Cosmological Radiative Transfer

Nick Gnedin

Reionization Inside a Computer

Easy physics: *fast, accurate*
- dark matter dynamics
- gas dynamics
- ionization/recombination/heating

Hard physics:
- star formation ← *phenomenology*
- radiative transfer ← *hard numerics*

Extra physics:
- magnetic fields
- dust
- cosmic rays

Perhaps, not needed **?**

Star Formation

Schmidt Law:

$$\frac{d\rho_*}{dt} = \epsilon_{\rm SF} \frac{\rho_{\rm gas}}{\tau_{\rm dyn}}$$

- Must be scale-dependent

Although, observed in large and small galaxies

Radiative Transfer Equation

$$\frac{\partial J_\nu}{\partial t} + c\vec{n}\frac{\partial J_\nu}{\partial \vec{x}} = -k_\nu J_\nu + S_\nu$$

Just another Boltzmann equation, but…

Major problems:

- 6-dimensional (or 5D + ν dependence)
- very high signal propagation speed (c/c_S~30,000)

Radiative Transfer Equation II

$$\frac{\partial f}{\partial t} + \vec{v}\frac{\partial f}{\partial \vec{x}} - \nabla\phi\frac{\partial f}{\partial \vec{v}} = 0$$

Yet another Boltzmann equation.

More Precisely

$$\frac{\partial J_\nu}{\partial t} + \frac{\partial}{\partial x^i}\left(\dot{x}^i J_\nu\right) - H\left(\nu\frac{\partial J_\nu}{\partial \nu} - 3J_\nu\right) = -k_\nu J_\nu + S_\nu$$

Looks rather long to me…
But, recall how we deal with dynamics:

$$\rho = \bar{\rho}\,(1+\delta)$$

Background, use full GR

Fluctuations, use Newtonian limit

More Precisely II

$$\bar{J}_\nu(t) \equiv \langle J_\nu(t,\vec{x},\vec{n})\rangle$$

Mean radiation background

Equation for the mean background includes GR effects (redshift) but no spatial gradients

$$\frac{\partial \bar{J}_\nu}{\partial t} - H\left(\nu\frac{\partial \bar{J}_\nu}{\partial \nu} - 3\bar{J}_\nu\right) = -\bar{k}_\nu\bar{J}_\nu + \bar{S}_\nu$$

$$\bar{k}_\nu \equiv \frac{\langle k_\nu J_\nu\rangle}{\bar{J}_\nu}$$

If you want to call a boy Peter, you at least have to have a boy!

More Precisely III

Let's define a fluctuation f_ν (like $1+\delta$):

$$J_\nu \equiv f_\nu \bar{J}_\nu$$

Then:

$$\frac{\partial f_\nu}{\partial t} + \frac{\partial}{\partial x^i}\left(\dot{x}^i f_\nu\right) = H\nu \frac{\partial f_\nu}{\partial \nu} - (k_\nu - \bar{k}_\nu + \frac{\bar{S}_\nu}{\bar{J}_\nu}) f_\nu + \frac{S_\nu}{\bar{J}_\nu}$$

Disgusting!!!

More Precisely IV

But, in the spirit of LSS theory:

$$\cancel{\frac{\partial f_\nu}{\partial t}} + \frac{\partial}{\partial x^i}\left(\dot{x}^i f_\nu\right) = H\nu \cancel{\frac{\partial f_\nu}{\partial \nu}} - (k_\nu - \bar{k}_\nu + \frac{\bar{S}_\nu}{\bar{J}_\nu}) f_\nu + \frac{S_\nu}{\bar{J}_\nu}$$

More Precisely V

$$\frac{a}{c}\frac{\partial f_\nu}{\partial t} + n^i \frac{\partial f_\nu}{\partial x^i} = -\hat{\kappa}_\nu f_\nu + \psi_\nu$$

