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Sunspot decay as a test of the eta-quenching concept

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A 2D model of the sunspot decay is constructed which under the assumption of axial symmetry follows the evolution of both the magnetic field and temperature. Magnetic-induced deformation and suppression of the tensors of turbulent heat conductivity and eddy diffusivity are explicitly included. In an older model without the η-quenching the decay law of spot area proved to be slightly convex with $\ddot{A} < 0$ in contrast to the results of new and detailed investigations of the observations. With the η-quenching involved, our models do not exhibit the convex decay laws but instead yield quasilinear decays for both the spot area as well as the magnetic flux (cf. Fig. 3).

Key words: Magnetic fields, Turbulence, Sun: magnetic fields, sunspots

1. Motivation

Some simulations of the influence of large-scale magnetic fields on turbulence yield the result that even weak fields may effectively suppress the turbulent diffusion. Even when the large-scale field is much weaker than the equipartition field

$$B_{\rm eq} = \sqrt{\mu_0 \rho}\; u_{\rm T} \tag{1}$$

– with the turbulence rms-velocity $u_{\rm T} = \sqrt{\langle \boldsymbol{u}'^2 \rangle}$ – the turbulent diffusion is reported as substantially reduced (Cattaneo & Vainshtein 1991, Cattaneo 1994). These conclusions are however controversial (e.g. Brandenburg 1993). In particular, the magnetic quenching concerns the turbulent electromotive force (EMF)

$$\boldsymbol{\mathcal{E}} = \langle \boldsymbol{u}' \times \boldsymbol{B}' \rangle, \tag{2}$$

which for non-helical turbulence describes the diffusive decay of a large-scale magnetic field. If such quenching were true, the concept of the mean-field dynamo becomes problematic.

A different simulation for MHD turbulence shows the opposite result. Turbulent diffusion in 3D magnetoturbulence is not quenched before the magnetic field reaches its equipartition value (1) (Nordlund et al. 1994). All the numerical simulations of MHD convection led to decreasing eddy diffusivity for increasing strength of the applied fields. This η-quenching obtained from the 3D simulations proved to be much weaker than the decay found for 2D MHD turbulence.

The analytical theory of eddy diffusivity and its magnetic modification results from the series expansion of the turbulent EMF,

$$\mathcal{E}_i = \alpha_{ij} \bar{B}_j + \beta_{ijk} \frac{\partial \bar{B}_j}{\partial x_k} + ... \tag{3}$$

(Krause & Rädler 1980). Under the influence of the magnetic field the eddy diffusivity loses its scalar character to become a tensor, i.e. β_{ijk}.

The diffusivity tensor for magnetized fluids has been derived by Kitchatinov et al. (1994) where the structure of the turbulent EMF in the presence of a non-uniform magnetic field is also given, i.e.

$$\boldsymbol{\mathcal{E}} = -\eta_{\rm T}\, {\rm rot}\, \bar{\boldsymbol{B}} + \boldsymbol{U}^{\rm mag} \times \bar{\boldsymbol{B}}, \tag{4}$$

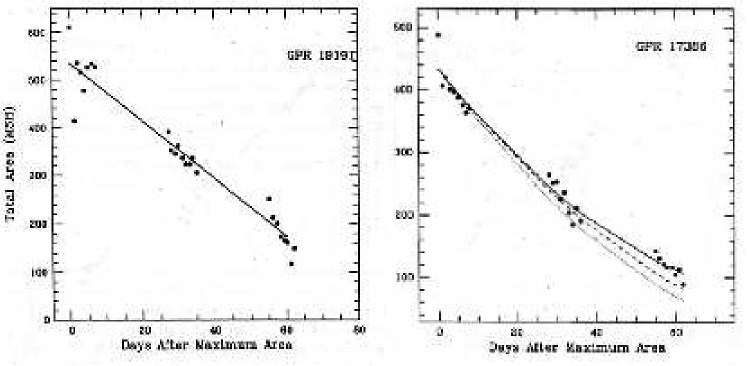

Fig. 1: Decay of spots with more than one transit with (LEFT) linear decay pattern and (RIGHT) parabolic decay pattern (Martínez Pillet et al. 1993).

