Astron. Nachr. / AN **326**, No. 5, 315–320 (2005) / **DOI** 10.1002/asna.200510360

The angular momentum transport in rotating turbulent convection

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Received 19 April 2005; accepted 5 May 2005; published online 20 May 2005

Abstract. The angular momentum transport in rotating turbulent convection is simulated with the NIRVANA code for Taylor numbers up to 10^6. The box consists of an unstable layer embedded in two stable overshoot layers. We find the expected anisotropies in the rotating anisotropic turbulence field: $\langle u_r'^2 \rangle$ exceeds $\langle u_\phi'^2 \rangle$, and $\langle u_\phi'^2 \rangle$ exceeds $\langle u_\theta'^2 \rangle$. The resulting *radial* angular momentum transport is directed inwards and peaks in the middle of the convective layer. The resulting *latitudinal* angular momentum transport is equatorwards, peaks at the surface and is highly concentrated to the equatorial region. Apart from a factor of 2–3 the total amplitudes of the cross-correlations are of the same order of magnitude. In the lower overshoot region (‘tachocline’) the cross-correlations are negative. It is argued that the concentration of the latitudinal angular momentum transport towards the surface and towards the equator does not too strongly reduce its potential to produce rotation laws with accelerated equators.

Key words: convection, turbulence – Sun: rotation

1. Motivation

In order to explain the internal rotation of convection zones (see Fig. 1) the theory of the Λ-effect has been developed (see Rüdiger, 1989, for the historical overview). It describes the angular momentum transport in rigidly rotating anisotropic fields of free turbulence. The preferred direction is the radial direction (in the sense of spherical coordinates) which is due to both the density stratification and turbulence intensity stratification.

The cross-correlations $\langle u_r' u_\phi' \rangle$ and $\langle u_\theta' u_\phi' \rangle$ of the one-point-correlation tensor

$$Q_{ij} = \langle u_i'(\boldsymbol{x}, t) u_j'(\boldsymbol{x}, t) \rangle \tag{1}$$

provide the radial and the latitudinal turbulent transport of angular momentum. For those terms the general formulation

$$Q_{ij} = \ldots + \Lambda_{ijk} \Omega_k \tag{2}$$

has been introduced, or, in more detail,

$$Q_{r\phi} = \nu_{\rm T} V \Omega \sin\theta, \tag{3}$$

$$Q_{\theta\phi} = \nu_{\rm T} H \Omega \cos\theta, \tag{4}$$

with $\nu_{\rm T}$ as the eddy viscosity. The dimensionless functions V and H are normalized expressions for the vertical and horizontal cross correlation. A quasilinear theory for a special turbulence model has been developed by Kitchatinov and Rüdiger (1993) to reveal their dependence on the basic angular velocity Ω and the colatitude θ. The main result of such calculations can be shortly written by relations such as

$$Q_{r\phi} \simeq -\hat{H} \cos^2\theta \sin\theta, \tag{5}$$

$$Q_{\theta\phi} \simeq \hat{H} \sin^2\theta \cos\theta, \tag{6}$$

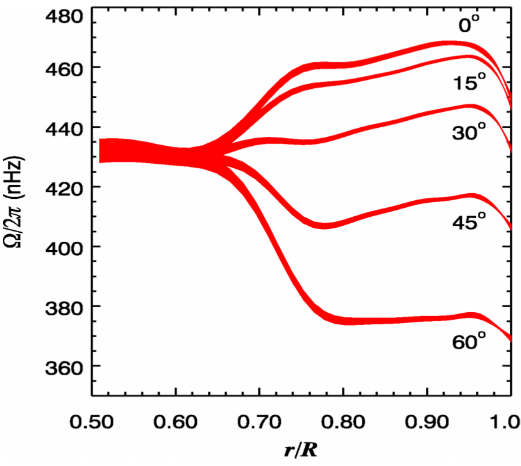

Fig. 1. (online colour at www.an-journal.org) The internal solar rotation law $\Omega = \Omega(x, \theta)$ as determined by the inversion of SOHO-data (Courtesy NSF National Solar Observatory, see also Kosovichev et al. 1997). The numbers marking the Ω-profiles are *latitudes* rather than colatitudes.

