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Diamagnetic pumping near the base of a stellar convection zone

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The property of inhomogeneous turbulence in conducting fluids to expel large-scale magnetic fields in the direction of decreasing turbulence intensity is shown as important for the magnetic field dynamics near the base of a stellar convection zone. The downward diamagnetic pumping confines a fossil internal magnetic field in the radiative core so that the field geometry is appropriate for formation of the solar tachocline. For the stars of solar age, the diamagnetic confinement is efficient only if the ratio of turbulent magnetic diffusivity $\eta_{\rm T}$ of the convection zone to the (microscopic or turbulent) diffusivity $\eta_{\rm in}$ of the radiative interior is $\eta_{\rm T}/\eta_{\rm in} \geq 10^5$. Confinement in younger stars requires larger $\eta_{\rm T}/\eta_{\rm in}$. The observation of persistent magnetic structures on young solar-type stars can thus provide evidence for the nonexistence of tachoclines in stellar interiors and on the level of turbulence in radiative cores.

1 Introduction

This paper concerns diamagnetism of stellar convective turbulence to show that the diamagnetic pumping can be important for formation of the solar tachocline.

Turbulent conducting fluids are known to expel large-scale magnetic fields in the direction of decreasing turbulent intensity. This effect of turbulent diamagnetism has been first found for inhomogeneous 2D turbulence (Zeldovich 1957). For this case, the effective velocity of magnetic field transport is proportional to the gradient of the turbulent magnetic diffusivity, $\boldsymbol{U}_{\rm dia} = -\boldsymbol{\nabla}\eta_{\rm T}$. The minus sign in the right shows the sense of turbulent magnetism: it is not para- but dia-magnetism so that magnetic fields are pushed away from the regions of relatively high turbulent intensity. The diamagnetism is closely related to the magnetic field expulsion from regions of circular motion (Weiss 1966). It was detected in 3D numerical simulations (Brandenburg et al. 1996; Tobias et al. 1998, 2001; Dorch & Nordlund 2001; Thomas et al. 2002; Ziegler & Rüdiger 2003) and laboratory experiments with turbulent liquid sodium (Spence et al. 2007). For 3D nearly isotropic turbulence the expression for the diamagnetic transport velocity becomes

$$\boldsymbol{U}_{\rm dia} = -\frac{1}{2}\boldsymbol{\nabla}\eta_{\rm T} \qquad (1)$$

(Krause & Rädler 1980). The influence of rotation and magnetic fields produce anisotropy and quenching of both turbulent pumping and diffusion (Kitchatinov 1988; Kitchatinov & Rüdiger 1992). In particular the strong magnetic quenching of the diamagnetism $\sim B^{-3}$ for super-equipartition fields has been found. The isotropic parts of the field advection and diffusion are still related by the Eq. (1) when rotation or strong magnetic field is imposed. Extensive numerical simulations – also as a test for the early theoretical SOCA-expressions – are due to Ossendrijver et al. (2002) and Käpylä et al. (2006).

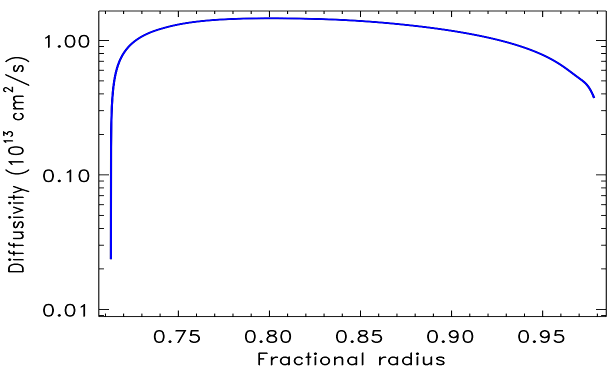

Fig. 1 (online colour at: www.an-journal.org) The radial profile of the turbulent magnetic diffusivity in the solar convection zone after the model of Stix & Skaley (1990). The diamagnetic pumping should be very strong near the base of the convection zone where the diffusivity almost jumps by orders of magnitude.