here for the case of uniform and nonrotating turbulence fields. For sufficiently high magnetic fields it includes new terms involved through the effective 'velocity'

$$\boldsymbol{U}^{\rm mag} = \hat{\eta} \boldsymbol{\nabla} \log \bar{B}^2 + \eta_z \, \frac{{\rm rot}\, \bar{\boldsymbol{B}} \times \bar{\boldsymbol{B}}}{\bar{B}^2}. \tag{5}$$

Note the similarity of the last term to the ambipolar diffusion. There are now three eddy diffusivity coefficients, $\eta_{\rm T}$, $\hat{\eta}$ and η_z, with different dependencies on the magnetic field strength (cf. Kitchatinov et al. 1994). In the 1D case of a magnetic field pointing in the z-direction and varying along x the only non-zero component of the mean EMF is

$$\mathcal{E}_y = \left(\eta_{\rm T} + \eta_z - 2\hat{\eta}\right) \frac{d\bar{B}}{dx}, \tag{6}$$

so that the total diffusivity,

$$\eta^{\rm tot} = \eta_{\rm T} + \eta_z - 2\hat{\eta}, \tag{7}$$

can be introduced. In the present paper we propose to use the observed decay of sunspots as a test for the magnetic suppression of the mean-field eddy diffusivity (7).

To this end detailed observations of the sunspots decay are necessary. There are demonstrations that the area A of the spots decays almost linearly with time for a large number of observed spots (Bray & Loughhead 1964, Thomas & Weiss 1972, Zwaan 1992, Skumanich et al. 1994). Bumba (1963) found

$$\dot{A} = -1.5 \cdot 10^{12}\ {\rm cm}^2/{\rm s} \tag{8}$$

for the constant value of the decay rate while Martínez Pillet (1994) obtained

$$\dot{A} = -1.99 \cdot 10^{12}\ {\rm cm}^2/{\rm s}. \tag{9}$$

Moreno-Insertis & Vázquez (1988) analyzed a big set of Greenwich data also with the result that a linear decay law is indeed compatible with the observations. For only a minority of 5% an exponential fit was more appropriate than the linear one.[1]

A very detailed inspection of the sunspot decay rates was also given by Martínez Pillet et al. (1993) where also the discrepancies between the decay rates found by different authors are discussed. They find no definite conclusion in favour of linear or nonlinear decay laws: their Fig. 11 displays a linear decay while their Fig. 10 shows a more concave pattern in the area vs. time diagram. The second derivative of the $A(t)$-profile, however, proves to be always non-negative, i.e. $\ddot{A} \gtrsim 0$ (see Fig. 1).

As pointed out by Stix (1989) a linear decay of the *flux*

$$\Phi = \int \bar{B}_z \, dA \tag{10}$$

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[1] With their own analysis of single spots with Greenwich data Gokhale & Zwaan (1972) already confirmed the constancy of $\dot{A}$ *independent of the area.*

can readily be reproduced by a solution of the 1D diffusion equation with constant eddy diffusivity,

$$\frac{\partial \bar{B}_z}{\partial t} = \eta_{\rm T} \, \frac{1}{\varpi} \, \frac{\partial}{\partial \varpi} \left(\varpi \, \frac{\partial \bar{B}_z}{\partial \varpi} \right). \tag{11}$$

With the same model Krause & Rüdiger (1975) showed that then the *spot area* decays like

$$\dot{A} \propto \log \frac{T}{t} - 1 \tag{12}$$

with T being the lifetime of the spot, i.e. with a convex profile $A(t)$ hence $\ddot{A} < 0$ – in contrast to the observational findings. We shall see below that the account for the magnetic feedback onto the eddy diffusivity tends to resolve the discrepancy. It yields a quasilinear decay of the spot area even for the early part of its life. As our model has now a 2D rather than a 1D geometry and also involves thermodynamics, the computations should have relevance to further sunspot decay discussions. Lustig & Wöhl (1995) report a significant increase of the daily decay rate of all decaying groups towards higher latitudes which could be a further test for the models probing the rotational influence onto the turbulence.

