

Oscillating α^2 -dynamos and the reversal phenomenon of the global geodynamo

A. GIESECKE, G. RÜDIGER, AND D. ELSTNER

Astrophysikalisches Institut Potsdam, An der Sternwarte 16, D-14482 Potsdam, Germany

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Abstract. A geodynamo-model based on an α -effect which has been computed under conditions suitable for the geodynamo is constructed. For a highly restricted class of radial α -profiles the linear α^2 -model exhibits oscillating solutions on a timescale given by the turbulent diffusion time. The basic properties of the periodic solutions are presented and the influence of the inner core size on the characteristics of the critical range that allows for oscillating solutions is shown. Reversals are interpreted as half of such an oscillation. They are rather seldom events because they can only occur if the α -profile exists long enough within the small critical range that allows for periodic solutions. Due to strong fluctuations on the convective timescale the probability of such a reversal is very small. Finally, a simple non-linear mean-field model with reasonable input parameters based on simulations of Giesecke et al. (2005) demonstrates the plausibility of the presented theory with a long-time series of a (geo-)dynamo reversal sequence.

Key words: physical data and processes – Earth – Magnetohydrodynamics

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1. Introduction

Paleomagnetic measurements show that the Earth's magnetic field exists for more than 10^9 years with nearly the same magnitude (Kono & Tanaka 1995). The process that is responsible for the production of this field is called the *geodynamo* and essentially takes place in the fluid outer part of the Earth's core. The magnetic field is dominated by a dipole which – as the most characteristic feature – from time to time “starts to oscillate” and changes its polarity from one sign to the other. This phenomenon has been called *reversals* and typically lasts $10^3 - 10^4$ years (Bogue & Merrill 1992). The duration of such “oscillating” phase is extremely short compared with the time between consecutive reversals. In fact the average time between two reversals is about 50 times longer than the duration of the reversal itself. The appearance of the reversals seems to be chaotic rather than periodic (Krause & Schmidt 1988), and the probability of a reversal during a certain time span can be described by a Poisson distribution. On geological time scales ($\sim 10^7 - 10^8$ years) the average rate of reversals changes (Merrill et al. 1996) and there exist even very long periods where no reversal occurs – so called *superchrons*.

Closely related to the reversal phenomenon are so called *excursions* a kind of aborted reversals, where the polarity begins to change but, instead of executing a full transition, the dipole returns to the original polarity. Excursions occur about ten times more often than reversals.

Deviations from a perfectly axisymmetric field become manifest in the tilt of the dipole axis with respect to the rotation axis (currently 11°) and in the non-axisymmetric field components in terms of localized flux patches. Such field patterns in average exhibit a common directed drift motion, the so called *westward drift* (see e.g. Bloxham & Jackson 1989; Bloxham & Jackson 1992). Roughly simplifying the value of this westward drift amounts approximately $0.3^\circ/\text{year}$.

Simulations of the 3D MHD-equations (that describe a thermal/chemical driven turbulent flow of a conducting fluid in the Earth's outer core and the magnetic field that is induced by this flow) have been able to reproduce some of the observed features of the Earth's magnetic field (Glatzmaier & Roberts 1996; Kageyama & Sato 1997; Christensen et al. 1998; Kuang & Bloxham 1999). Unfortunately such calculations are computationally very expensive. The time periods that can be covered are rather short compared to the time scales on which for example changes of the mean reversal rate occur. In order to examine geodynamo-models in matters of the

Correspondence to: agiesecke@aip.de

statistics of the reversal phenomenon mean-field models remain indispensable.

A further unsolved issue is the influence of the small scales. Global MHD simulations are restricted in the achievable parameter regime and in the affordable spatial resolution. These limits prevent from resolving the actual scales of the turbulence in the fluid outer core, and for reasons of numerical stability unphysical large values for the viscous losses have to be adopted. Therefore the smaller scales are artificially damped, and properties and influence of the small scale turbulence remains unsure.

Sarson & Jones (1999) developed a 2.5D model to examine the reversal mechanism in detail. Their general picture is a “large scale” $\alpha\Omega$ -dynamo mechanism, where a strong zonal flow and meridional circulations are responsible for the dynamo action. Reversals are induced by fluctuations of the meridional flow. The authors reclaim that a opposite dipole polarity can evolve if the velocity remains long enough in a regime that allows for oscillating magnetic fields.

Statistical properties in a one-dimensional $\alpha\Omega$ -dynamo model where reversals are triggered by a fluctuating α -effect have been examined by Hoyng et al. (2002). Their model was able to predict some basic features of the Sint-800 data and although there have been some contradictions in general their mean-field approach seems to be selfconsistent.

Other models, based on the absence of differential rotation (or any shear flows) in the Earth’s core, interpreted the geodynamo as an α^2 -dynamo (Steenbeck & Krause 1966). It is known that the spherical α^2 -dynamo “almost always” possesses stationary axisymmetric magnetic field solutions (for scalar α -effect) or longitudinally drifting non-axisymmetric solutions (Rüdiger et al. 2003) and therefore such models have difficulties to explain the reversal phenomenon. There are few exceptions of the rule that α^2 -dynamos with scalar α -effect are stationary.

