SUBROUTINE hpsort(n,ra)
INTEGER n
REAL ra(n)
Sorts an array ra(1:n) into ascending numerical order using the Heapsort algorithm. n is input; ra is replaced on output by its sorted rearrangement.
INTEGER i,ir,j,l
REAL rra
if (n.lt.2) return
The index l will be decremented from its initial value down to 1 during the “hiring” (heap
creation) phase. Once it reaches 1, the index ir will be decremented from its initial value
down to 1 during the “retirement-and-promotion” (heap selection) phase.
l=n/2+1
ir=n
10 continue
if(l.gt.1)then
Still in hiring phase.
l=l-1
rra=ra(l)
ext else
In retirement-and-promotion phase.
r=ra(ir)
ra(ir)=ra(l)
ir=ir-1
if(ir.eq.1)then
The least competent worker of all!
done
return
endif
i=l
Whether in the hiring phase or promotion phase, we here
setup to sift down element rra to its proper level.
j=1+l
20 if(j.le.ir)then
Do while j.le.ir:
if(j.lt.ir)then
if(ra(j).lt.ra(j+1))j=j+1
Compare to the better underling.
ext else
ra(j)=ra(j)
i=j
j=j+j
endif
endif
if(ra(j).lt.ra(j+1))j=j+1
Demote rra.
i=j
j=ir+1
else
This is rra’s level. Set j to terminate the sift-down.
j=ir+1
endif
goto 20
endif
ra(i)=rra
Put rra into its slot.
goto 10
END

CITED REFERENCES AND FURTHER READING:

8.4 Indexing and Ranking

The concept of keys plays a prominent role in the management of data files. A data record in such a file may contain several items, or fields. For example, a record in a file of weather observations may have fields recording time, temperature, and
wind velocity. When we sort the records, we must decide which of these fields we want to be brought into sorted order. The other fields in a record just come along for the ride, and will not, in general, end up in any particular order. The field on which the sort is performed is called the *key* field.

For a data file with many records and many fields, the actual movement of $N$ records into the sorted order of their keys $K_i$, $i = 1, \ldots, N$, can be a daunting task. Instead, one can construct an *index table* $I_j$, $j = 1, \ldots, N$, such that the smallest $K_i$ has $i = I_1$, the second smallest has $i = I_2$, and so on up to the largest $K_i$ with $i = I_N$. In other words, the array

$$K_{I_j}, \quad j = 1, 2, \ldots, N$$  \hfill (8.4.1)

is in sorted order when indexed by $j$. When an index table is available, one need not move records from their original order. Further, different index tables can be made from the same set of records, indexing them to different keys.

The algorithm for constructing an index table is straightforward: Initialize the index array with the integers from 1 to $N$, then perform the Quicksort algorithm, moving the elements around as if one were sorting the keys. The integer that initially numbered the smallest key thus ends up in the number one position, and so on.

**SUBROUTINE indexx(n,arr,indx)**

**INTEGER n, indx(n), M, NSTACK**

**REAL arr(n)**

**PARAMETER (M=7, NSTACK=50)**

Indexes an array arr(1:n), i.e., outputs the array indx(1:n) such that arr(indx(j)) is in ascending order for $j = 1, 2, \ldots, N$. The input quantities n and arr are not changed.
INTEGER i, indxt, ir, itemp, j, jstack, k, l, istack(NSTACK)
REAL a