2. The model

The model solves the equations for the magnetic field and entropy with account for the magnetically induced anisotropy and quenching of the effective transport coefficients.

The diffusion equation for the mean magnetic field reads

$$\frac{\partial \bar{\boldsymbol{B}}}{\partial t} \ = \ {\rm rot} \left(-\eta_{\rm T} {\rm rot} \bar{\boldsymbol{B}} \ + \ \boldsymbol{U}^{\rm mag} \times \bar{\boldsymbol{B}} \right), \tag{13}$$

with $\boldsymbol{U}^{\rm mag}$ defined by Eq. (5). The effective diffusivities,

$$\eta_{\rm T} = \ \eta_0 \varphi(\beta), \quad \hat{\eta} \ = \ \eta_0 \hat{\varphi}(\beta), \quad \eta_z = \ \eta_0 \varphi_z(\beta), \tag{14}$$

depend on the normalized magnetic field strength

$$\beta \ = \ 2\pi \, \frac{\bar{B}}{B_{\rm eq}}, \tag{15}$$

where $B_{\rm eq}$ is the field equipartition value (1). The non-magnetic reference value for the diffusivity is

$$\eta_0 = \tau_{\rm corr} \langle \boldsymbol{u}'^2 \rangle / 3, \tag{16}$$

and the quenching functions were derived by Kitchatinov et al. (1994)

$$\begin{aligned} \varphi \ &= \ \frac{3}{2\beta^2} \left(-\frac{1}{1+\beta^2} + \frac{1}{\beta} \arctan \beta \right), \\ \varphi_z \ &= \ \frac{3}{8\beta^2} \left(1 + \frac{2}{1+\beta^2} + \frac{\beta^2 - 3}{\beta} \arctan \beta \right), \\ \hat{\varphi} \ &= \ \frac{3}{8\beta^2} \left(-\frac{5\beta^2 + 3}{(1+\beta^2)^2} + \frac{3}{\beta} \arctan \beta \right). \end{aligned} \tag{17}$$

For weak fields they approach the following values

$$\varphi(0) = 1, \qquad \hat{\varphi}(0) = \varphi_z(0) = 0, \tag{18}$$

while for strong fields the functions φ and $\hat{\varphi}$ run like β^{-3}, and φ_z decreases like β^{-1}.

The brightness of the spot is governed by the entropy equation,

$$\rho T \, \frac{DS}{Dt} + \ {\rm div} \, \boldsymbol{F}^{\rm conv} \ = \ 0, \tag{19}$$

with the convective heat flux

$$F_i^{\rm conv} \ = \ -\rho T \chi_{ij} \, \frac{\partial S}{\partial x_j}. \tag{20}$$

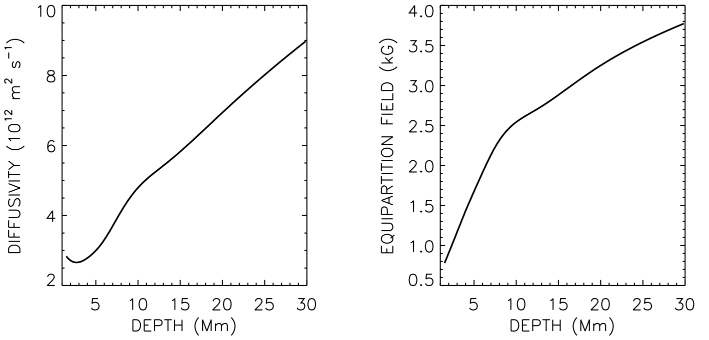

Fig. 2: The depth profiles of the diffusivity (16), η_0 (LEFT), and the equipartition magnetic field (1) (RIGHT).