Fearn & Rahman (2004) solved the Navier-Stokes equation and a mean-field induction equation for an α^2 -dynamo model with a radial dependence of the α -effect given by $\alpha \propto \sin \pi(r - R_{\text{in}})$. This radial profile leads to a vanishing α -effect at the boundaries of the fluid outer core but the α -effect does not show any zero within the interior. They obtained non-linear periodic solutions if the α was larger than a certain critical value. In contrast to the solutions of Sarson & Jones (1999) their results were strongly influenced by the non-linear back-reaction of the Lorentz force on the flow which serves as a saturation mechanism for the magnetic field.

Without considering any mean flow, Stefani & Gerbeth (2003) found oscillating α^2 -dynamos in case that the α -effect (uniform in θ) changes its sign in radius. Already Soward (1974) with his quasi-linear approximation for rotating convection in layers with uniform density found that the α -effect strongly varies with depth: it is negative (positive) in the lower (upper) part of the convection layer – well described by a radial sinus-function. Giesecke et al. (2005) have shown that such profiles indeed result from numerical simulations of the convection in the outer fluid core where the density stratification is very small (see Sect. 3).

In the present paper we shall show that radial sin-profiles of the α -effect lead to oscillating α^2 -dynamos but already a slight deviation from this profile provides stationary modes. Combining the principle properties of the α -effect from the calculations of Giesecke et al. (2005) with a simple α^2 -dynamo, a mean-field model is constructed that exhibits irregular reversals induced by a fluctuating α -effect.

2. The equations

Taking the induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \quad (1)$$

and split magnetic field \mathbf{B} and velocity \mathbf{u} in a mean part, $\langle \mathbf{B} \rangle$, $\langle \mathbf{u} \rangle$ and a fluctuating component \mathbf{B}' , \mathbf{u}' the mean magnetic field $\langle \mathbf{B} \rangle = \mathbf{B} - \mathbf{B}'$ is determined by

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times (\langle \mathbf{u} \rangle \times \langle \mathbf{B} \rangle + \mathcal{E} - \eta \nabla \times \langle \mathbf{B} \rangle) \quad (2)$$

with $\mathcal{E} = \langle \mathbf{u}' \times \mathbf{B}' \rangle$ as the mean electromotive force (EMF) and η the (molecular) magnetic diffusivity. The components of the EMF are usually given by

$$\mathcal{E}_i = \alpha_{ij} \langle B_j \rangle + \beta_{ijk} \partial_k \langle B_j \rangle. \quad (3)$$

The tensor α_{ij} correlates the EMF due to turbulent motions with the large-scale magnetic field, including the effects of anisotropy. In the simplest case the tensor β_{ijk} is related to the turbulent diffusivity by $\beta_{ijk} = \eta_T \epsilon_{ijk}$ which is the case that we shall discuss here. In the following all mean flows $\langle \mathbf{u} \rangle$ are neglected and we end up with a mean-field induction equation that describes an α^2 -dynamo:

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times (\alpha \langle \mathbf{B} \rangle - \eta_T \nabla \times \langle \mathbf{B} \rangle), \quad (4)$$

where $\eta_T \gg \eta$ is implied. Equation (4) together with a prescribed α -effect that depends on the radius r and the latitude angle θ is solved numerically using an explicit finite difference scheme in two dimensions in spherical coordinates. The standard resolution is 64×64 grid points in both radial and latitudinal directions. A perfect conductor is assumed to exist at the inner core boundary which is justified because of the absence of turbulent motions in the solid inner core so that the diffusivity is significant smaller inside the solid core than inside the fluid outer core. However, a finitely conducting inner core affects the behavior of the magnetic field as it has been shown e.g. by Hollerbach & Jones (1993). At the outer boundary a vacuum is simulated by increasing the magnetic dissipation by a factor of 10. The details of the numerical realizations of these boundary conditions are described by Rüdiger et al. (2003). To estimate general properties of the linear mean-field model at first a quenching mechanism is abandoned to avoid the complicated questions that are associated with the non-linearities. Only for the long time simulations in Sect. 4 an equilibration mechanism is used to prevent the field from growing to infinity.

3. Geodynamo α -effect

3.1. General properties

Fig. 1 shows a typical radial dependence of the α -effect computed from local simulations of weak stratified and fast rotating magnetoconvection by Giesecke et al. (2005). The α -

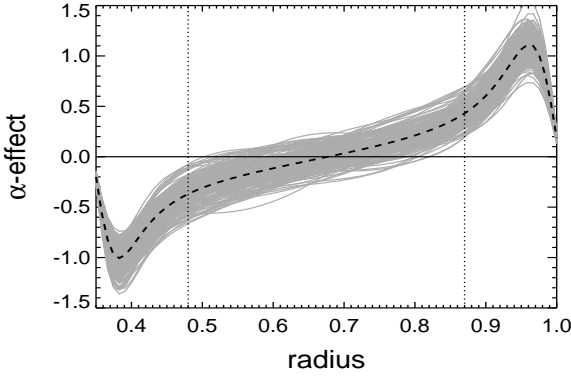


Fig. 1. Radial dependence of the α -effect for $\Lambda = \frac{B^2}{2\mu\rho\Omega\eta} = 1$. The dashed thick line denotes the time average and the grey lines show fluctuations in time.

effect on the northern hemisphere shows a maximum (minimum) close to the upper (lower) boundary and the cross-over takes place exactly in the middle of the convective unstable layer. The radial profile is almost perfectly antisymmetric with respect to the middle of the layer so that the net α -effect (integrated over r) approximately vanishes.