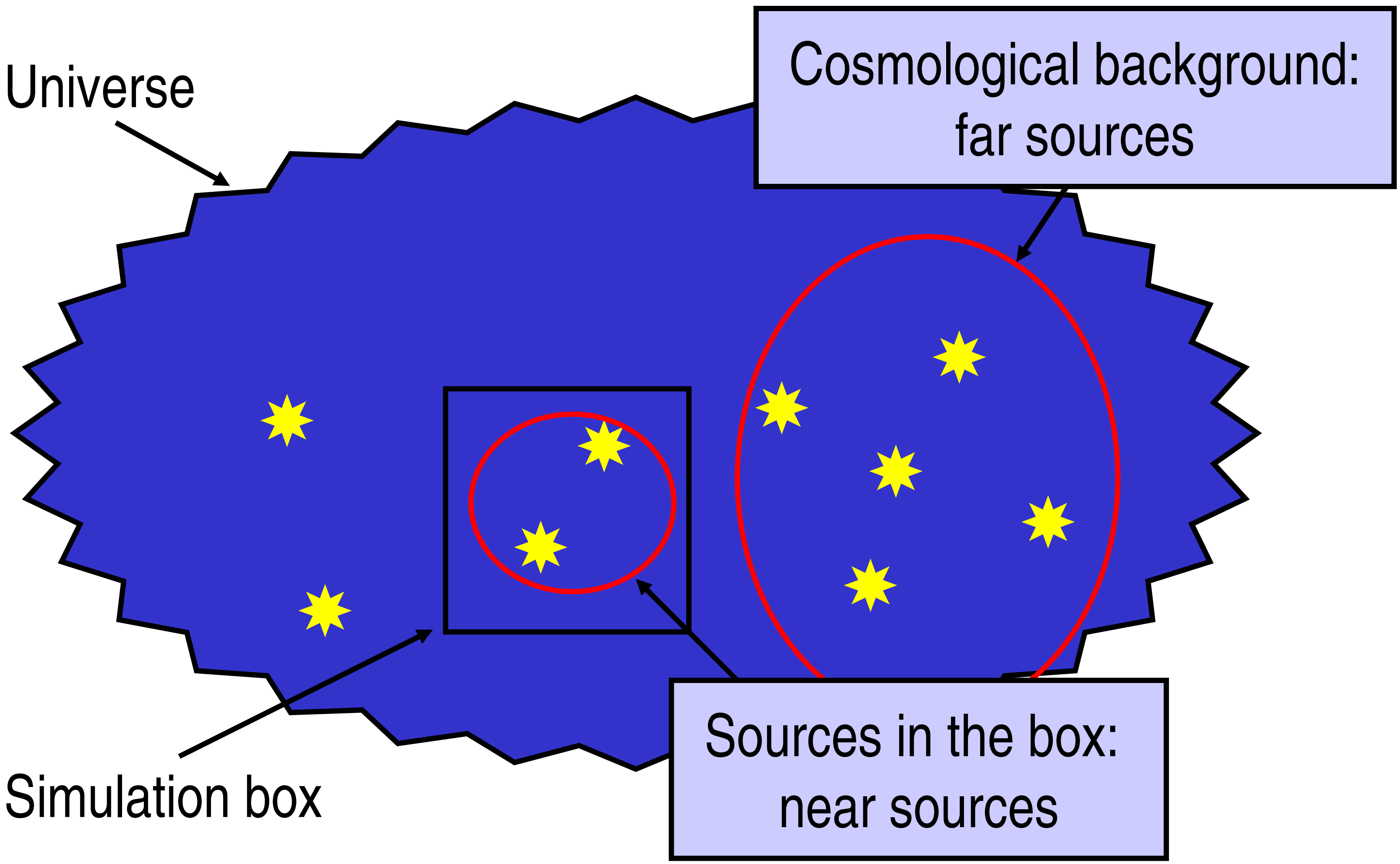
Can be omitted

$$\kappa_\nu \equiv \frac{a}{c} k_\nu$$

$$\hat{\kappa}_\nu \equiv \kappa_\nu - \bar{\kappa}_\nu + \frac{\bar{S}_\nu}{\bar{J}_\nu}$$

$$\psi_\nu \equiv \frac{a}{c}\frac{S_\nu}{\bar{J}_\nu}$$

Sources: Near vs Far



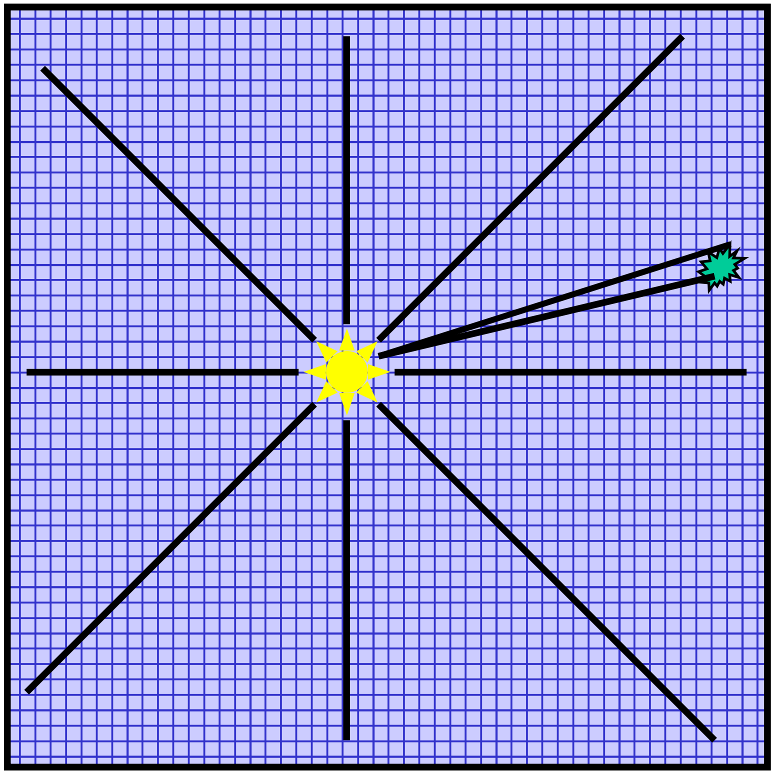
Using PM For Radiative Transfer

We cannot use the method of characteristics for RT:

$c/c_S \sim 30{,}000$ kills us

Also, we would need many more photon particles than the dark matter particles.

Ray Tracing



If we want to preserve the resolution of gas solver:

N(rays) ~ N(surface cells)