magnetic field influences the energy transport through the conductivity tensor,

$$\chi_{ij} = \chi_T \delta_{ij} + \chi_z \frac{\bar{B}_i \bar{B}_j}{\bar{B}^2}, \tag{21}$$

where

$$\chi_T = \chi_0 \varphi_\chi(\beta), \quad \chi_z = \chi_0 \varphi_z(\beta) \tag{22}$$

and $\chi_0 = \eta_0$. The function φ_χ reads (Kitchatinov et al. 1994),

$$\varphi_\chi = \frac{3}{8\beta^2}\left(\frac{\beta^2-1}{\beta^2+1} + \frac{\beta^2+1}{\beta}\arctan\beta\right), \tag{23}$$

while φ_z is identical to $(17)_2$.

Eqs. (13) and (19) were solved numerically in a plane layer assuming axial symmetry. The layer is bounded in the vertical direction by two horizontal planes, and by the cylinder surface, $\varpi = R$, in the horizontal direction. For an ideal gas the entropy is given by $S = C_v \log(p/\rho^\gamma)$, where the relation between pressure and density is

$$\frac{\partial}{\partial z}\left(p + \rho\langle u_z'^2\rangle\right) + \frac{B_\varpi}{\mu_0}\left(\frac{\partial B_\varpi}{\partial z} - \frac{\partial B_z}{\partial \varpi}\right) = -\rho g. \tag{24}$$

For the density stratification we have taken the model by Stix & Skaley (1990).

The bottom boundary is placed at a depth of $H = 30.000$ km below the photosphere. The requirement for the magnetic field to be vertical, $B_\varpi = 0$, and the constant heat flux, $F_z = 6.35 \cdot 10^{10}$ erg/cm^2/s, constitute the bottom boundary conditions. The near-photospheric region with its very steep stratification is excluded from the computation domain by placing the top boundary 1500 km below the photosphere. The vacuum boundary condition for the magnetic field and the black-body radiation for the heat transport are applied there. At the side boundaries we assume both the horizontal heat flux and the horizontal component of the magnetic field to equal zero (super conductor boundary). The aspect ratio is R/H=3.

The initial entropy profile is the steady solution of the heat transport equation (19) for the non-magnetic case. The initial magnetic field has vertical orientation and an exponential distribution over ϖ,

$$B_z(\varpi, t=0) = B_0 \exp\left(-\varpi^2/L^2\right), \tag{25}$$

where the scale L is defined by the magnitude of the total magnetic flux Φ: $L = \sqrt{\Phi/\pi B_0}$ (cf. Petrovay & Moreno-Insertis 1997). Starting from this initial state, Eqs. (13) and (19) were stepped forward in time. The magnetically-induced flow-system is neglected (see, however, Meyer et al. 1974).

The 'undisturbed' diffusivity, η_0, and the equipartition magnetic field, $B_{\rm eq}$, are specified after the solar model by Stix & Skaley (1990). The corresponding depth profiles are shown in Fig. 2.

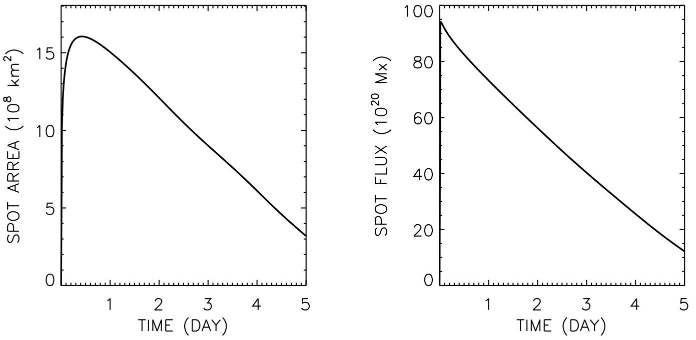

Fig. 3: The time dependencies of the spot area (LEFT) and the magnetic flux (RIGHT) for the run with the total flux of $\Phi = 10^{22}$ Mx.