The dotted lines divide the domain in three parts. In the outer zones the exact determination of the α -effect is difficult because close to the boundaries strong gradients of the magnetic field require the exact knowledge of the turbulent diffusivity η_T for an calculation of the α -effect from Eq. (3). In the central part – between $r = 0.48$ and $r = 0.87$ – the field gradients are negligible and thus the result should be more reliable. But even if the presented profile comes with some uncertainties, the qualitative overall behavior can quite well be described by $\alpha \sim \sin r$, where the argument of the sin must be chosen in a way that the α -effect disappears at the inner and the outer core boundary. If a density stratification is included in the simulations, the α -effect cross-over moves more and more towards the bottom of the box (see Rüdiger & Hollerbach 2004 their Fig. 4.23).

3.2. Oscillating α^2 -dynamoes

The general properties of the above presented α -effect are used as an input for a global axisymmetric α^2 -dynamo. The model is a spherical shell with the inner radius R_{in} and the outer radius R_{out} . For the present date Earth the radius of the solid inner core is given by $R_{in} = 1222$ km and the radius of the fluid outer core is given by $R_{out} = 3480$ km. In all simulations the radius of the outer core is scaled to 1 and to maintain the correct ratio, the radius of the inner core is

scaled to 0.35. In the following the α -tensor is antisymmetric with respect to the equator ($\sim \cos \theta$). The “standard profile” of the α -effect is given by

$$\alpha(r, \theta) = \alpha_0 \cos \theta \sin \left(2\pi \frac{r - R_{in}}{R_{out} - R_{in}} \right). \quad (5)$$

This equation is slightly modified to vary the amplitude in the upper (lower) half of the sphere and the zero-crossing of the α -effect. We start with a strict sin-profile in radius of the α -effect, i.e. with a cross-over R_0 in the middle between $R_{in} = 0.35$ and $R_{out} = 1$ and equal amplitudes. If supercritical the field grows exponentially and the resulting dynamo oscillates.

The condition to R_0 for the excitation of the oscillation of α^2 -dynamoes is very strict. The oscillations only exist for $R_0 \approx 0.67 \dots 0.70$. If the radial α -profile does not lie in the narrow area indicated in Fig. 2 (top) then the dynamo does not oscillate. The periodic solution due to the strict radial sin-

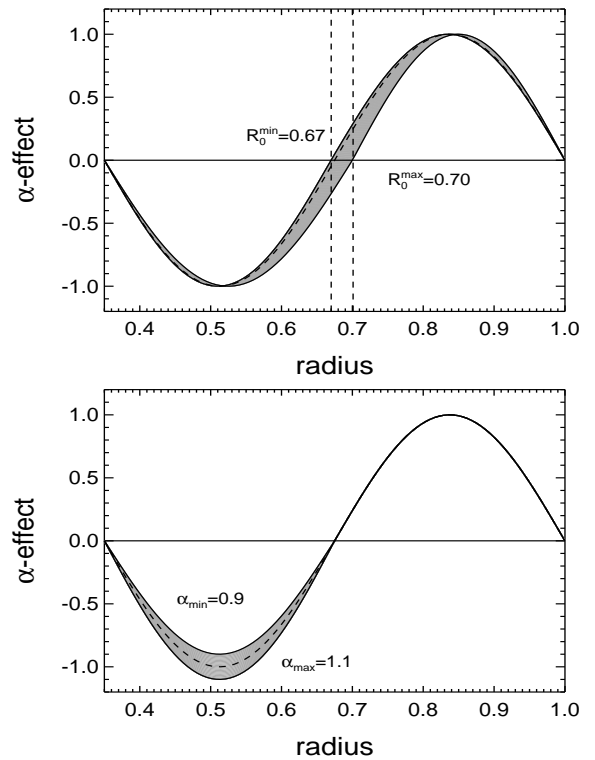


Fig. 2. Critical domains for the radial profile of the α -effect that lead to oscillating solutions. Top: critical interval for the location of the zero. Bottom: critical interval for the magnitude of the lower amplitude

profile of the α -effect can also be suppressed by a variation of one of the amplitudes. Numerical experiments were made by multiplication of the lower part of the sin-profile with a factor A . Oscillating solutions are found with $\delta A \simeq 0.1$ (see bottom of Fig. 2). Of course, a similar result would hold for the upper part of the radial sin-profile but the general property of the model is now known: The oscillating solution only exists if the deviations of the actual α -effect profile from the profile given by Eq. (5) are very small.

Due to fluctuations a simple radial sin-profile of the α -effect is rather seldom. A possible oscillation of the dynamo only happens if the profile of the α -effect lasts long enough within the critical range that allows for periodical solutions. The minimum time which must be covered by the (critical) radial α -profile in order to excite (half of) an oscillation has been estimated from the simulations and is given approximately by

$$t_{\min} \approx 0.3 \cdot \tau_{\text{diff}} \quad (6)$$

with the diffusion time τ_{diff} defined by

$$\tau_{\text{diff}} = \frac{R_{\text{out}}^2}{\eta}. \quad (7)$$

If this minimum time is not reached by the radial profile of the α -effect the stationary solution would not start to become oscillating.