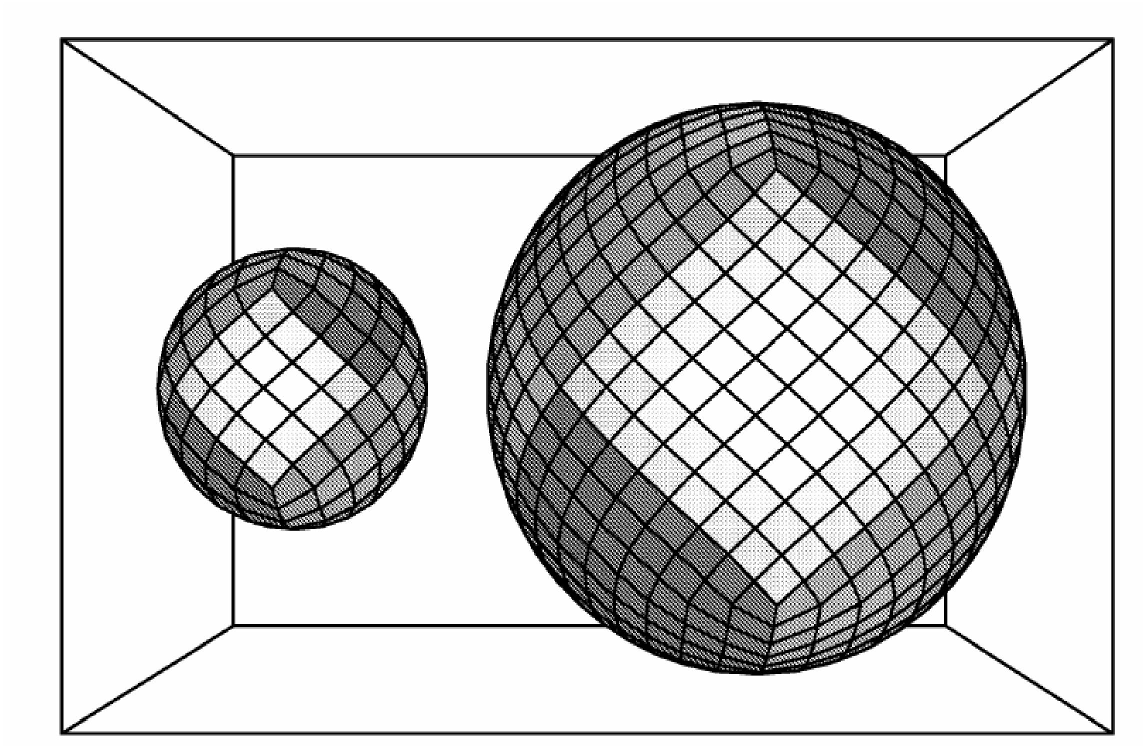
↓

Far sources: ~ $N^{5/3} \times N_\nu$

Near sources: ~ $N \times N_S \times N_\nu$

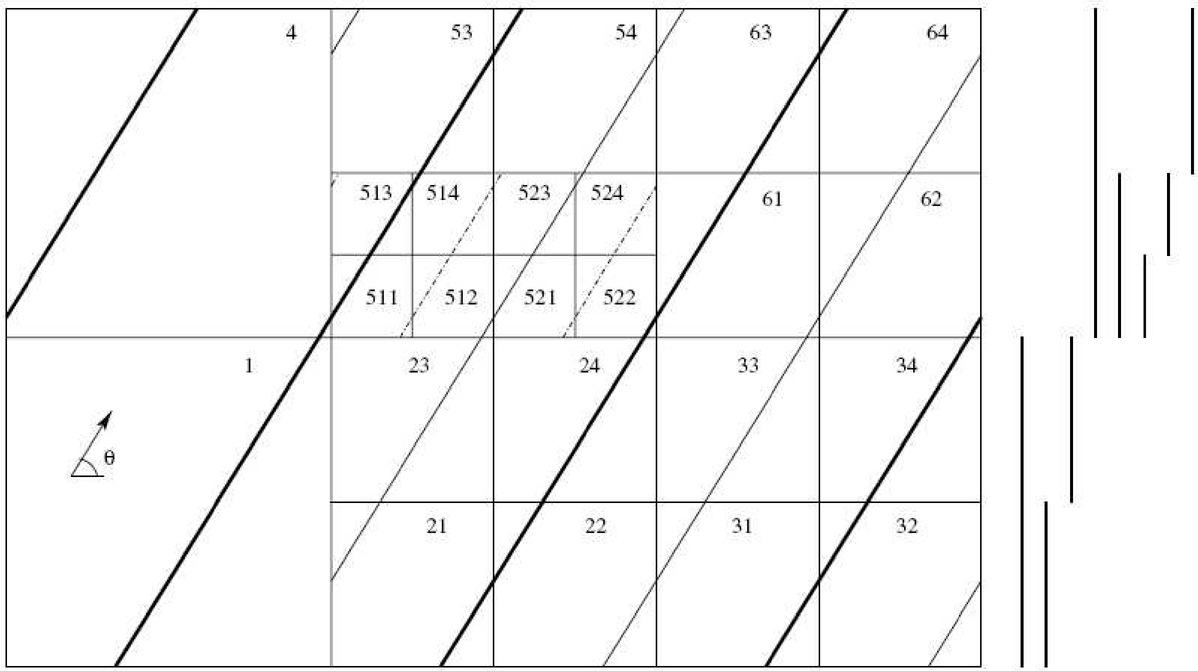
No way!

Ray Tracing II



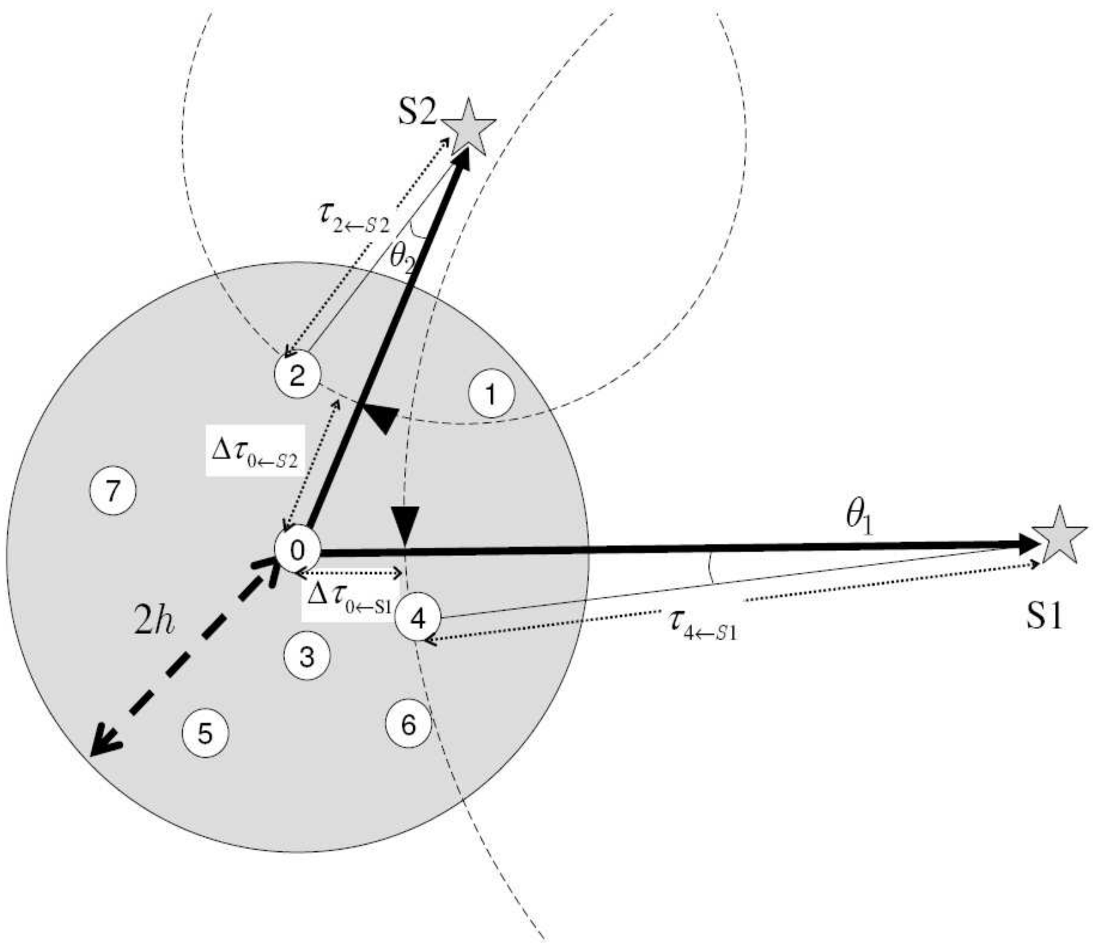
Smart ray-tracing a-la Abel & Wandelt 2002 (MNRAS 330, L53):
Put more rays at larger radii, where they are needed.