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which, however, only hold for rapid rotation (see Rüdiger & Hollerbach 2004). While the vanishing of the expression at the poles (and also the vanishing of $Q_{\theta\phi}$ at the equator) is a simple consequence of the prevailing symmetries the vanishing of $Q_{r\phi}$ at the equator is a surprising and nontrivial result. Also, the signs of the cross-correlations are nontrivial results of the calculations. For fast rotation the angular momentum transport is always inwards ($V < 0$) and it is always equatorwards ($H > 0$). By a direct inspection of Fig. 1 one finds that at least in the uppermost layers of the solar convection zone the vertical angular momentum transport (3) is directed inwards. Provided the outer boundary is stress-free, the corresponding boundary condition reads

$$\left.\frac{\mathrm{d}\Omega}{\mathrm{d}x}\right|_{x=1} = V(1,\theta)\Omega_{\mathrm{s}} \tag{7}$$

(Ω_{s} surface value of Ω, x the fractional radius) from which V results as negative. One finds from the observations $V < 0$ even for slow rotation contrary to the earlier results of Kitchatinov and Rüdiger (1993). We shall see in the following that all the presented numerical simulations lead to $V < 0$ for both slow and fast rotation.

Since the early work of Pulkkinen et al. (1993) and Rieutord et al. (1994) it has been tried to simulate rotating convection in spherical shells to compute the cross-correlations of the resulting turbulence. Very clear results came from Chan (2001) who used f-plane simulations to compute the main parts of the correlation tensor. For a fixed rotation rate Fig. 2 presents his results for the normalized functions V and H. There are three main properties:

i) V is always negative; H is always positive,
ii) $|V|$ is minimum at the equator,
iii) H is highly concentrated to the equator ($\sim \sin^6\theta$).

The amplitudes of the functions are of the same order. Most of these results comply with the SOCA-expressions (6). Surprising, however, is the strong concentration of the factor H to the equatorial region. This phenomenon also occurs in the recent calculations of Käpylä, Korpi and Tuominen (2004) and Hupfer, Käpylä and Stix (2005). In the following the NIRVANA code is used to probe the predicted close relation between anisotropies in the rotating (nonmagnetic) convection and the Λ-effect.

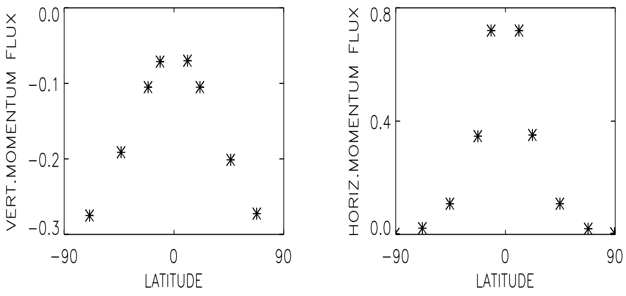

Fig. 2. $V(\theta)$ (left) and $H(\theta)$ (right) obtained with f-plane simulations by Chan (2001).

2. The turbulent transport of angular momentum

The ability of turbulence to transport angular momentum is reflected in the structure of the correlation tensor Q_{ij} of the fluctuating velocity $\boldsymbol{u}'$. The angular momentum fluxes in radius and latitude are proportional to the off-diagonal components $Q_{r\phi}$, $Q_{\theta\phi}$ of the tensor in spherical coordinates. The fluxes are finite even for the case of rigid rotation, and they are conventionally parameterized as

$$Q^{\Lambda}_{r\phi} = \Lambda_{\mathrm{V}}\Omega\,\sin\theta, \qquad Q^{\Lambda}_{\theta\phi} = \Lambda_{\mathrm{H}}\Omega\,\cos\theta \tag{8}$$

(with $\Lambda_{\mathrm{V}} = V\nu_{\mathrm{T}}$ and $\Lambda_{\mathrm{H}} = H\nu_{\mathrm{T}}$).

Negative Λ_{V} is required to reproduce the angular velocity increase with depth in the solar subsurface layer. Positive Λ_{H} is required for the equatorial acceleration. For a non-uniform rotation, however, the correlations (1) include the diffusive part, Q^{ν}, which is defined by the eddy viscosity tensor $\mathcal{N}_{ijkl}$, i.e.

$$Q_{ij} = Q^{\Lambda}_{ij} + Q^{\nu}_{ij}, \qquad Q^{\nu}_{ij} = -\mathcal{N}_{ijkl}\frac{\partial\bar{u}_k}{\partial x_l}, \tag{9}$$

where $\bar{\boldsymbol{u}}$ is the mean velocity. It is difficult to separate the non-diffusive momentum fluxes from their viscous counterparts in the results of 3D simulations. Käpylä et al. (2004) suggest that the diffusive momentum fluxes may be small in their box simulations.