In all simulations the time the dipole needs for the transition from one polarity to the other is of the order of τ_{diff} . If we assume $\eta \simeq 2 \cdot 10^4 \text{ cm}^2/\text{s}$ (for molten iron under conditions in the fluid outer core) we retrieve $\tau_{\text{diff}} \sim 10^5$ years. To keep the model consistent with the observed duration of a reversal (10^4 years) we have to assume a turbulence-induced enhancement of the magnetic diffusivity by one order of magnitude:

$$\eta_{\text{T}} \approx 10 \cdot \eta. \quad (8)$$

With $\eta_{\text{T}} \approx 20 \text{ m}^2/\text{s}$ the diffusion time reduces to 10^4 yrs, the typical duration of a reversal. The ratio $\eta_{\text{T}}/\eta \approx 10$ seems not to be totally devious and was also the result of a rough estimation of Giesecke et al. (2005). A comparable value has been presented by Hoyng et al. (2002) who independently determined a turbulent diffusion time of 10000 – 15000 years from the analyzation of autocorrelation functions from the Sint-800 observations. However, it should be kept in mind, that this are very rough estimations.

3.3. Field pattern of a reversal

Figure 3 shows the temporal behavior of the magnetic field projected on a meridional plane during one oscillation. The left-hand side of each panel shows the isolines of the toroidal field component where solid (dashed) lines denote field directions clockwise (counterclockwise). The arrows on the right hand side represent the poloidal field component. The length of the arrows is scaled with the field length. The magnetic field is concentrated near the rotation axis which is a consequence of the latitudinal $\cos \theta$ -dependence of the α -effect. The solution is of dipolar parity. Regarding the toroidal component in the northern hemisphere the considered reversal cycle starts close to the rotation axis with a clockwise oriented toroidal field in the upper half of the outer core and a field of opposite sign starts in the lower part of the outer core. The outer part weakens (1,2) and is replaced by a growing toroidal field of opposite sign (2,3,4). Between the two belts of equally counterclockwise (clockwise) oriented components that determine the appearance of the magnetic field in the northern (southern) hemisphere in the middle of the reversal sequence (5,6) a new opposite directed field appears (7) and pushes away the lower toroidal component (8). The

reversal is completed in panel (9) when the lower part of the sphere is completely filled out by this new reversed oriented field. Note that the poloidal field component already shows the reversed polarity in part (6) of Fig. 3 and only undergoes minor changes in the remainder of the sequence.

3.4. Oscillation period and critical dynamo number

Figure 4 shows the oscillation period (dashed line, in units of τ_{diff}) and the critical dynamo number (solid line)

$$C_{\alpha}^{\text{crit}} = \frac{\alpha^{\text{crit}} R_{\text{out}}}{\eta_{\text{T}}} \quad (9)$$

in dependence of the location of the zero of the α -effect (R_0). C_{α}^{crit} determines the minimum amplitude of the α -effect at which dynamo-action occurs. The vertical dotted lines indicate the transition between oscillating and stationary solutions. The critical dynamo number is always larger for the oscillating solutions. This is not a surprising result. The oscillating solutions have smaller scales than the stationary solutions so that the Ohmic losses are larger for the oscillating solutions. The immediate consequence is that for given amplitude of α the oscillating solutions are less nonlinear than the non-oscillating solutions. In the center of the crit-

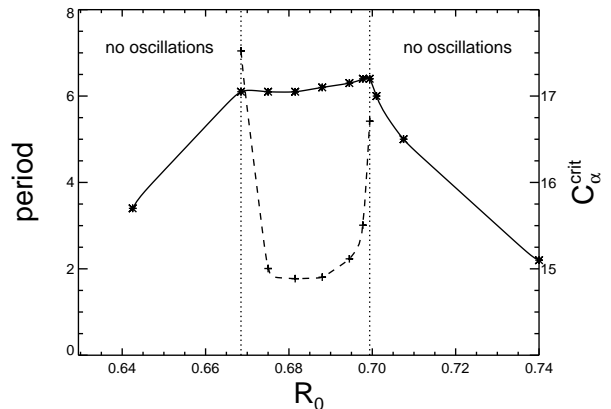


Fig. 4. The characteristics of the linear α^2 -dynamo model in dependence of the location of the cross-over point R_0 in the radial α -profile.

ical interval the oscillation-period is nearly constant for R_0 and increases strongly if the zero of the α -effect is close to the upper or lower boundary that separates the oscillating solutions from the stationary states. The values of the critical dynamo number for the symmetric and antisymmetric (with respect to the equator) axisymmetric modes (S0, A0) and for the first non-axisymmetric modes (A1, S1) are specified in Table 1. Since the dominant part of the Earth's magnetic field is a dipole the A0 mode is of profound interest. Note that the A0-mode (leads to a dipole solution) and the S0-mode (leads to a quadrupole solution) coincide within our numerical accuracy.