Ray Tracing in AMR



Razoumov & Cardall 2005 (MNRAS 362, 1413)

Ray Tracing in SPH



Moments of Boltzmann Equation

Massive particles:

- Density
- Momentum
- Pressure tensor
- …

Photons:

- Energy density
- Flux
- Radiation Pressure tensor
- …

The hierarchy needs to be closed.

But: photons move with the speed of light!

Eddington Tensor

$$P_R^{ij} = \frac{4\pi}{c}\langle J_\nu n^i n^j \rangle = E_\nu h^{ij}$$

Eddington tensor

$$\mathrm{Trace}(h^{ij}) = 1$$

5 independent components, not 6!

Diffusion Approximation

$$F_\nu^i = -D\nabla^i E_\nu$$

- Valid in the regime when radiation slowly diffuses through the medium, like in stellar atmospheres.
- Not very useful in cosmology.

OTVET Approximation

Simple idea:

• use the Eddington tensor from the optically thin regime.

Optically thin regime:

• collect $1/r^2$ contributions from all sources – just like gravity!

Result:

• **O**ptically **T**hin **V**ariable **E**ddington **T**ensor approximation

(Gnedin & Abel 2001, New Astronomy, 6, 437)

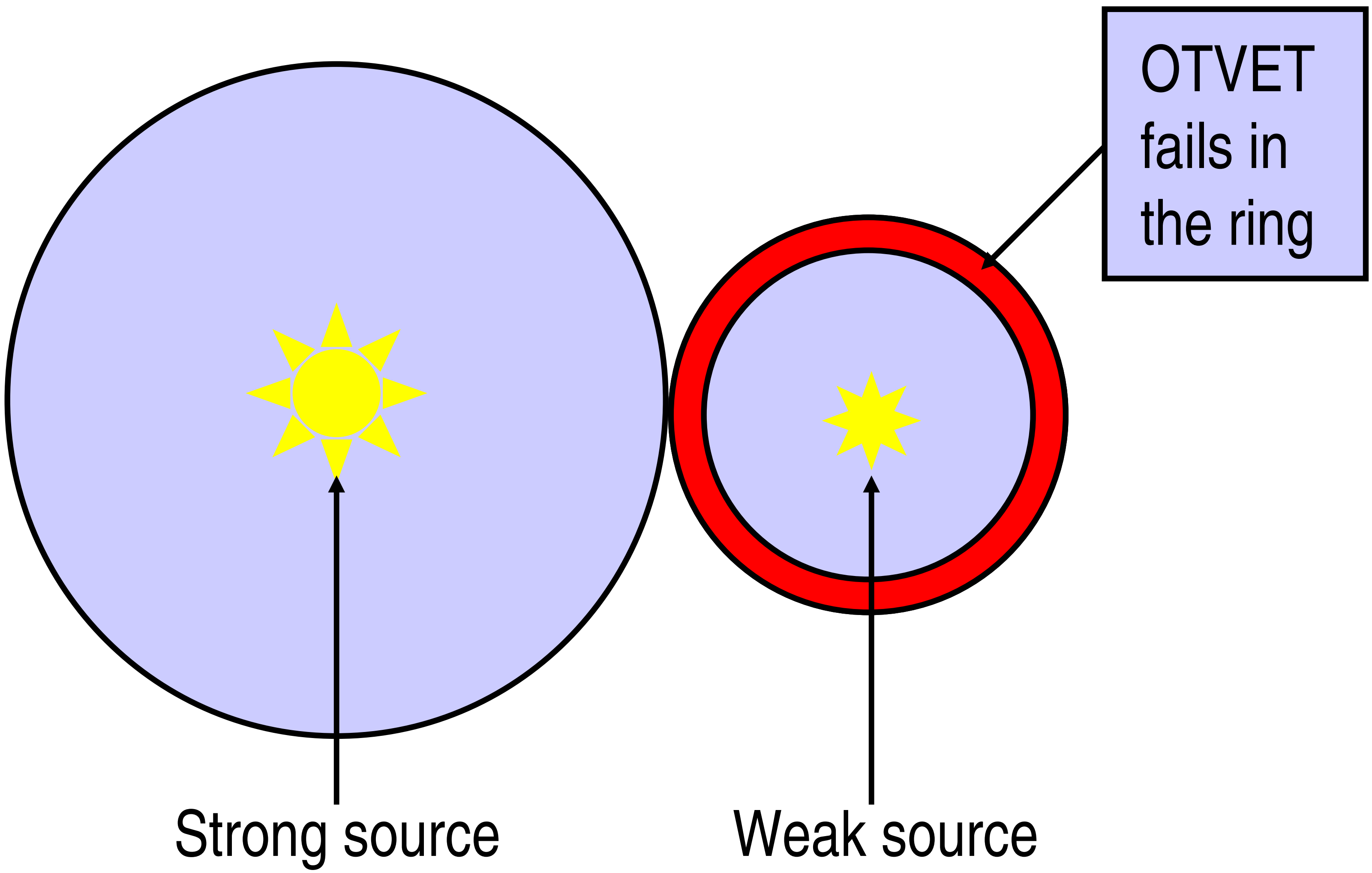
Scaling

- Moments equations (like hydro) ~ $N \times N_\nu$
- Eddington tensor (like gravity) ~ $N \times \log(N)$