The turbulent pumping can be significant in various astrophysical contexts. Its strong effect is known in models of the galactic dynamo where the pumping may exceed the azimuthal α-effect so that the dynamo can even switch off. In the present paper we suggest its importance also for the Sun and solar-type stars with external convection zones where it can participate in formation of the tachocline immediately below the base of the convection zone.

Figure 1 shows the radial profile of the magnetic diffusivity in the solar convection zone estimated with the mixing-length relation $\eta_{\rm T} = u'\ell/3$ (u' is the rms velocity

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and ℓ is the mixing length). The diffusivity varies sharply near the base of convection zone. Its gradient represents a velocity of the downward diamagnetic pumping (1). The amplitude of the velocity is up to 50 m/s exceeding both the amplitude of the α-effect (1–10 m/s) and the meridional flow in the convection zone ($\lesssim 10$ m/s).

The tachocline can be explained by an effect of a weak internal magnetic field of the solar radiative core (Rüdiger & Kitchatinov 1997, 2007). The magnetic tachocline theory does not strongly restrict the magnetic field amplitude. Even a weak field well below 1 Gauss can produce the tachocline. The tachocline theory is, however, very sensitive to the field structure. The internal field should be almost totally confined inside the radiative core in order to produce the tachocline (MacGregor & Charbonneau 1999; Kitchatinov & Rüdiger 2006). The downward diamagnetic pumping near the convection zone base can keep the internal magnetic field confined in the radiative core thus providing the field geometry necessary for the tachocline formation (Garaud 2007; Garaud & Rogers 2007). The diamagnetic confinement is estimated in this paper almost linear and on large scales. The main astrophysical question is how long the turbulent pumping would need to produce a confined magnetic field from an open field configuration. Note that the diffusion time in the solar interior is longer by many orders of magnitude than the diffusion time in the convection zone. If the time to produce a confined magnetic geometry would, e.g., equal 10 Myr then – if the tachocline is magnetic by origin – the tachocline would only exist in older stars. On the other hand, the confinement mechanism via meridional flow with amplitudes of order 10 m/s works much faster (Kitchatinov & Rüdiger 2006). There is thus the hope that future asteroseismology observations will decide which of both the mechanisms is really acting.

Also the evolution of external fields in the convection zone under the presence of the diamagnetic effect must be reconsidered. This process may have consequences for the overshoot dynamo models (see Gilman 1992; Belvedere, Lanzafame & Proctor 1991; Rüdiger & Brandenburg 1995) although much stronger dynamo-fields may suppress turbulent advection of the field.

2 Equations

Consider the magnetic field in a sphere with an inner core where magnetic diffusivity $\eta_{\rm in}$ is relatively low and an outer spherical shell with the diffusivity $\eta_{\rm T} \gg \eta_{\rm in}$. The diffusivity varies smoothly between these two values around the interface at $r = R_{\rm in}$,

$$\eta = \eta_{\rm in} + \frac{1}{2}\left(\eta_{\rm T} - \eta_{\rm in}\right)\left(1 + {\rm erf}\left(\frac{r - R_{\rm in}}{h_{\rm d}}\right)\right), \quad (2)$$

where erf is the error function and $h_{\rm d}$ defines the width of the transition layer. The evolution of the large-scale magnetic field is governed by the diffusion equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\boldsymbol{\nabla} \times \left(\sqrt{\eta}\boldsymbol{\nabla} \times \left(\sqrt{\eta}\boldsymbol{B}\right)\right), \quad (3)$$

which includes the diamagnetic transport with the effective velocity (1). A vacuum boundary condition is applied at the top, $r = R$.