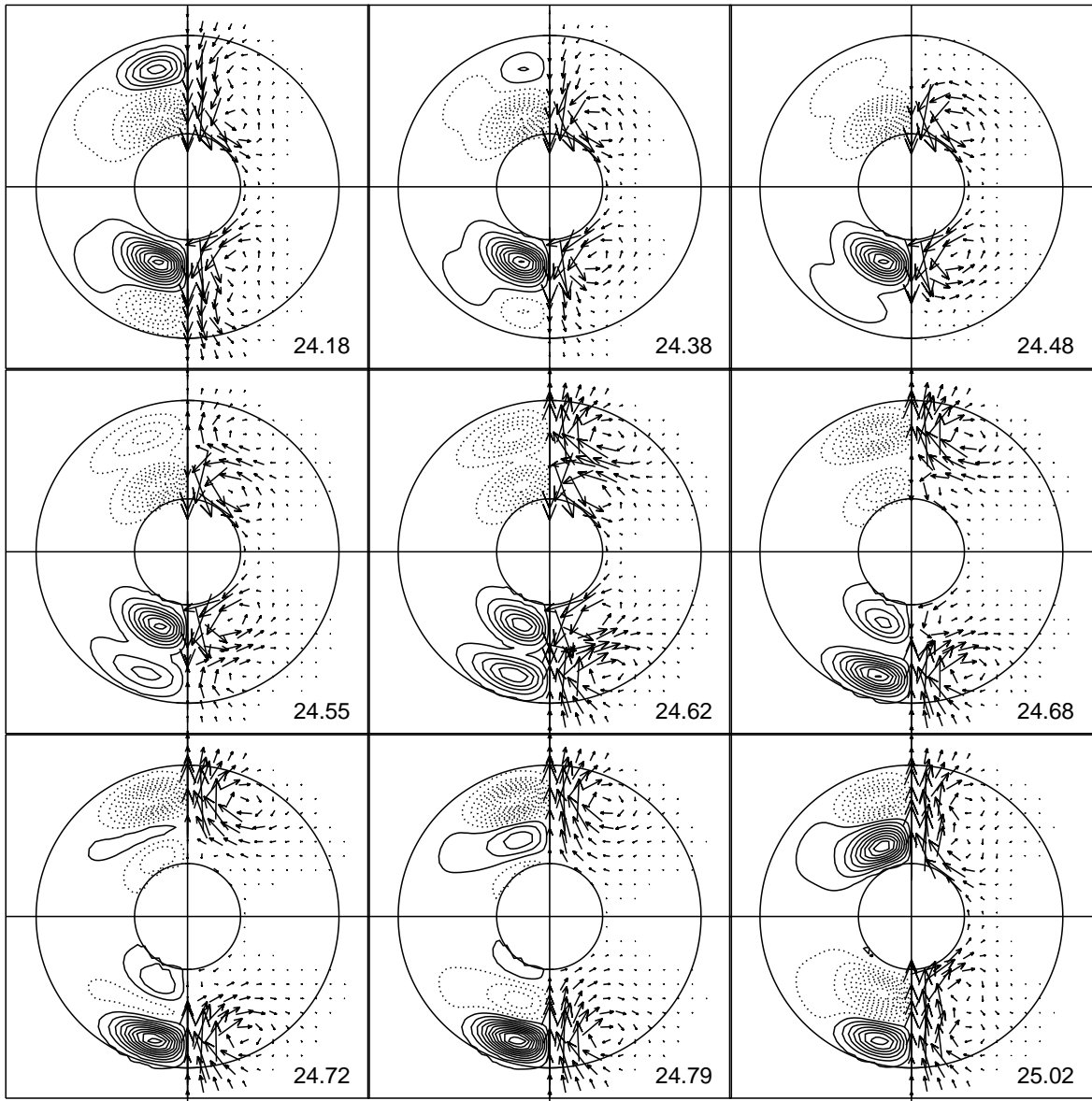


Fig. 3. The magnetic field configuration of the cyclic solution with a strict radial sin-profile of the α -effect. The time in units of the diffusion time is printed in the lower right corner of each snapshot.

Table 1. Critical dynamo number for different basic dynamo modes for α taken from Eq. (5) ($R_0 = 0.675$)

	A0	A1	S0	S1
C_α^{crit}	17.04	17.67	17.04	17.67

3.5. Influence of the inner core size

The solid inner core of the Earth is growing on geological time scales (hundreds of millions of years). Due to the specific thermodynamic conditions in the Earth's interior the liquid iron first freezes out at the center of the Earth because the melting point of iron decreases faster with increasing pres-

sure than the temperature increases towards the center. The appearance of a solid inner core few billion years ago resulted in important changes of the physical conditions and processes that dominate the flow in the fluid core. It is obvious that the size of the solid inner core also should have a strong influence on the oscillations of the α^2 -model that has been presented above. We restrict our examination to the geometric effects that arise from different sizes of the inner core and do not consider the changes in the turbulence that are associated e.g. with the emerging compositional convection. Figure 5 shows the critical profiles of the α -effect that lead to oscillating solutions for different ratios of $R_{\text{out}}/R_{\text{in}}$. For increasing size of the inner core – indicated by the solid vertical line on the left side – the critical interval becomes smaller (indicated by the dashed line). At the same time the center of this interval

moves closer to the center of the fluid outer core (denoted by the dotted line). For $R_{\text{in}} = 0.1$ (bottom curve) the critical cross-over-points of the α -profiles are clearly located in the upper half of the fluid outer core, whereas for $R_{\text{in}} = 0.7$ (top curve) the critical profile is nearly a perfect sin-profile as given by Eq. (5) where the zero is located exactly in the middle of the fluid outer core and the width of the critical interval has become very small.