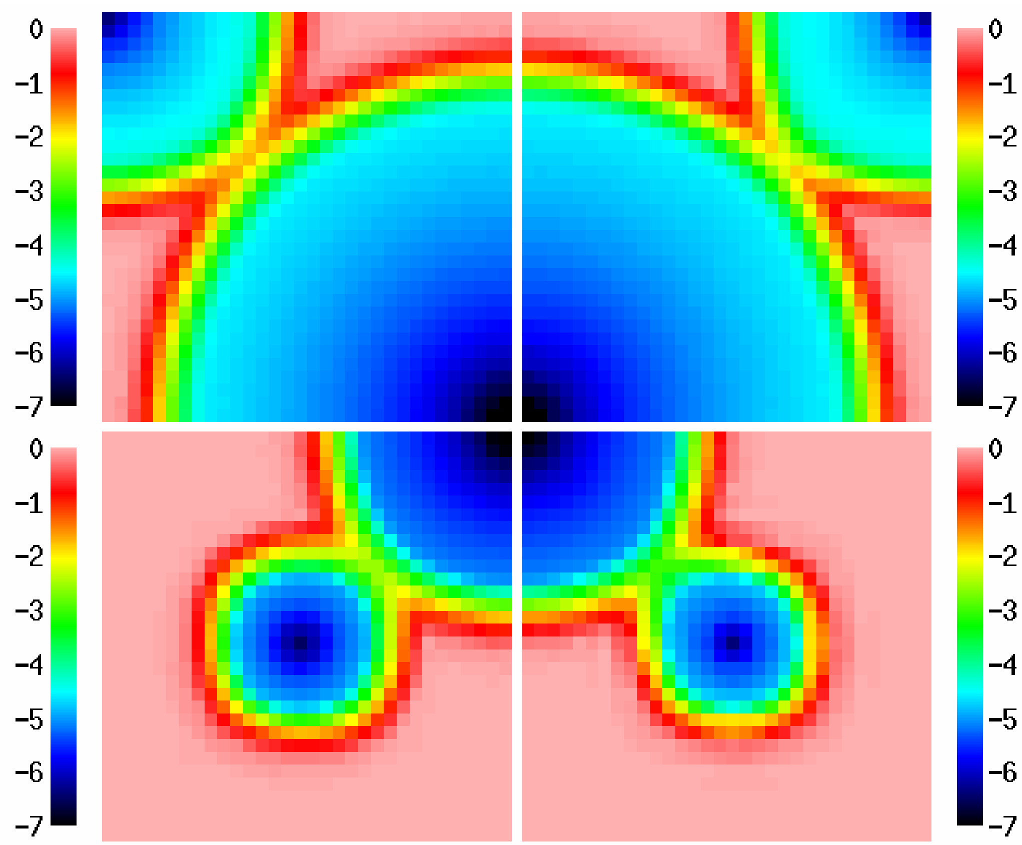
Advantages:

- **fast**
- controlled: the error of the approximation can be measured posteriori
- no scaling with number of sources: works for any N_S
- energy density and flux are conserved **exactly**

Where It Fails



Where It Fails II



One column is the exact solution, another one is OTVET

Reduced Speed of Light Approximation

- We still have a problem that the speed of light is too large: moments equations need to be integrated on a light crossing time-scale.
- But who said the speed of light must be 300,000 km/s?
- From the point of view of gas dynamics:

$$c/c_S\ (\sim 30{,}000) >> 1$$

- But:

$$10 >> 1 \text{ also}$$

- Can we use $c = 10\ c_S$ (100 km/s)?

Reduced Speed of Light Approximation II

Formal derivation: expand in powers of 1/c

$$2\frac{a}{c}\frac{\partial E}{\partial t} = \frac{\partial}{\partial x^j}\left(\frac{1}{\hat{\kappa}}\frac{\partial E h^{ij}}{\partial x^i}\right) - \hat{\kappa}E + \psi$$

Formally the next order, retained for numerical convenience

Frequency Dependence

- With logarithmically-spaced frequency bins, we need about 20 bins per e-folding to compute photo-ionization and photo-heating rates to better than 1%.
- That implies ~ 200 frequency bins between ~10eV and ~200keV.
- Way too many!

Frequency Dependence II

The case study: optically thin regime:

$$\langle J_\nu \rangle_\Omega(\vec{x}) = \bar{J}_\nu + \frac{a}{4\pi c} \int d^3x_1 \frac{S_\nu(x_1^i) - \bar{S}_\nu}{(\vec{x} - \vec{x}_1)^2}$$

Mean background (far sources)

Local sources

Frequency Dependence III

$$\langle J_\nu \rangle_\Omega = \bar{J}_\nu E_1 + \bar{S}_\nu D_2$$

$$Q_\nu = Q_{\rm OT} \exp\left(- \sum_\alpha \sigma_\nu^{(\alpha)} N_{\rm eff}^{(\alpha)} \right)$$

(Q is either E_1 or D_2, OT means Optically Thin, α=HI, HeI, HeII)