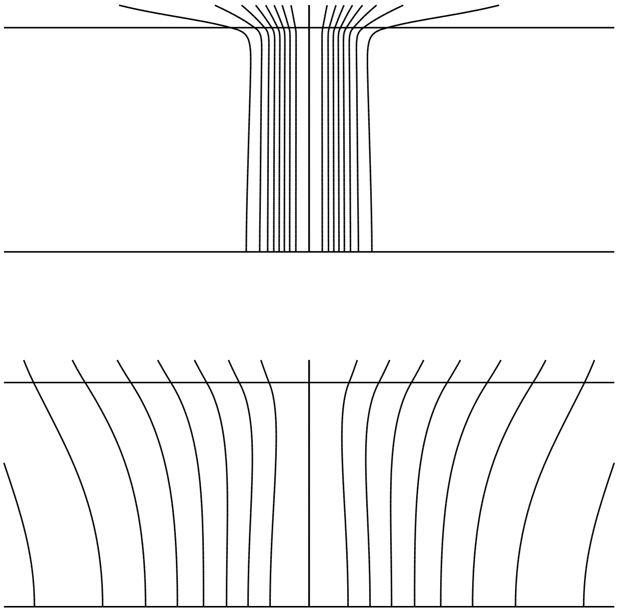

Fig. 4: The field line structure at the beginning of the run ($t \simeq 3$ hr, TOP) and after about 5 days (BOTTOM).

3. Results and discussion

In the numerical simulations, we define the ‘spot’ as the dark (circular) region where the luminosity is reduced below 75% of its undisturbed value. With this definition, there is no spot at the initial moment in spite of the strong magnetic field being present. However, the dark ‘spot’ develops in several hours. After that, the decay starts and lasts for about ten days. A typical example is shown in Fig. 3. The total flux $\Phi = 10^{22}$ Mx, and $B_0 = 3$ kG for this case. The main observation with this picture is that *the decrease of both the spot area and the magnetic flux with time are close to linear* laws. It seems to be also the case with real spots, see, however, Petrovay & van Driel-Gesztelyi (1997).

The decay rate, $\dot{A} \simeq 3 \cdot 10^{13}$ cm^2 s^{-1} of Fig. 3 is, however, considerably larger compared to the observational values of (8) and (9). In any case, the decay law loses the convex form ($\ddot{A} < 0$) characteristic for the solution of the linear diffusion equation (Krause & Rüdiger 1975).

The total magnetic flux through the top boundary does not change with time. Figure 4 shows the field line structure at the beginning of the run and at about 5 days after. The field distribution becomes much broader but the field strength in the center of the spot is reduced only slightly.

Figure 5 shows the profile of the surface luminosity normalized to its undisturbed value with the distance to the spot center. No bright ring can be found. The heat flux blocked by the magnetic structure is ‘smoothed’ over the simulation domain by the turbulent diffusion (Spruit 1992).

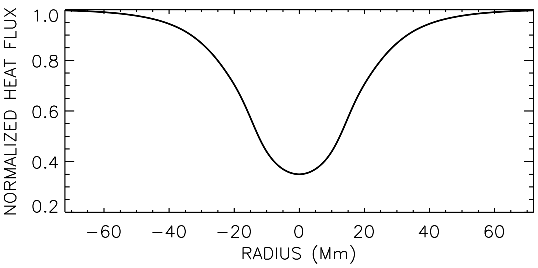

Fig. 5: The distribution of the normalized surface luminosity over the distance ϖ to the spot center. No bright ring can be observed. $t \simeq 5$ hr.

The diffusivity quenching concept serves well to reproduce the observed decay laws for both spot flux and area. The computed decay rates are somewhat too high, however. More detailed consideration the fluid motion may be noted as a prospect for the future model development. Probably the barocline and magnetic forces are not in balance leading to a meridional circulation. The allowance for the internal flow patterns – even under the influence of the basic rotation – may be hoped to reconcile the discrepancy between the computed and observed decay rates.

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