A linear analytic calculation of the angular momentum transport of a prescribed turbulence yields

$$\Lambda_{\mathrm{V}} = \frac{2\tau_{\mathrm{corr}}}{K^2}\left(\left(K_\theta^2 + K_\phi^2\right)\langle u'^2_\phi\rangle - \left(K_r^2 + K_\theta^2\right)\langle u'^2_r\rangle\right) \tag{10}$$

$$\Lambda_{\mathrm{H}} = \frac{2\tau_{\mathrm{corr}}}{K^2}\left(\left(K_r^2 + K_\phi^2\right)\langle u'^2_\phi\rangle - \left(K_r^2 + K_\theta^2\right)\langle u'^2_r\rangle\right) \tag{11}$$

for the vertical and the horizontal components of the Λ-effect (Rüdiger 1989). $\boldsymbol{K} = (K_r, K_\theta, K_\phi)$ is the vector of the characteristic wave numbers. Obviously, the Λ-effect vanishes for isotropic turbulence, its existence is based on the presence of rotating *and* anisotropic turbulence.

Equations (10) and (11) only take nonzero values when the turbulence intensities and/or the wave number components are different. Equation (10) can also be written as

$$\Lambda_{\mathrm{V}} = \frac{2\tau_{\mathrm{corr}}}{K^2}$$
$$\left(\left(K_\theta^2 + K_\phi^2\right)\left(\langle u'^2_\phi\rangle - \langle u'^2_r\rangle\right) + \left(K_\phi^2 - K_r^2\right)\langle u'^2_r\rangle\right). \tag{12}$$

For spherical cells (i.e. isotropic components of K)

$$\Lambda_{\mathrm{V}} \propto \langle u'^2_\phi\rangle - \langle u'^2_r\rangle \tag{13}$$

(i.e. negative for dominating vertical turbulence intensity); and for isotropic turbulence intensity we find

$$\Lambda_{\mathrm{V}} \propto K_\phi^2 - K_r^2 \tag{14}$$

(i.e. negative for cells which are flat in radial direction). In summary, vertically-dominating turbulence fields with vertically-flat cells lead to negative Λ_{V}, i.e. inwardly directed angular momentum transport.

On the other hand, after eq. (11) turbulence fields with $\langle u'^2_\phi\rangle > \langle u'^2_\theta\rangle$ and/or large K_ϕ (i.e. small correlation length in azimuthal direction) provide positive $\Lambda_{\rm H}$. We shall see that indeed rotating convection provides the relation

$$\langle u_r'^2 \rangle > \langle u_\phi'^2 \rangle > \langle u_\theta'^2 \rangle, \tag{15}$$

so that the signs in eqs. (5) and (6) (with positive $\hat{H}$) appear to be plausible results.

There are only indirect indications for the amplitude of the solar correlations. Ward (1965) found a positive cross-correlation $\langle u_\theta' u_\phi' \rangle$ of the order of 10^7 cm^2/s^2 which well complies with the value of $\nu_{\rm T}\Omega$. The same is true for the vertical correlation $\langle u_r' u_\phi' \rangle$ for which the rotation law of the supergranulation layer yields $V \simeq -1$ also corresponding to the numerical value of $\nu_{\rm T}\Omega$ (see Rüdiger, Küker & Chan 2003). With radial velocity fluctuations of 100 m/s one finds a correlation coefficient $\langle u_r' u_\phi' \rangle / \langle u_r'^2 \rangle$ of only $\lesssim 0.1$. This rather plausible value has also been reproduced by the simulations of Chan (2001, his Table 2).

3. Basic equations, the model

3D numerical simulations of compressible, thermal convection under the influence of rotation are made with the finite-difference, fractional-step code NIRVANA (Ziegler 1998, 1999) in a small rectangular box defined on a Cartesian grid. It can be considered as a small piece of a spherical shell. The domain is placed tangentially at different colatitudes from $\theta = 0°$ (at the north pole) to $180°$ (at the south pole), and it contains a convectively unstable layer, surrounded by stable stratified layers with overshooting convection. The height of the convection zone d is chosen as the unit length.

The box rotates around the polar axis from west to east. The geometry of the computational domain is $(x, y, z) \in [0, 8d] \times [0, 8d] \times [-2d, 0]$. This volume is discretized by $100 \times 100 \times 128$ grid points which are uniformly distributed in each coordinate direction.

The governing equations describing thermal convection in a rotating stratified medium are

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \boldsymbol{u}), \tag{16}$$

$$\frac{\partial \rho \boldsymbol{u}}{\partial t} = -\nabla \cdot (\rho \boldsymbol{u}\boldsymbol{u}) - \nabla P + \nabla \cdot \pi + \rho \boldsymbol{g} - 2\rho \boldsymbol{\Omega} \times \boldsymbol{u}, \tag{17}$$

$$\frac{\partial \rho U}{\partial t} = -\nabla \cdot (\rho U \boldsymbol{u}) - P \nabla \cdot \boldsymbol{u} + Q_{\rm vis} + \nabla \cdot (\rho C_{\rm p} \chi \nabla T). \tag{18}$$

The notation for physical variables is standard (U thermal energy density, π the viscous stress tensor and $Q_{\rm vis}$ the viscous heating term).