Equation (3) describes the decay of the field. We expect the decay being sufficiently slow for the internal fields to survive in the stellar radiation cores on time scales of Gyrs (Cowling 1945). With the diamagnetism included the magnetic fields are expected to be confined in the core. A solution of the eigenvalue problem

$$\boldsymbol{B}(\boldsymbol{r}, t) = \exp(\sigma t)\boldsymbol{b}(\boldsymbol{r}) \quad (4)$$

provides both the decay time $\tau = -\sigma^{-1}$ and the field geometry. The eigenmode with the smallest decay rate is the most significant one. The degree of confinement of an axisymmetric poloidal field,

$$\boldsymbol{b} = \boldsymbol{\nabla} \times \left(\boldsymbol{e}_\phi \frac{A}{r\sin\theta}\right), \quad (5)$$

can be described by the escape parameter

$$\delta\phi = \frac{\max|A|_{r=R_{\rm in}}}{\max|A|_{r\le R_{\rm in}}} \quad (6)$$

(Kitchatinov & Rüdiger 2006). Here, r, θ, ϕ are standard spherical coordinates, and $\boldsymbol{e}_\phi$ is the azimuthal unit vector. The parameter (6) measures the ratio of the characteristic values of magnetic flux through the interface to the flux in the core. The maximum value of $\delta\phi = 1$ corresponds to an open field structure. The smaller $\delta\phi$ the more confined to the core the internal field is. A tachocline can be formed by the internal field if the escape parameter is sufficiently small, i.e.

$$\delta\phi \lesssim 10^{-2} \quad (7)$$

(Rüdiger & Kitchatinov 2007).

The eigenvalue problem for the potential A of the poloidal field formulated in terms of the spherical harmonics, $A = A_l(r)P_l^1(\cos\theta)\sin\theta$, leads to the equation

$$\sigma A_l = \sqrt{\eta}\frac{{\rm d}}{{\rm d}r}\left(\sqrt{\eta}\frac{{\rm d}A_l}{{\rm d}r}\right) - \eta\frac{l(l+1)}{r^2}\, A_l\,, \quad (8)$$

with the vacuum boundary condition

$$\frac{{\rm d}A_l}{{\rm d}r} = -\frac{l}{R}\, A_l \quad (9)$$

at $r = R$.

We can solve Eq. (8) numerically for the continuous diffusivity profile (2). But also an analytical solution can be found for a discontinuous profile of η ($h_{\rm d} \to 0$). In this case the solution of Eq. (8) for constant η can be used, i.e.

$$A_l = a\sqrt{r}\; J_{l+\frac{1}{2}}\left(\frac{r}{\sqrt{\eta\tau}}\right) + b\sqrt{r}\; J_{-l-\frac{1}{2}}\left(\frac{r}{\sqrt{\eta\tau}}\right), \quad (10)$$

where J_ν is the Bessel function, $\tau = -\sigma^{-1}$ is the inverse eigenvalue, and a and b are free constants. Solutions with different sets of constants a and b apply to the inner core and the envelope. The boundary conditions

$$[A] = 0, \quad \left[\sqrt{\eta}\frac{{\rm d}A}{{\rm d}r}\right] = 0 \quad {\rm at}\;\; r = R_{\rm in} \quad (11)$$

can be applied at the interface. In the inner core, b must be zero to avoid a singularity at $r = 0$. The other constant a can freely be chosen, e.g. $a = 1$, to normalize the linear solution. Then, Eqs. (9) and (11) provide three conditions to define both the constants a and b in the outer shell and the eigenvalue.

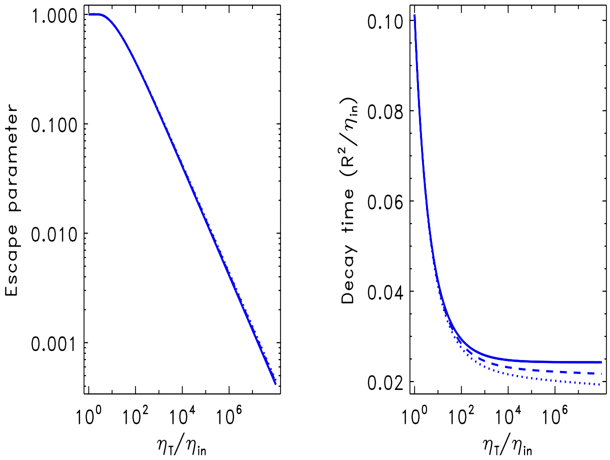

Fig. 2 (online colour at: www.an-journal.org) Dependence of the escape parameter (6) and the decay time of the most long-living dipolar eigenmode on the diffusivity contrast $\eta_{\rm T}/\eta_{\rm in}$. Full line corresponds to analytical solution for discontinuous change of η. Dashed and dotted lines show the numerical results for smooth profiles (2) with $h_{\rm d} = 0.01R$ and $h_{\rm d} = 0.02R$ respectively. All three lines overlap in the left panel. $R_{\rm in} = 0.7R$.