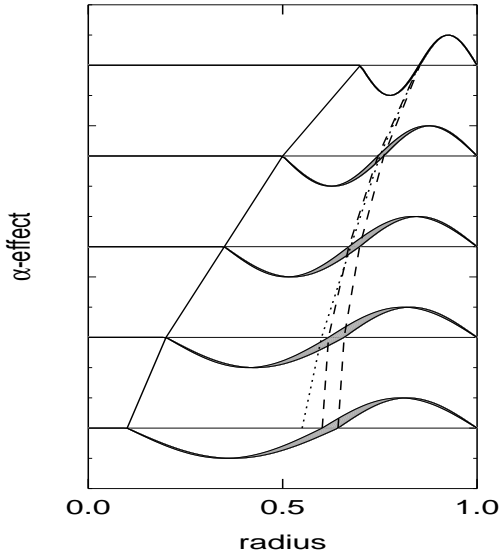


Fig. 5. Critical α -profiles for oscillating solutions for different sizes of the inner core. From top to bottom: $R_{\text{in}} = 0.7, 0.5, 0.35, 0.2, 0.1$. The dashed lines confine the critical interval and the dotted line denotes the center of the fluid outer core.

Although the critical interval becomes smaller with increasing size of the inner core – making it more difficult to excite an oscillation – one can speculate that the overall probability for a reversal in case of a fluctuating α -profile might be approximately constant (or at least > 0) for all sizes since the center of the interval moves towards the middle of the fluid outer core which is the preferred location for the zero cross-over of the α -effect (see Fig. 1).

3.6. Non-axisymmetric modes

The α -effect is a nontrivial tensor if the rotation is fast: $\alpha_{zz} \rightarrow 0$ for $\Omega \rightarrow \infty$ (in cylindrical coordinates, see Moffatt 1970; Rüdiger 1978; Busse & Miin 1979) where α_{zz} refers to the α -effect in cylindrical coordinates and z denotes the direction parallel to the rotation axis. The tensorial structure of the α -effect is now taken into account, i.e. relation the $\alpha_{zz} = 0$ is used in the dynamo equation.

Figure 6 shows C_{α}^{crit} (solid line) and the drift period (dashed line) for the lowest non-axisymmetric modes. Again the results for the symmetric mode (S1) and the antisymmetric mode (A1) coincide in both quantities. The dotted vertical lines denote the critical interval for oscillating solutions for the A0- respective S0-mode for the (scalar) isotropic α -tensor as described in the previous section.

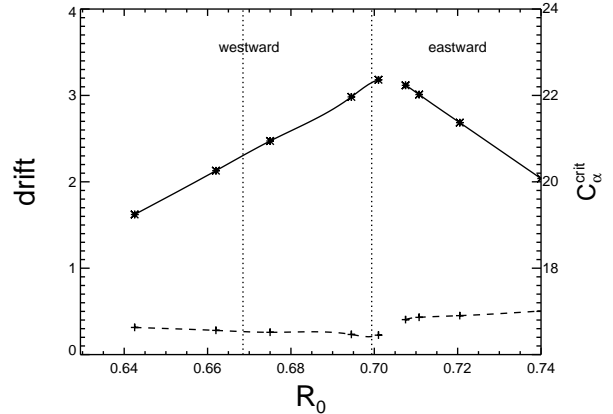


Fig. 6. C_{α}^{crit} of the non-axisymmetric modes for the case $\alpha_{zz} = 0$. The corresponding drift periods are given by the dashed line.

For the zero of the α -effect below $R_0 \approx 0.705$ we obtain a westward drift. Above $R_0 \approx 0.71$ both modes show an eastward directed drift motion. The characteristic drift time scale is approximately $(0.20 \dots 0.30) \cdot \tau_{\text{diff}}$ for the westward drifting modes and $(0.40 \dots 0.50) \cdot \tau_{\text{diff}}$ for the eastward drifting modes. Note that the ratio of the drift timescale ($0.2\tau_{\text{diff}}$) to reversal timescale (τ_{diff}) obtained from the simulations coincides with the same ratio observed for the Earth where the timescale of the westward drift is ~ 2000 years and the duration of a reversal ~ 10000 years. However, these timescales are estimations with a large uncertainty (e.g. data for the reversal time reach from 100 to several 10000 years).

The critical dynamo numbers for the axisymmetric modes (S0, A0) are larger than C_{α}^{crit} of the A1/S1-modes so that the solution would be dominated by these non-axisymmetric modes. The axisymmetric modes with the higher eigenvalues are oscillating. We know that this constellation is in contradiction to the observations of the Earth's magnetic field which is dominated by a stationary dipole-part. The non-axisymmetric modes only lead to smaller contributions that are manifested in the dipole tilt and the drifting field patterns. The interaction of the nonaxisymmetric modes (for anisotropic α) and the oscillating modes (for isotropic α with cross-overs) is still an open question.