Equations (16)–(18) are closed through the ideal gas equation $P = (\mathcal{R}/\mu)\rho T$. The initial distribution of the physical quantities represents a 3-layer polytrophic stratification. We assume all quantities to be periodic in the horizontal directions. At the bottom ($z = -2$) and top ($z = 0$) of the box impermeable conditions are imposed for the vertical velocity, while the horizontal velocities satisfy stress-free boundary conditions. The temperature and density are fixed at the top of the domain and a constant heat-flux is injected at the bottom.

In the calculations $\mathrm{Ra} = 3 \cdot 10^5$ and $\mathrm{Pr} = 0.1$, while Ta varies from 10^4 to 10^6 which may represent here the realization of slow rotation and fast rotation. The value $\mathrm{Ta} = 10^6$ does not describe the real solar convection but it seems to be high enough to reveal the relation between anisotropy and angular momentum transport in rotating convection.

4. The radial flow field

All velocities are normalized with the speed of sound ($c_{\rm ac}$) at the upper layer of the unstable domain. The solar value of this quantity is about 6.4 km/s. In Fig. 3 the maximum velocity in the box is given with 0.044 which would mean about 300 m/s for the solar case. This value corresponds to the velocity amplitudes which are characteristic in the bulk of the solar convection zone. Figure 3 also reveals that for Taylor numbers of order 10^6 the turbulence intensity $\langle u_r'^2 \rangle$ grows from the pole to the equator. This is an unexpected result which is also present in the simulations of convection in boxes with very weak density stratification. More references to this situation and the resulting consequences are given in a foregoing publication (Rüdiger et al. 2005).

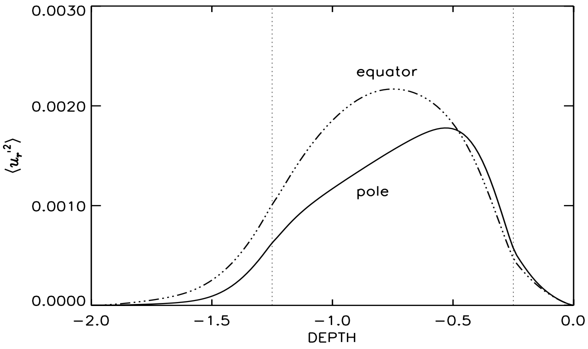

Fig. 3. The radial turbulence intensity for $\mathrm{Ta} = 10^6$. Except the top layer the turbulence at the equator exceeds the polar values (see also Chan 2001, Table 2). The flow amplitudes are normalized with the sound velocity at the surface of the unstable domain. Solar values adopted the maximal rms velocity approaches 300 m/s.

5. The radial angular momentum transport

We start with a discussion of the anisotropy between vertical and azimuthal turbulence intensities (Fig. 4). Without rotation the turbulence is vertically dominated except in the top layer. This is also true for the lower transition zone between the unstable and the stable layer ('tachocline'). After eq. (13) we therefore expect the occurrence of negative $\Lambda_{\rm V}$ in the bulk of the convection zone.

In the representation of the numerical simulations the Λ-effect is renormalized in accordance to

$$\Lambda_{\rm V}^* = \frac{\Lambda_{\rm V}\Omega}{c_{\rm ac}^2}, \qquad \Lambda_{\rm H}^* = \frac{\Lambda_{\rm H}\Omega}{c_{\rm ac}^2}. \tag{19}$$