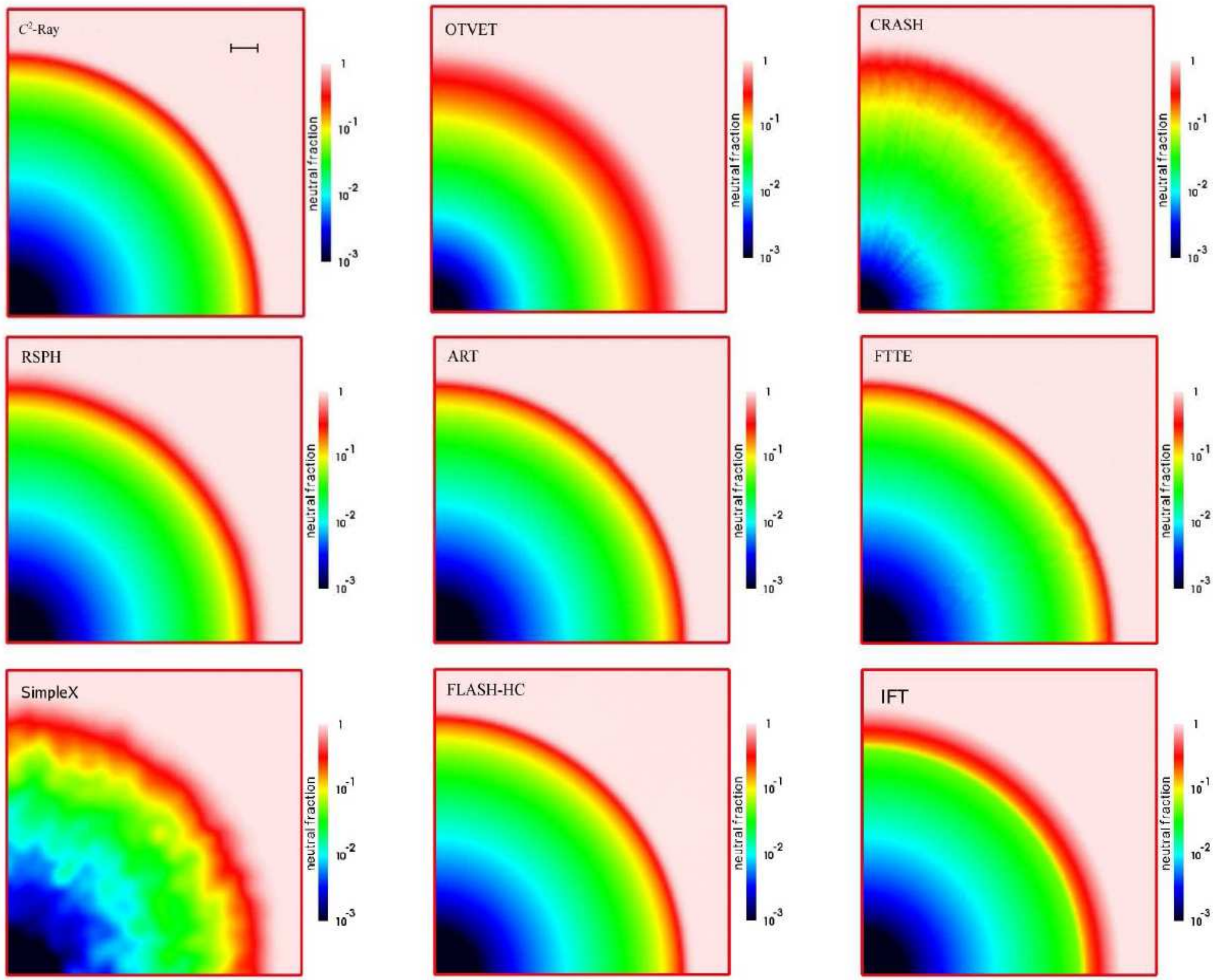
Frequency Dependence IV

Finally:

$$N_{\rm eff} = N_1 + X\,nL$$

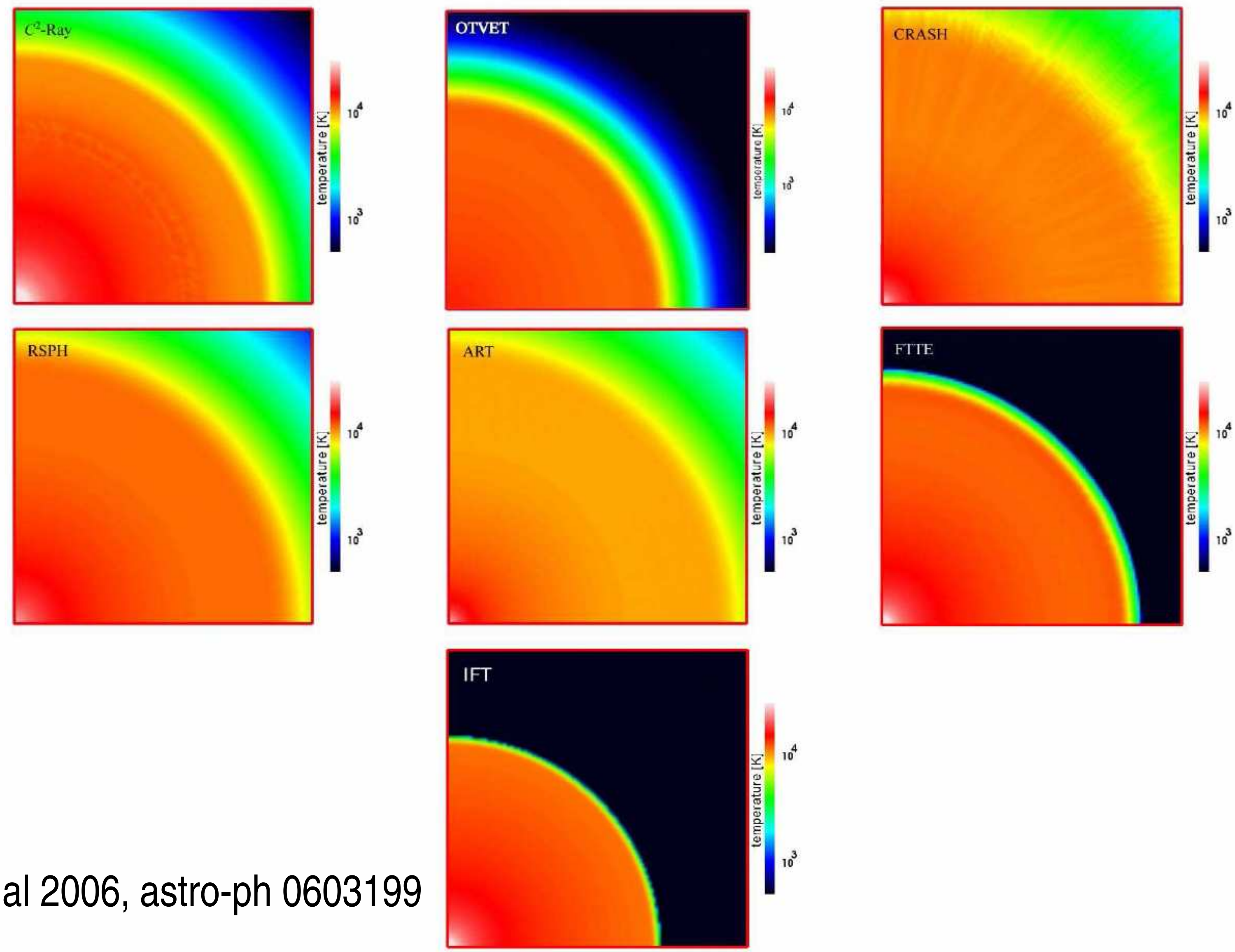
Where X is the abundance of whatever, n is the number density of baryons, and L is the resolution length.

Does It Work?