4. Irregular reversals induced by a fluctuating α -effect

In the following the complications that appear from the non-axisymmetric modes are ignored. The effects of the full non-trivial α -tensor will be treated in a subsequent paper.

It is known from numerous calculations that the α -coefficients are rather noisy quantities (e.g. Ossendrijver et al. 2001; Giesecke et al. 2005). Figure 7 shows the maximum (minimum) of the α -effect in dependence of the time in units of the turnover time or advective timescale taken from the simulations of Giesecke et al. (2005)

$$\tau_{\text{adv}} = \frac{R_{\text{out}} - R_{\text{in}}}{u'} \quad (10)$$

(where u' is the turbulent rms-velocity).

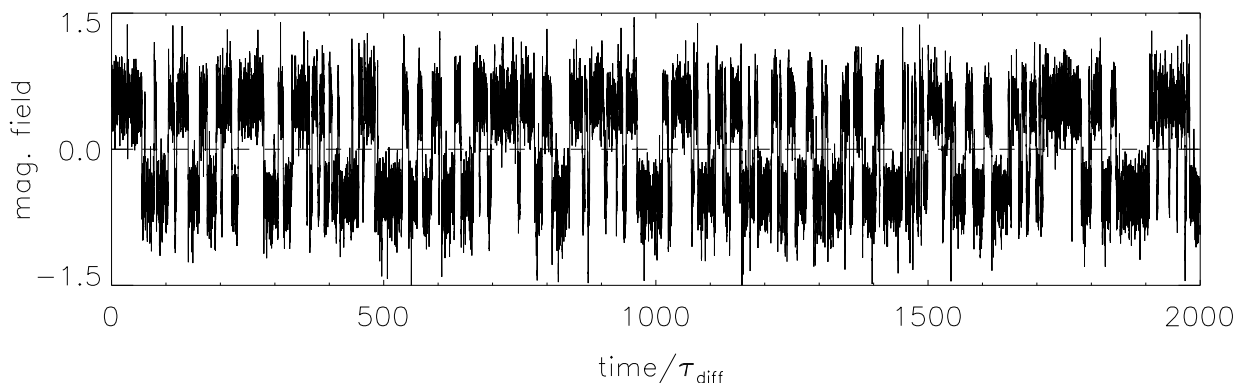


Fig. 8. Reversals of the magnetic field. 173 events have been identified as reversals which leads to a mean polarity life-time of approximately 11 diffusion times.

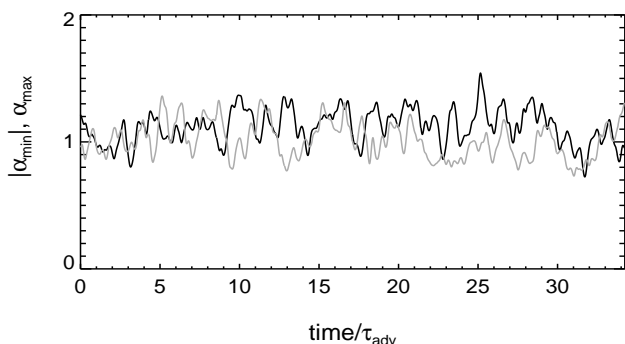


Fig. 7. Fluctuations of the maximum and the minimum of the α -effect in dependence of time (in units of the turnover-time τ_{adv}) taken from the results shown in Fig. 1.

The amplitude of the α -effect in the outer part of the shell is slightly larger than in the inner part and the strength of the fluctuations amounts approximately 10% of the average. It is obvious that the timescale of the fluctuations is given by τ_{adv} . If we assume a typical value for the Earth’s fluid outer core: $u' \approx 5 \cdot 10^{-4} \text{m/s}$ the timescale of the fluctuations τ_{adv} is related to the diffusive timescale τ_{diff} by

$$\tau_{adv} \approx (0.01 \dots 0.02) \cdot \tau_{diff} \quad (11)$$

where τ_{diff} is estimated by Eq (8). Thus the duration which is necessary for the α -effect to stay in a critical state – given by Eq. (6) – is about 15...30 times longer than the time scale on which the α -effect fluctuates. This indicates that a reversal clearly must be a very seldom event. It is also typical for the presented theory of the reversal phenomenon of the geodynamo that practically never a realization of the α -profile may exist so long that a complete oscillation can happen.

For the long time calculations we adopt an isotropic α -effect given by Eq. (5) and add fluctuations for the magnitude and for the location of the zero. For simplicity we assume equal averages for the upper and lower amplitude which both vary independently. The fluctuations are described by a Gaussian distribution with a standard deviation σ of 10% of the

average. The fluctuations of the zero cross-over are slightly larger. We adopt an average value of $R_0 = 0.675$ (the middle of the fluid layer) with a standard deviation of $\sigma = \pm 0.100$. According to Eq. (11) the actual values for the fluctuating quantities are updated each $0.01 \tau_{diff}$.