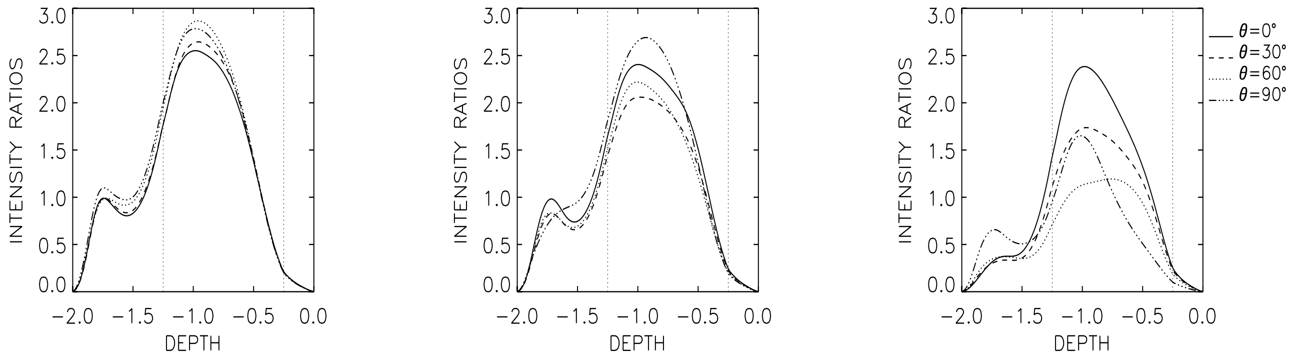

Fig. 4. The ratio $\langle u_r'^2\rangle/\langle u_\phi'^2\rangle$ of the turbulence intensities for rotating turbulence fields with $\mathrm{Ta}=10^4, 10^5, 10^6$ (from left to right).

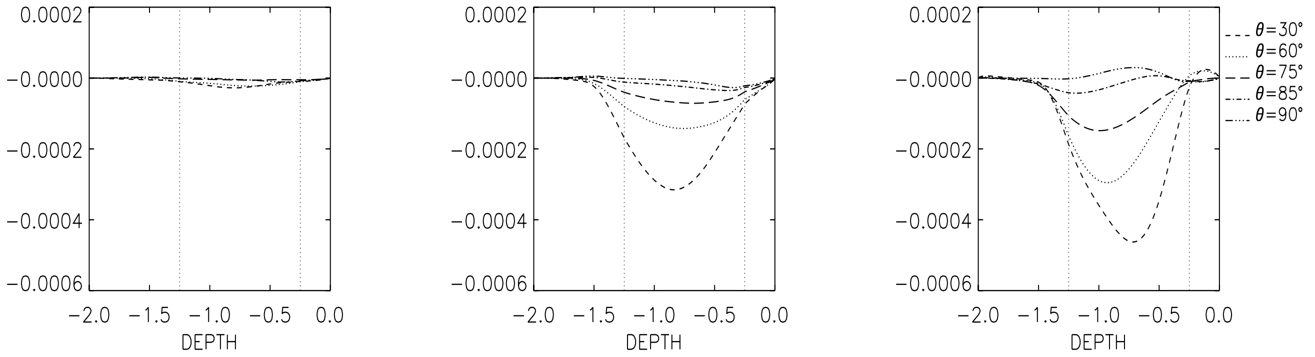

Fig. 5. The function Λ_{V}^* for $\mathrm{Ta}=10^4, 10^5, 10^6$ (from left to right).

The relation between the old normalization (with $\nu_{\mathrm{T}}\Omega$) and the new one (with c_{ac}^2) is

$$V=\Lambda_{\mathrm{V}}^*\frac{c_{\mathrm{ac}}^2}{\nu_{\mathrm{T}}\Omega}, \qquad\qquad H=\Lambda_{\mathrm{H}}^*\frac{c_{\mathrm{ac}}^2}{\nu_{\mathrm{T}}\Omega}, \tag{20}$$

where for the Sun the value of $c_{\mathrm{ac}}^2/\nu_{\mathrm{T}}\Omega$ is of order 10^4. In order to get V and/or H of order unity the simulations for Λ^* should yield rather small values.

For $\mathrm{Ta}=10^4$ the resulting Λ_{V} is indeed still small. Already for $\mathrm{Ta}=10^5$, however, the results are already very clear (Fig. 5, middle). The function Λ_{V}^* is zero at the equator and it is negative in both hemispheres. This result complies with the findings of Chan (2001). The vanishing of Λ_{V}^* at the equator is *not* trivial and allows formulations such as $Q_{r\phi}\propto\cos^{2l}\theta\sin\theta$ with $l>0$. Note also that the Λ_{V}^* vanishes in the upper overshoot layer while it remains basically negative in the tachocline layer. Comparing the Figs. 3 and 5 (right) one finds a value of approximately 0.01 for the correlation coefficient which indeed for the Sun appears to be smaller than 0.1 (see above). The amplitude of Λ_{V}^* for $\mathrm{Ta}=10^6$ is $4\cdot10^{-4}$ which leads to values for V of order 10 which might be too large by a factor of (only) 10.

Of similar interest is the behavior of the anisotropy parameter $\langle u_r'^2\rangle/\langle u_\phi'^2\rangle$ for fast rotation. At the poles the influence of the rotation is rather small. It is much stronger in the equatorial region. There, we find a tendency of return-to-isotropy as a consequence of the Taylor-Proudman theorem. The vanishing of the vertical angular momentum transport at the equator might be due to this phenomenon. The vanishing of Λ_{V} at the equator has been derived first by Kitchatinov and Rüdiger (1993) with analytical methods.