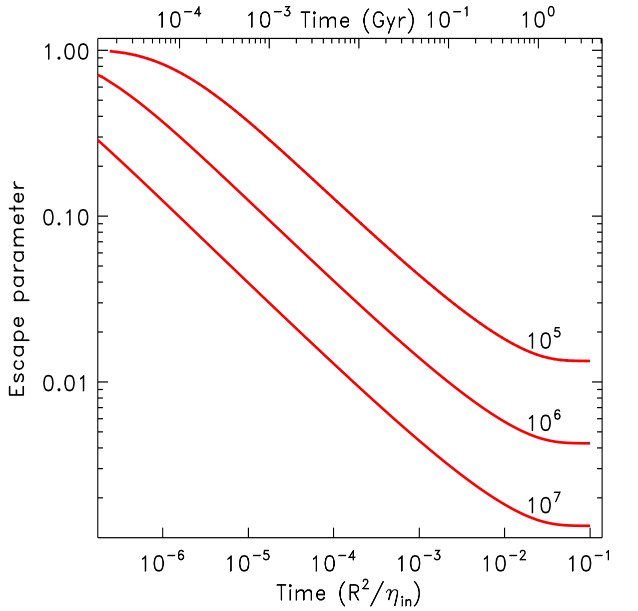

Fig. 4 (online colour at: www.an-journal.org) Escape parameter (6) as function of time for the runs starting from an open field structure. The lines are marked by the corresponding values of the diffusivity contrast. The upper scale gives the physical time for the Sun computed for the microscopic diffusivity $\eta_{\rm in} = 3\times10^3$ cm^2/s.

3 Results and discussion

3.1 Normal modes of the internal field

Figure 2 shows the dependencies of the escape parameter (6) and the decay time of the slowest decaying eigenmode of dipolar symmetry as functions of the diffusivity contrast $\eta_{\rm T}/\eta_{\rm in}$ between the inner core and outer envelope. The decay times are given separately for the analytical solution and numerical solutions for two values of the transition width $h_{\rm d}$. In all computations to follow, $h_{\rm d} = 0.02R$. The trapping of the field in the radiative core grows with the diffusivity contrast. Figure 3 shows the structure of the eigenmodes.

The tachocline can be formed by the internal field if the escape parameter is sufficiently small (7). After Fig. 2, this is the case if $\eta_{\rm T}/\eta_{\rm in} > 10^5$. For an eddy diffusivity $\eta_{\rm T} \sim 10^{13}\ {\rm cm^2s^{-1}}$ in the convection zone, this inequality means that

$$\eta_{\rm in} < 10^8\ {\rm cm^2s^{-1}} \qquad (12)$$

in the tachocline. Hence, the tachocline can be only mildly turbulent if it is magnetic by origin.

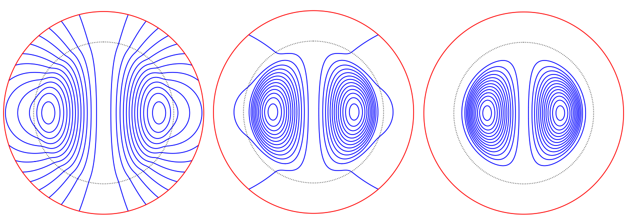

Fig. 3 (online colour at: www.an-journal.org) Poloidal field eigenmodes. The diffusivity contrast varies as $\eta_{\rm T}/\eta_{\rm in} = 10$, 10^3, 10^5 from the left to the right. The dotted circle shows the interface at $R_{\rm in} = 0.7R$.