The dynamo number is given by $C_\alpha = 20$. This is clearly overcritical and therefore the non-linearities introduced by a local quenching function for the α -effect given by

$$\alpha(\mathbf{B}) = \alpha_0 \frac{1}{1 + \mathbf{B}^2} \quad (12)$$

might also have some influence on the solution. Indeed test calculations show that increasing C_α – corresponding to a stronger driven and thus more non-linear dynamo – reduces the probability of a reversal.

Figure 8 shows the time dependence of the radial magnetic field at some point in the spherical shell for a long time calculations that spans 2000 diffusion times (with the actual time scaling this corresponds to 20 million years). The magnetic field reverses irregularly. Both polarity states occur with nearly the same probability and field strength. The distribution of time-periods between each reversal can be described by an exponential function $e^{-\Delta t/t}$, thus the reversals are independent randomly occurring events (see Fig. 9). The number of short life-times between consecutive reversals is overestimated because it is difficult to distinguish between excursions and reversals. Within the presented 2D-simulations the only possibility to characterize a reversal is a sign change of a field component. Other properties, e.g. an slight increase of the tilt of the dipole axis followed by an immediate decrease back to the original state as an indication for an excursion, are intrinsically not available. Here we filtered out all events where the field changes its sign only “slightly” and for a very short time ($\lesssim 0.5 \tau_{diff}$). This means that an original dipole-state recovers very fast after the magnetic field just touches the zero in Fig. 8.

For the chosen set of parameters the mean reversal rate is about a factor of 5 higher than the rate observed for the present date Earth. Short-time test-calculations also have been performed with slightly different values for η_T/η . This is the crucial relation that specifies the α -effect fluctuation timescale with respect to the turbulent diffusion time. The

choice of this timescale fixes the time span for which the α -effect remains in a certain state and therefore determines the probability of a reversal. Decreasing η_T/η increases the (turbulent) diffusion time which results in a decrease of the relation τ_{adv}/τ_{diff} (assuming that τ_{adv} is fixed for the Earth's case). Therefore the timescale of the fluctuations of the α -effect is shorter with respect to the diffusive timescale and the probability of a reversal decreases. The results show that already a slightly reduced value for τ_{adv}/τ_{diff} significantly increases the mean time between consecutive reversals.

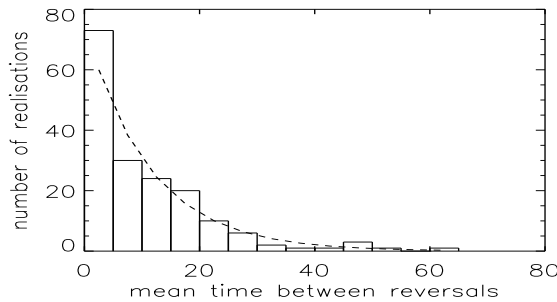


Fig. 9. Number of realizations of life-times of a single polarity state.

5. Discussion

We have shown that an α^2 -dynamo is able to exhibit periodic solutions where the dipole polarity changes on a timescale of the order of the diffusion time τ_{diff} . A crucial contribution for the existence of an oscillating α^2 -dynamo is an α -effect that strongly depends on the radial coordinate r . The most important property of such α -effect is a sign change within the field producing domain and a certain symmetry between positive and negative magnitude. Only a highly restricted class of (radial) α -profiles leads to oscillating solutions. It turned out that the radial profiles of the α -effect that have been calculated by Giesecke et al. (2005) under conditions that are suitable for the Earth's fluid core provide the essential characteristics that are necessary to construct such oscillating dynamos.

Long time simulations over 20 million years retrieve solutions which show irregular reversals. The model is very sensitive to the choice of parameters that describe the statistical properties. In principle it should easily be possible to adopt values for the fluctuating quantities (C_α , σ and τ_{adv}/τ_{diff}) in a way that would result in a time series which exhibit the basic properties of the geodynamo. The most important characteristics are the mean reversal rate, the distribution of polarity states, the distribution of the mean time between reversals and the ratio between number of excursions to number of reversals. An exact adjustment of the model at this stage is beyond the purpose of this work because further effects that affect the behavior of the field as well have not been considered. We neglected the influence of a finitely conducting inner core. Test calculations have shown that this does not prevent the solutions from oscillating, only some properties like the positions

of the critical intervals might be slightly changed. Possible changes in the behavior of the convection (changes in amplitude and strength of the fluctuations) have been ignored just as the (possible) existence of large-scale flows.

An unsolved issue is the inclusion of the non-trivial components of the α -effect. Currently it is not possible to retrieve the characteristics of the non-axisymmetric parts of the Earth's magnetic field. The Ansatz $\alpha_{zz} = 0$ – based on theoretical reasons – resulted in a dynamo that is dominated by the non-axisymmetric modes (and the axisymmetric modes, dipole and quadrupole are strongly suppressed). A resulting realistic solution should be described by a combination of different modes as it seems to be the case for the geodynamo. Indeed, observations of the magnetic fields of other planets or moons in the Solar system show, that various manifestations of field configurations are possible. The nearly perfect dipole field of Saturn or the non-axisymmetric dominated fields from Uranus and Neptune are some extraordinary examples.

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