6. The horizontal angular momentum transport

The horizontal angular momentum transport bases on the anisotropy in the turbulence field between both the horizontal direction θ and ϕ – which only exists for rotating stars (Fig. 6). It is therefore not surprising that (in contrast to the radial angular momentum transport) only for rapid rotation ($\mathrm{Ta}=10^6$) a considerable effect exists (Fig. 7, right). The anisotropy between $\langle u_\phi'^2\rangle$ and $\langle u_\theta'^2\rangle$ mainly exists in the equatorial region. There, $\langle u_\theta'^2\rangle$ is strongly reduced so that $\langle u_\phi'^2\rangle$ dominates leading to positive cross-correlations close to the equator. The result is an extreme concentration of the horizontal cross-correlation in the vicinity of the equator. The maximal values of Λ_{H}^* are shown for $\theta=85°$[1]. The latitudinal profile of Λ_{H}^* at the top of the convective domain is shown in Fig. 8 (bottom). It mainly exists for $\theta\gtrsim60°$. The profile $\sin^{20}\theta$ is given for comparison. The corresponding results of Chan (2001) are less concentrated to the equator while those of Käpylä et al. are even more concentrated. The reason for this jet-like behavior is shown in Fig. 6 (right). The turbulence is only anisotropic in the latitudinal direction in the near-equator region.

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[1] Note that from reasons of symmetry the correlation $Q_{\theta\phi}$ must vanish at $\theta=90°$.

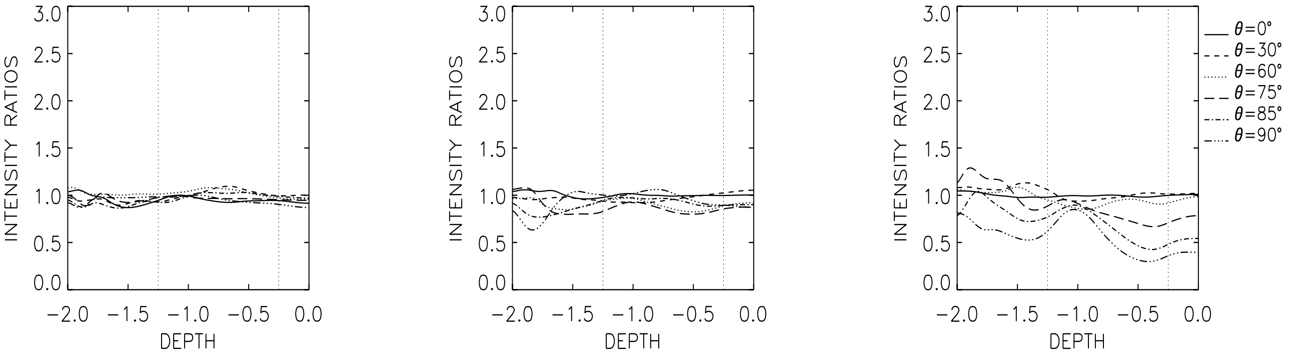

Fig. 6. The same as in Fig. 4 but for the intensity ratio $\langle u'^2_\theta\rangle/\langle u'^2_\phi\rangle$.

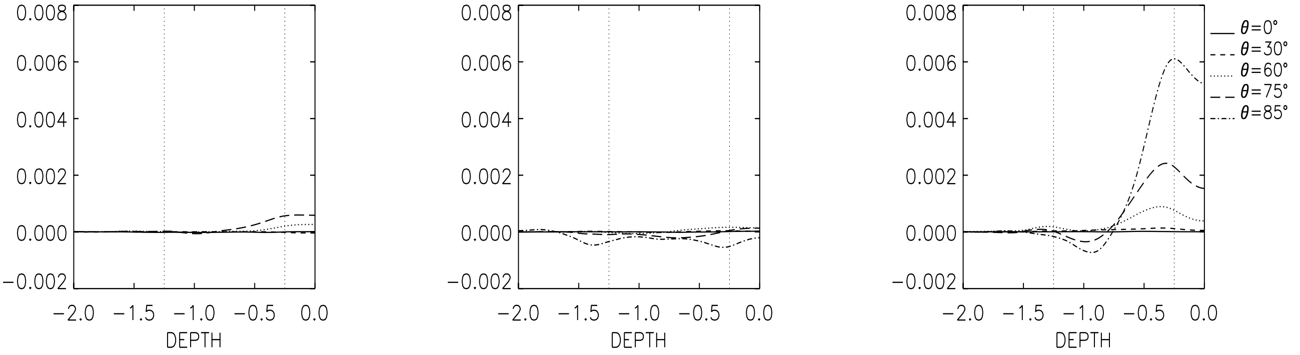

Fig. 7. The same as in Fig. 5 but for the function $\Lambda^*_{\rm H}$.

