$\boldsymbol{\mathcal{B}} = \frac{1}{\rho^2}\ \langle \mathrm{div}\, \tilde{\boldsymbol{m}} \cdot \mathrm{rot}\, \boldsymbol{m}' \rangle. \tag{10}$$

With the Fourier transform

$$\boldsymbol{m}'(\boldsymbol{x}, t) = \int \hat{\boldsymbol{m}}(\boldsymbol{k}, \omega) e^{i(\boldsymbol{k}\boldsymbol{x} - \omega t)} d\boldsymbol{k}\, d\omega \tag{11}$$

we obtain

$$\rho^2\ \mathcal{B}_i = -\epsilon_{ijn} \int k'_j (k_f - (\boldsymbol{g}^\circ \boldsymbol{k}) g^\circ_f)$$

$$\langle \hat{m}_f(\boldsymbol{k}, \omega) \hat{m}_n(\boldsymbol{k}', \omega') \rangle e^{i(\boldsymbol{k}+\boldsymbol{k}')\boldsymbol{x} - (\omega+\omega')t}\, \boldsymbol{dk}\, \boldsymbol{dk'} d\omega\, d\omega', \tag{12}$$

where we have used the mass conservation law in an anelastic medium

$$\mathrm{div}\, \boldsymbol{m}' = 0. \tag{13}$$

In a turbulent medium the spectral tensor in (12) is

$$\langle \hat{m}_i(\boldsymbol{k}, \omega)\, \hat{m}_j(\boldsymbol{k}', \omega') \rangle = \frac{E(K, \omega, \kappa)}{16\pi k^2}$$
$$\left( \delta_{ij} - \frac{K_i K_j}{K^2} + \frac{\kappa_i K_j - \kappa_j K_i}{2K^2} + \frac{\kappa_i \kappa_j}{4K^2} \right) \delta(\omega + \omega'), \tag{14}$$

where $\boldsymbol{K}$ and $\boldsymbol{\kappa}$ are the wave vectors for large as well as small scales (Kitchatinov & Rüdiger 1993). It is simply $\boldsymbol{K} = (\boldsymbol{k} - \boldsymbol{k}')/2$ and $\boldsymbol{\kappa} = \boldsymbol{k} + \boldsymbol{k}'$.

Insertion of (14) into (12) does not provide any nonvanishing values. Only the influence of the basic rotation onto the turbulence produces a finite effect. In Kitchatinov & Rüdiger (1993) the rotational influence is computed in full detail. The results must be introduced into (12), which leads to

$$\mathcal{B}_i = -\frac{3\Omega_j}{70\rho^2}\ \frac{\partial^2}{\partial x_i \partial x_j}\ \int \nu k^2 E(k, \omega, \boldsymbol{r}) \frac{\nu^2 k^4 + 7\omega^2}{(\omega^2 + \nu^2 k^4)^2} dk d\omega -$$
$$-\frac{G_i \Omega_j}{35\rho^2}\ \frac{\partial}{\partial x_j} \int E(k, \omega, \boldsymbol{r}) \frac{\nu k^2}{\omega^2 + \nu^2 k^4} dk d\omega\ -$$
$$-\frac{2}{105} G_{ij} \Omega_j \int \nu k^2 E(k, \omega, \boldsymbol{r}) \frac{\nu^2 k^4 - \omega^2}{(\omega^2 + \nu^2 k^4)^2} dk d\omega. \tag{15}$$

The result written here is of first order in the rotation rate. After adopting the mixing length approximation the result is

$$\mathcal{B}_i = -\Bigg[ \frac{3}{70\rho^2} \Omega_j\ \frac{\partial^2}{\partial x_i \partial x_j} (\rho^2 \langle u'^2 \rangle \tau_{\mathrm{corr}}) +$$
$$+ \frac{G_i \Omega_j}{35\rho^2}\ \frac{\partial}{\partial x_j} (\rho^2 \langle u'^2 \rangle \tau_{\mathrm{corr}}) + \frac{2}{105} G_{ij} \Omega_j \langle u'^2 \rangle \tau_{\mathrm{corr}} \Bigg]. \tag{16}$$

When the inhomogeneity of the turbulence intensity can be neglected compared with that of the density stratification,

$$\mathcal{B}_i = -\left[\frac{8}{35}G_i(\boldsymbol{G\Omega}) + \frac{11}{105}G_{ij}\Omega_j\right]\langle u'^2\rangle\tau_{\rm corr} \tag{17}$$

is obtained. Its $z$-component,

$$\mathcal{B} = -\frac{8}{35}\frac{\alpha_{\rm MLT}^2}{\gamma^2}\frac{\Omega}{\tau_{\rm corr}}\cos\theta, \tag{18}$$

proves to be *negative* on the northern hemisphere – as expected from (5). Here, $\alpha_{\rm MLT}$ is the standard mixing length parameter and $\gamma$ is the ratio of specific heats.