The internal field can also be confined by a global meridional flow penetrating the radiative core from convection zone (Kitchatinov & Rüdiger 2006). This type of confinement also can be efficient only if $\eta_{\rm in}$ is sufficiently low with the same upper bound (12) as for the diamagnetic confinement. The reason for this coincidence might be that the confinements by meridional flow or inhomogeneous turbulence represent basically the same mechanism on different spatial scales. The condition (12) for the field expulsion from the region of motion to be efficient in the Sun does not, however, depend on the scale at all.

3.2 Internal field evolution towards a core-confined geometry

A freely decaying field eventually approaches the most long-living eigenmode. To find the characteristic time scale for the field convergence to confined geometry of the eigenmodes the field was evolved in time with Eq. (3) starting from an initial field of open structure. Figure 4 shows the dependencies of the escape parameter (6) on time for several diffusivity contrasts. The $\delta\phi$ decreases in time but eventually approaches a constant escape parameter of the corresponding eigenmode. The confinement proceeds on the long time scale $R^2/\eta_{\rm in}$ of the internal diffusion despite the diamagnetic pumping being fast. Why the confinement is so slow can be understood by observing the evolution of the magnetic field structure.

Figure 5 shows the initial phases of the confinement. After one (short) diffusion time, $R^2/\eta_{\rm T}$, the strong concentration of the field in the upper radiative core is already

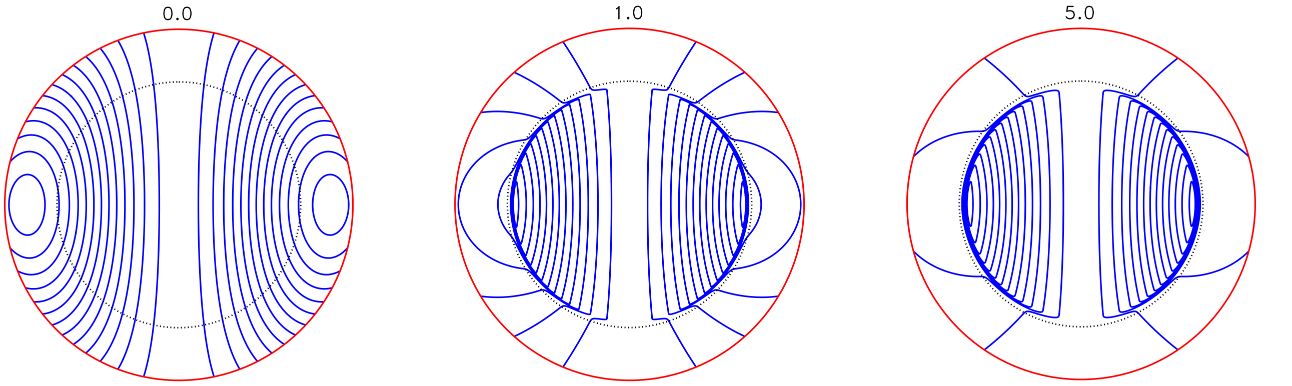

Fig. 5 (online colour at: www.an-journal.org) Initial field (*left*) and the structure of the field after it was evolved with Eq. (3) to later instants (*middle and right*). Time in *external* diffusion units $R^2/\eta_{\rm T}$ is shown on the top, all for $\eta_{\rm T}/\eta_{\rm in} = 10^5$.

formed. The strong downward increase of the poloidal field across the interface is already in balance with diamagnetic pumping. The confinement slows down after this balance is achieved. The high concentration of the field immediately beneath the interface does not allow a high degree of confinement even with a strong decrease of the field from the upper core to the lower convection zone. The field must be further smoothed over the entire core to attain the eigenmode structure with high confinement. This smoothing is, of course, slow. Nevertheless, the internal fields can be confined already in young stars of the age of several tens of million years if the diffusivity contrast between the convection zone and the radiative core is sufficiently high (Fig. 4). Our calculations, therefore, predict that even rather young stars may already possess tachoclines.

Observations can probe whether the solar-type stars of young clusters possess persistent magnetic structures which do not considerably change for years. If the fields rooted in the radiative core are not confined there, they will penetrate to the surface with no remarkable variations. If persistent magnetic structures are found then the internal field is not confined and a tachocline cannot be formed. Simultaneously, that would indicate a certain level of turbulence in the stellar cores.

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