The suppression of $\langle u'^2_\theta\rangle$ compared with $\langle u'^2_\phi\rangle$ at the equator is not easy to understand. For rotating isotropic but free turbulence one would expect the opposite as a consequence of the Taylor-Proudman theorem. Rüdiger et al. (2005) have discussed how the rotation and the radial character of the stellar turbulence interact to produce the resulting rotation-induced anisotropy. The related terms of the spectral tensor at the equator (where $\boldsymbol{g}\cdot\boldsymbol{\Omega}=0$) are

$$\hat{Q}_{ij} = \ldots\delta_{ij} + \left(\phi_\parallel + a\left(\frac{\phi'_\parallel}{2}+\phi_3\right)\right)\frac{\Omega_i\Omega_j}{\Omega^2} \tag{21}$$

with $a \geq -1$. Negative a means turbulence with dominant radial intensity. It is

$$\phi_\parallel(\Omega^*) = \frac{1}{5}\Omega^{*2}, \quad \phi'_\parallel = -\frac{8}{35}\Omega^{*2}, \quad \phi_3 = \frac{19}{35}\Omega^{*2}. \tag{22}$$

Here

$$\Omega^* = 2\tau_{\rm corr}\Omega \tag{23}$$

is the Coriolis number (or inverse Rossby number) of the turbulence. It follows for slow rotation

$$\langle u'^2_\theta\rangle - \langle u'^2_\phi\rangle \simeq \frac{7+15a}{35}\Omega^{*2}, \tag{24}$$

which is indeed negative for $a < -7/15$. For fast rotation, however, the expression (24) is always positive. Here is a striking difference between the results of box simulations and quasilinear calculations for rotating free turbulence.

Contrary to the $Q_{r\phi}$ the $Q_{\theta\phi}$ peaks at the top of the convective layer. In Fig. 8 the latitudinal profiles of the cross-correlations $Q_{r\phi}$ and $Q_{\theta\phi}$ taken at the depth where they are maximal are plotted. The amplitude of the horizontal cross-correlation $Q_{\theta\phi}$ exceeds the amplitude of the radial cross-correlation $Q_{r\phi}$ by a factor of ~ 3.

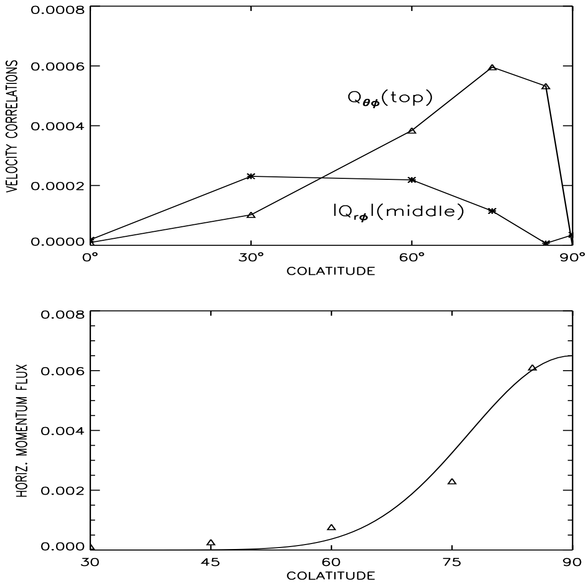

Fig. 8. Top: The latitudinal profile of $|Q_{r\phi}|$ in the middle of the convective layer and of $Q_{\theta\phi}$ at the top of the convective layer for $\mathrm{Pr}=0.1$ and $\mathrm{Ta}=10^6$. Bottom: The same for $\Lambda^*_{\rm H}$ at the top of the convective layer. Solid: the profile $\sin^{20}\theta$.

7. Discussion

The relation of the auto-correlations and the cross-correlations in rotating convection is probed in the present paper. The clear relation between the differences of the turbulence intensities $\langle u_r'^2\rangle, \langle u_\phi'^2\rangle$ and $\langle u_\theta'^2\rangle$ and the cross-correlations $\langle u_r' u_\phi'\rangle$ and $\langle u_\theta' u_\phi'\rangle$ (which are responsible for the angular momentum transport in the turbulence) is the main result of the numerical simulations. The dominance of the radial rms velocity over the longitudinal rms velocity produces the (negative) radial angular momentum transport already for slow rotation.