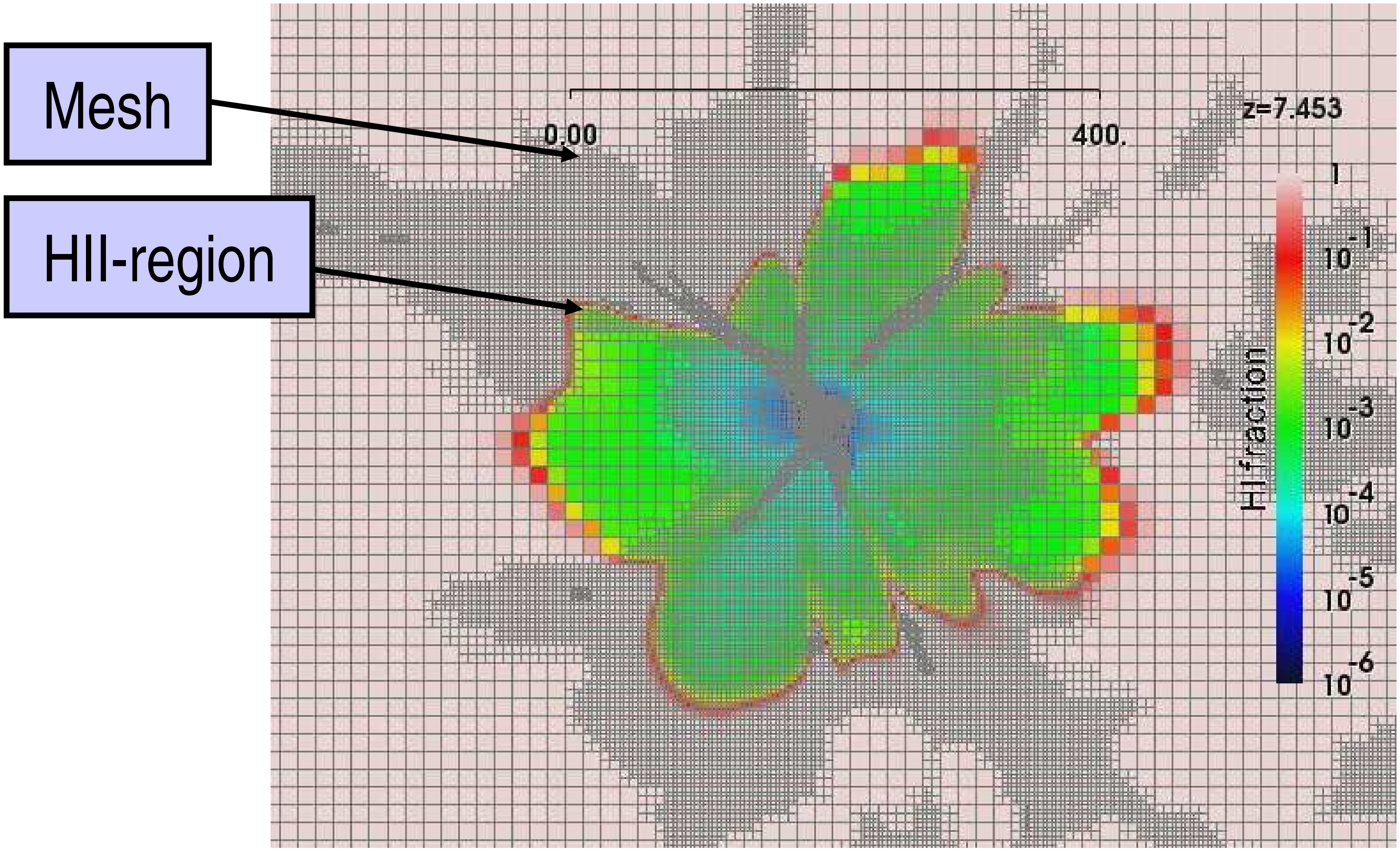
C²-Ray
OTVET
CRASH
RSPH
ART
FTTE
SimpleX
FLASH-HC
IFT
neutral fraction
1
10⁻¹
10⁻²
10⁻³

Does It Work?