## 3. Simulations

In order to test the feasibility of estimating $\mathcal{H}$ from horizontal velocity measurements we now analyse data from numerical simulations of stratified shear flow turbulence. Those data are particularly interesting, because the flow supports large scale dynamo action. All details about this simulation can be found in Brandenburg et al. (1995). For the present purpose it suffices to say that the simulation covers part of an accretion disc, where the density varies by a factor of thirty in the $z$-direction between the midplane and the top and bottom surfaces. The inverse Rossby number, $\Omega^*$, is around 20 and the Reynolds number with respect to the turbulence is around 100, and with respect to the shear flow around 750. The relative helicity of the flow, i.e. the helicity normalized with respect to the root-mean-square values of vorticity and velocity is less than 5%. We mention this, because it is important to realise that even though the system is rapidly rotating and stratified, the helicity is well below the maximum possible value. This is partly because significant cancellation is taking place from flow regions where the helicity may locally be large, but of either sign.

We now proceed by estimating the helicity from horizontal velocity measurements. We adopt two technically different approaches. First we construct a correlation plot between horizontal divergence and vorticity; see Figure 1. This corresponds to the plus signs in Figure 2, where we have plotted the dependence of the correlation coefficient on $z$. We also compute the quantity $\mathcal{B}$ as in Eq. (7). This corresponds to the asterisks in Figure 2.

We find that $\mathcal{B}$ is positive in the lower part of the midplane and negative above. This is consistent with the expected, and directly measured, sign of the helicity. However, the scatter is very large. Thus, in order to obtain a reliable result from observations of solar mesogranulation, it is important to

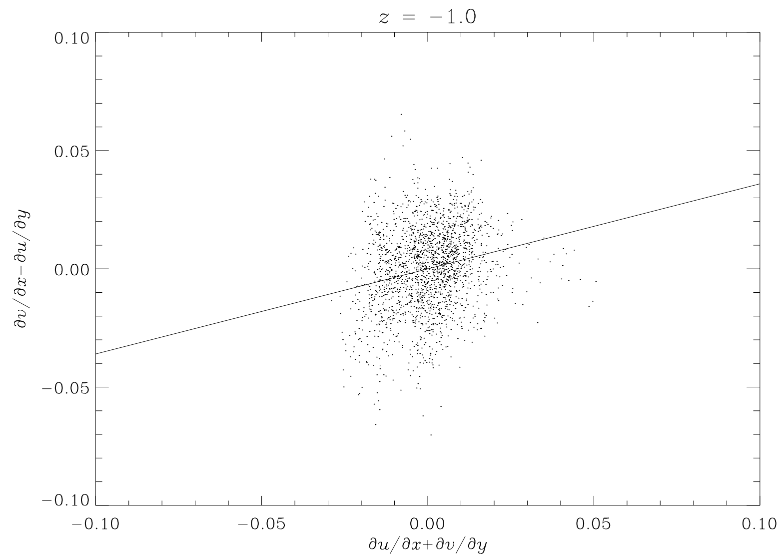


*Figure 1.* Scatter plot between horizontal divergence and horizontal vorticity an a horizontal plane for $z = -1.0$. The positive correlation indicates positive helicity for negative values of $z$. Note also that the scatter is relatively large.

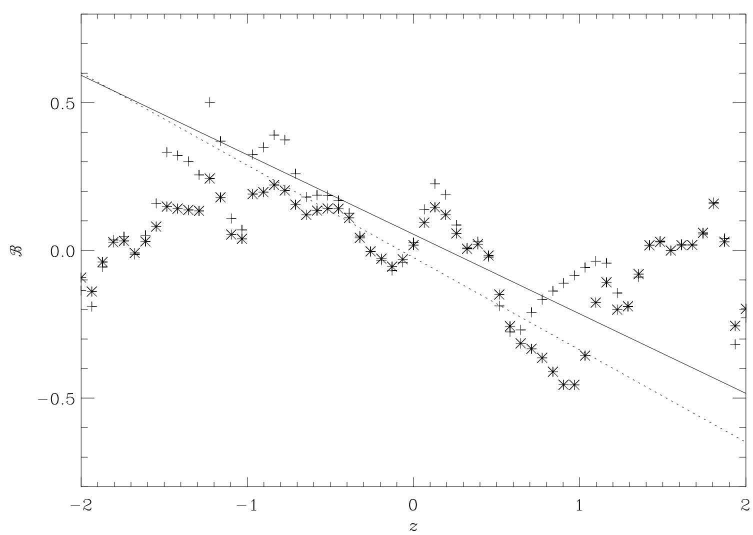


*Figure 2.* Correlation coefficients as a function of $z$. Plus signs denote the correlation coefficient obtained from a least-square fit to a scatter plot between horizontal divergence and vorticity. The asterisks were obtained by calculating $\mathcal{B}$. The solid line gives a least-square fit through the plus signs and the dotted line is the fit for the asterisks.

produce good averages using data at many different times to eliminate the naturally occurring fluctuations of the turbulence.

Finally, it is worth noting that in this simulation the sign of the $\alpha$-effect (important in dynamo theory) is opposite to that expected from the sign of the helicity; see Brandenburg et al. (1995) and Brandenburg & Donner (1997). Thus, while estimates of the solar helicity may be of interest in its own right, they may not be directly relevant to the solar dynamo.

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