Much more complicated is the situation with the horizontal cross-correlation which – as known – does not exist for slow rotation (see Fig. 7, middle). It bases on the dominance of $\langle u_\phi'^2\rangle$ over $\langle u_\theta'^2\rangle$ which, however, in the simulations only exists in the equatorial region of the upper half of the convective layer. There, indeed, also the horizontal cross-correlation exists. Its amplitude is higher than the amplitude of $\langle u_r' u_\phi'\rangle$ and it is positive in the upper half of the convection zone. The amplitude of H exceeds the amplitude of V by a factor of about 10 – similar to Chan's results (see Fig. 2) obtained with a completely different code. In the lower half of the convective domain it is much smaller and negative and also highly concentrated to the equator.

The situation in the overshoot regions is of particular interest. Note that the turbulence in both (rotating) overshoot regions is horizontal rather than vertical, i.e. $\langle u_\phi'^2\rangle > \langle u_r'^2\rangle$. At the top of the convection box the situation is more complicated. For slow rotation ($\mathrm{Ta} = 10^5$) there is no H but a negative V which only slightly depends on the latitude. The same is true for the negative slope of the outer solar rotation law in the supergranulation layer (see Fig. 1). For faster rotation ($\mathrm{Ta} = 10^6$) one finds the opposite. V goes to zero and H is a positive and large number (in the equatorial region). This is a very basic process which exists in the entire box.

At the bottom of the box and in its lower overshoot region both transport coefficients are negative[2]. It is thus clear that the region with $V = H = 0$ which is only able to produce rigid rotation is located deeper in the lower overshoot region. Hence, our box simulations suggest that turbulence alone is not able to solve the tachocline problem.

In order to probe the consequences of a Λ_H-effect which mainly exists in the surface layers, i.e. which strongly *decreases* downwards, we have to solve the angular momentum equation

$$\frac{\partial}{\partial x}\left(x^4\frac{\partial\Omega}{\partial x}\right) - Gx^3\frac{\partial\Omega}{\partial x} + \frac{x^2}{\sin^3\theta}\frac{\partial}{\partial\theta}\left(\sin^3\theta\frac{\partial\Omega}{\partial\theta}\right) = \frac{x^2}{\sin^3\theta}\frac{\partial}{\partial\theta}(H\cos\theta\sin^2\theta\,\Omega_\odot) \tag{25}$$

with $G = -\mathrm{dlog}\rho/\mathrm{dlog}x$. Here the vertical Λ-effect is ignored. For H the ansatz

$$H = x^\lambda \sin^{2l}\theta\, H_\mathrm{s} \tag{26}$$

is used with high values of l and λ (positive for a downward decrease). As in Rüdiger (1989, p. 130) the surface rotation law is written as

$$\frac{\Omega}{\Omega_\odot} \simeq \mathrm{const} + \hat{\Psi}\frac{H_\mathrm{s}}{2l}\sin^{2l}\theta. \tag{27}$$

$\hat{\Psi}$ is given in Fig. 9 for a convection zone with the lower boundary at $x_\mathrm{in} = 0.7$ and for the two cases with and without density stratification. For rather high values of λ there is a reduction of the pole-equator difference of Ω at the surface but the reduction is not too dramatic.

Only detailed calculations with meridional flows included may reveal the consequences of the rather high values of λ which are suggested by the numerical simulation of the horizontal Λ-effect. On the other hand, also for $\lambda = 0$ and $x_\mathrm{in} \to 1$ the prefactor $\hat{\Psi}$ remains unity. Therefore, even if Λ_H only exists in a thin surface layer a distinct surface rotation law still exists. The results of our simulations do thus *not* contradict the findings of Reiners et al. (2005) of a strong equatorial acceleration at the surface of a rapidly rotating A-star with a very thin outer convection zone.

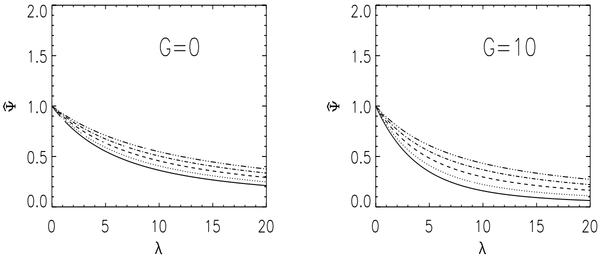

Fig. 9. The amplitude function $\hat{\Psi}$ from eq. (27) for various λ and for a convection zone without (left) and with (right) density stratification. The various lines belong to various Legendre polynomials (not important here). $x_\mathrm{in} = 0.7$.

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[2] Käpylä et al. (2004) find a positive peak of H there.