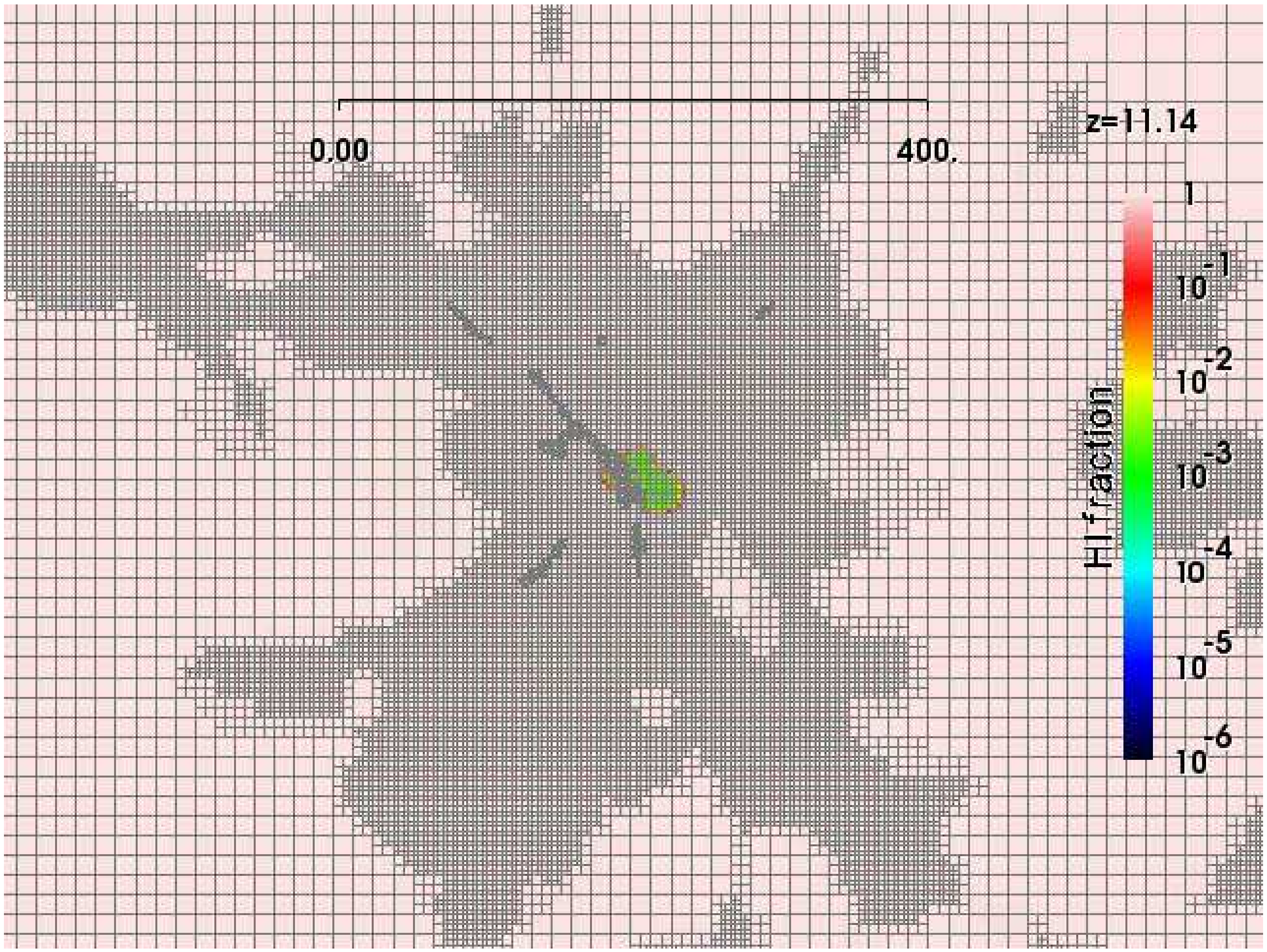
Iliev et al 2006, astro-ph 0603199

AMR & OTVET



OTVET is mesh-based, just like hydrodynamics!

AMR & OTVET



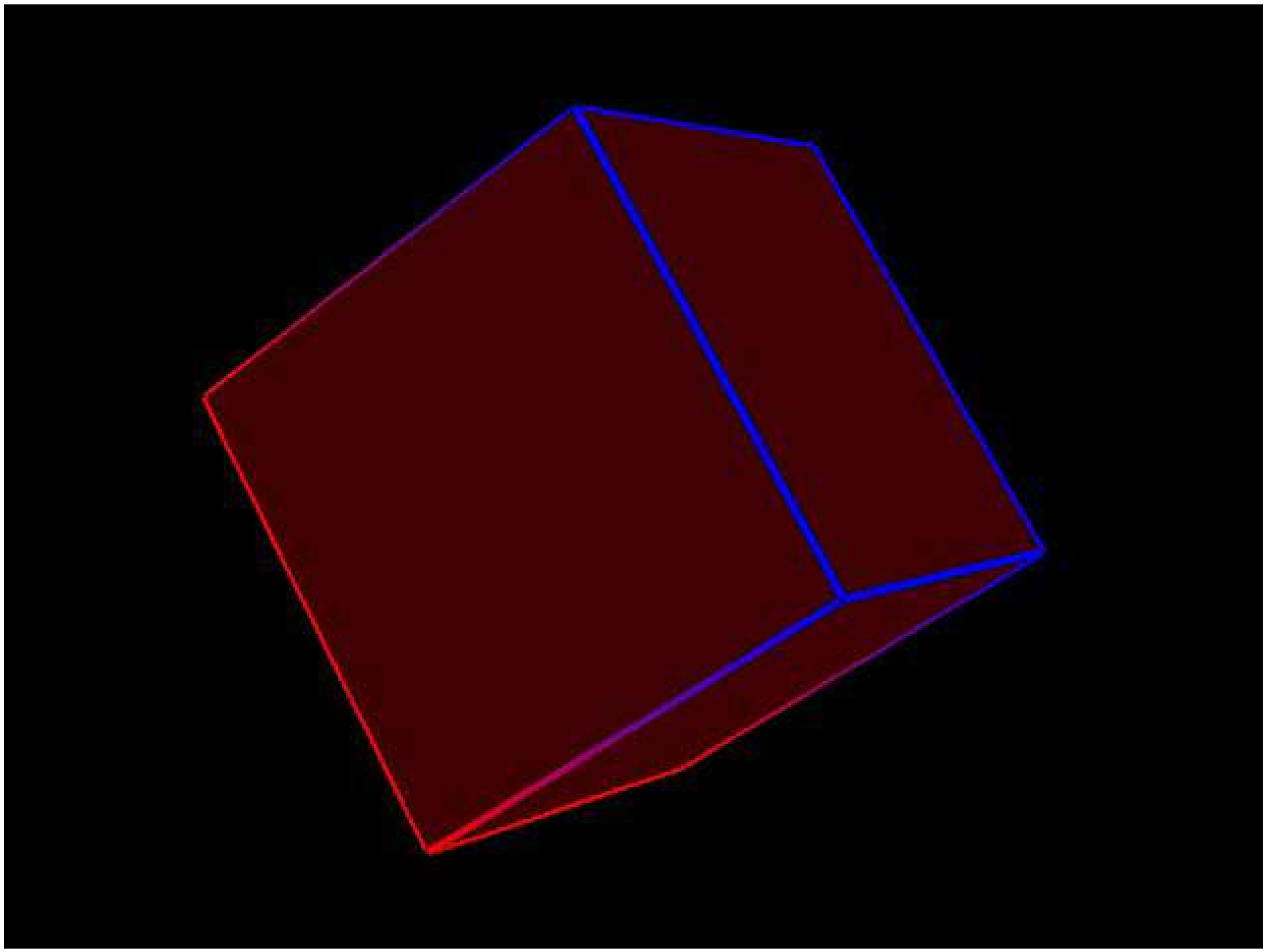
RT vs N-body

Radiative Transfer		N-body
Ray tracing	⇦⇨	Direct summation
OTVET	⇦⇨	Hydrodynamics
?	⇦⇨	PM
???	⇦⇨	P^3M, Tree, AMR

Conclusions

- Reionization is a complex process: its complete understanding will crucially depend on our ability to simulate radiative transfer with a fast and accurate solver.
- Large computational boxes will be required, because H II regions are biased and non-trivially shaped.
- As AMR takes over, we are left with few options for doing RT on adaptive meshes.
- Graduate students: ***YOU ARE THE ONLY HOPE!***

A Bonus



The End

Title