DO 11 j=1,n
   indx(j)=j
ENDDO

jstack=0
l=1
ir=n

1 IF(ir-l.lt.M)THEN
   DO 13 j=l+1,ir
      indxt=indx(j)
      a=arr(indxt)
      DO 12 i=j-1,l,-1
         IF(arr(indx(i)).le.a)GO TO 2
         indx(i+1)=indx(i)
      ENDDO
      i=l-1
   ENDDO
   2 indx(i+1)=indxt
   ENDDO
   IF(jstack.eq.0)RETURN
   ir=istack(jstack)
   l=istack(jstack-1)
   jstack=jstack-2
   ELSE
      k=(l+ir)/2
      itemp=indx(k)
      indx(k)=indx(l+1)
      indx(l+1)=itemp
      IF(arr(indx(l)).gt.arr(indx(ir)))THEN
         itemp=indx(l)
         indx(l)=indx(ir)
         indx(ir)=itemp
      ELSE
         itemp=indx(l+1)
         indx(l+1)=indx(ir)
         indx(ir)=itemp
      ENDIF
      IF(arr(indx(l+1)).gt.arr(indx(l+1)))THEN
         itemp=indx(l+1)
         indx(l+1)=indx(l)
         indx(l)=itemp
      ELSE
         itemp=indx(l)
         indx(l)=indx(l+1)
         indx(l+1)=itemp
      ENDIF
      IF(j.lt.i)GO TO 5
      itemp=indx(i)
      indx(i)=indx(j)
      indx(j)=itemp
      GO TO 3
   ENDIF
   3 I=i+1
   IF(arr(indx(i)).lt.a)GO TO 3
   4 CONTINUE
   J=j-1
   IF(arr(indx(j)).gt.a)GO TO 4
   5 CONTINUE
   GOTO 3
   6 I=indxt
   7 J=istack(jstack)
   8 Jstack=jstack+2
   IF(jstack.gt.NSTACK)PAUSE 'NSTACK too small in indexx'
   IF(ir-1.ge.j-1)THEN
      istack(jstack)=ir
   ELSE
If you want to sort an array while making the corresponding rearrangement of several or many other arrays, you should first make an index table, then use it to rearrange each array in turn. This requires two arrays of working space: one to hold the index, and another into which an array is temporarily moved, and from which it is redeposited back on itself in the rearranged order. For 3 arrays, the procedure looks like this:

SUBROUTINE sort3(n, ra, rb, rc, wksp, iwksp)
INTEGER n, iwksp(n)
REAL ra(n), rb(n), rc(n), wksp(n)
C USES indexx
Sorts an array ra(1:n) into ascending numerical order while making the corresponding rearrangements of the arrays rb(1:n) and rc(1:n). An index table is constructed via the routine indexx.
INTEGER j
call indexx(n, ra, iwksp) Make the index table.
do ii j=1, n
   wksp(j)=ra(j)
endo ii

do ii j=1, n
   ra(j)=wksp(iwksp(j)) Copy it back in the rearranged order.
endo ii

do ii j=1, n
   wksp(j)=rb(j)
endo ii

do ii j=1, n
   rb(j)=wksp(iwksp(j)) Ditto rb.
endo ii

do ii j=1, n
   wksp(j)=rc(j)
endo ii

do ii j=1, n
   rc(j)=wksp(iwksp(j)) Ditto rc.
endo ii
return
END

The generalization to any other number of arrays is obviously straightforward.

A rank table is different from an index table. A rank table’s jth entry gives the rank of the jth element of the original array of keys, ranging from 1 (if that element was the smallest) to N (if that element was the largest). One can easily construct a rank table from an index table, however:
8.5 Selecting the Mth Largest

Selection is sorting’s austere sister. (Say that five times quickly!) Where sorting demands the rearrangement of an entire data array, selection politely asks for a single returned value: What is the \( k \)th smallest (or, equivalently, the \( m = N + 1 - k \)th largest) element out of \( N \) elements? The fastest methods for selection do, unfortunately, rearrange the array for their own computational purposes, typically putting all smaller elements to the left of the \( k \)th, all larger elements to the right, and scrambling the order within each subset. This side effect is at best innocuous, at worst downright inconvenient. When the array is very long, so that making a scratch copy of it is taxing on memory, or when the computational burden of the selection is a negligible part of a larger calculation, one turns to selection algorithms without side effects, which leave the original array undisturbed. Such in place selection is slower than the faster selection methods by a factor of about 10. We give routines of both types, below.

The most common use of selection is in the statistical characterization of a set of data. One often wants to know the median element in an array, or the top and bottom quartile elements. When \( N \) is odd, the median is the \( k \)th element, with \( k = (N + 1)/2 \). When \( N \) is even, statistics books define the median as the arithmetic mean of the elements \( k = N/2 \) and \( k = N/2 + 1 \) (that is, \( N/2 \) from the bottom and \( N/2 \) from the top). If you accept such pedantry, you must perform two separate selections to find these elements. For \( N > 100 \) we usually define \( k = N/2 \) to be the median element, pedants be damned.

The fastest general method for selection, allowing rearrangement, is partitioning, exactly as was done in the Quicksort algorithm (§8.2). Selecting a “random” partition element, one marches through the array, forcing smaller elements to the left, larger elements to the right. As in Quicksort, it is important to optimize the inner loop, using “sentinels” (§8.2) to minimize the number of comparisons. For sorting, one would then proceed to further partition both subsets. For selection, we can ignore one subset and attend only to the one that contains our desired \( k \)th element. Selection by partitioning thus does not need a stack of pending operations, and its operations count scales as \( N \) rather than as \( N \log N \) (see §11). Comparison with \texttt{sort} in §8.2 should make the following routine obvious: