TRIANGULATION OF THE UNIVERSE

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Abstract: The use of the uniform cosmic population of cool hydrogen clouds, observed by the absorption forests in quasar spectra, for cosmometry is discussed. The assumption of a uniform configuration of absorbers yields a Friedmann-Lemaître universe with essential cosmological constant, and small, but positive curvature.

1 The curvature of space

H.J.Treder was my teacher. Together with my fellow-students we were most impressed by his ability to reduce scientific questions to their basic content, backed by an incredibly deep knowledge of the older literature, never reached by ourselves. This paper shall recall the contributions of H.-J.Treder to cosmology. I can cite only some, [28], [29], [30], [31], [33], [34], [35], [36], [37], [38], [39], [40], and add a small increment.

Einstein's General Relativity Theory solved the problem of an internally consistent model for a homogeneous universe. In this model, the universe is curved by gravitation, and generically evolving. Its evolution is to lowest order an evolution of size, i.e. an expansion, supported by the observation of the microwave background radiation and the cosmic abundances of helium and deuterium, produced in a early hot phase of the universe. Its curvature concerns the four-dimensional space-time, not necessarily the three-dimensional space, because one of the components of the four-dimensional curvature consists just in the expansion itself. The question to be considered here is the large-scale curvature of space.

Curvature is defined by the rotation of an object transported parallelly along a closed path. For a triangle on a surface, this is equivalent to the excess of the sum of angles against the euclidean value of π. If space has more than two dimensions, the different orientations of the triangle define different components of curvature. The most famous experiment to determine the curvature of space is that of Gauß to evaluate the triangle between the mounts Brocken, Inselsberg, and Kahler Asten. Even the curvature produced by the gravitational field of the earth is too small to be measureable that way. For the purpose of measuring the large-scale curvature the direct triangulation cannot be used: We are not able to leave our observation point.

We may, however, use spacecraft. Interpreting the definition operationally, we have to consider the parallel transport of the angular momentum vector of a gyroscope freely falling around the earth along a closed orbit. H.-J.Treder contributed to this question a small paper [32], and at that time a lot of effort to state the ability of the late academy of sciences to enter corresponding programs. We only mention, that freely falling motion can be used only in the solar system, which is infinitesimally small for the scale in question.

We might triangulate starting from a base in the solar system, built by different points on the earth or rockets or other planets. The baseline is at most of the order of the astronomical unit. By this method we can infer the apparent distance (parallax) of an object, supposing the euclidean geometry. If the true size of the object is known, we have an expectation for its apparent size. If the observed apparent size exceeds this value, we have a triangle with positive excess and positive curvature component. In this form, the method cannot be used because of the lack of objects of known size far enough for cosmic purposes and near enough to have a measureable parallax.

At last, we have to substitute the parallax by other means to determine an apparent distance. For cosmological scales, this is the redshift (requiring for existence the expansion in time and for evaluation the knowledge of the expansion law), the apparent size and the apparent magnitude (requiring objects not evolving in size or luminosity respectively), and volume in the sphere of the redshift of the object (requiring only the existence of the expansion, not its law, but supposing in addition some uniform population to measure the volume by number counts. The comparison of the observed angular size with the expected dependence on the redshift has been tried recently by Kellermann [14].

Compared with the distances in the universe, light is slow. Any distance in space means also distance in time. This is the reason why most of the determinations of the curvature of space are intrinsically connected with the determination of the expansion law. The redshift measures the distance of an object not only in space, but also in time.

2 The evolution of the universe

A homogenous and isotropic universe may be described by the line element

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - R^2[t](\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2)). \tag{1}$$

the radial coordinate is defined here by the surface of a sphere around the origin,

$$O = 4\pi r^2 R[t]^2. \tag{2}$$

The function $R[t]$ describes the expansion. If the curvature of space does not vanish ($k \neq 0$), it is usually chosen to be the curvature radius, and k is an index taking the values -1, 0, and +1. The coordinates r, θ, φ are comoving coordinates (expansion reduced), change in the position in (r, θ, φ) is peculiar motion.

For triangulation, we define the comoving radial distance χ,

$$\chi = \int_0^r \frac{\mathrm{d}r}{\sqrt{1-kr^2}}. \tag{3}$$

The basic formula is the comoving radial distance χ crossed by a light ray:

$$c^2\mathrm{d}t^2 = R^2[t]\mathrm{d}\chi^2. \tag{4}$$

Its first consequence is the cosmological redshift: If observer and emitter are supposed to have no peculiar motion, in an expanding universe wavelengthes are shifted to the red, and the redshift z is defined by

$$1 + z = \frac{R[\lambda_{\mathrm{observed}}]}{R[\lambda_{\mathrm{emitted}}]} = \frac{\nu_{\mathrm{emitted}}}{\nu_{\mathrm{observed}}} = \frac{R[t_{\mathrm{observed}}]}{R[t_{\mathrm{emitted}}]}. \tag{5}$$

The evolution of the expansion parameter $R[t]$ is ruled by Einstein's equations. In the case of an homogeous isotropic universe, they yield the Friedmann equation. The matter filling the universe may consist of different components, but for our consideration we have to take into account only two: pressure-free "cold" matter and "hot" radiation. Einstein's equations require

$$(\frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}t})^2 + \frac{kc^2}{R^2} = \frac{\Lambda c^2}{3} + \frac{8\pi G\varrho_{\mathrm{m0}}}{3}\frac{R_0^3}{R^3} + \frac{8\pi G\varrho_{\mathrm{r0}}}{3}\frac{R_0^4}{R^4}. \tag{6}$$

Λ is the cosmological constant (dynamically equivalent to a vacuum density, as long as the latter does not change in phase transitions), $\frac{kc^2}{R^2}$ the curvature of space. As a consequence of the Friedmann equation, the Hubble expansion rate, $H = \frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}t}$, is the square root of a polynomial in the redshift,

$$H^2[z] = H_0^2 h^2[z], \tag{7}$$

$$h^2[z] = \lambda_0 - \kappa_0(1+z)^2 + \Omega_{\mathrm{m0}}(1+z)^3 + \Omega_{\mathrm{r0}}(1+z)^4. \tag{8}$$

The parameters λ, κ, Ω are the entries of the right-hand side of eq.(6) in terms of the critical density defined by the expansion rate, $\varrho_{\mathrm{crit}} = 3H_0^2/8\pi G$, i.e. normalized cosmological constant, normalized actual curvature, normalized density. For $z < 10$, the contribution of radiation is less than 5% of the cold matter contribution, and will be omitted in the considerations to follow. The basic formula reads now

$$\chi[z] = \frac{c}{R_0 H_0}\int_0^z \frac{\mathrm{d}z}{h[z]}. \tag{9}$$

3 The curvature of an expanding space

In any dimension, the simplest indication of curvature consist in the comparison of volume and surface of a sphere. If the volume corresponds to the euclidean expectation, the space might be flat, if the volume exceeds that expectation, the space is positively curved, if the excess is negative, the space is negatively curved. A homogenous and isotropic space knows of no other component of curvature, and for the purpose of presenting the question about large-scale curvature, we consider such spaces only. The point is that this definition of curvature works independently of the expansion of the space in a cosmological model. Taking the redshift z as a radial coordinate, we get for $k = 1$ the formulas

$$O[z] = (1+z)^2\ o[z], \quad o[z] = 4\pi R_0^2 \sin^2[\chi[z]] \tag{10}$$

for the physical surface, and

$$V[z] = (1+z)^3\ v[z], \quad v[z] = 4\pi R_0^3 \int_0^{\chi[z]} \mathrm{d}\chi \sin^2[\chi] \tag{11}$$

for the physical volume. The defining equation for the comoving volume,

$$\mathrm{d}v[z] = o[z] R_0 \mathrm{d}\chi[z] \tag{12}$$

can be transformed into

$$\mathrm{d}v = \frac{\sqrt{o}\mathrm{d}o}{\sqrt{16\pi}\sqrt{1 - \frac{o}{4\pi R_0^2}}} \tag{13}$$

The only entry which modifies the euclidean expectation is the present curvature radius R_0. In comoving coordinates, we got rid of the influence of expansion. Translating equation (13) into physical volumes and surfaces, we have to substitute $v[z] = V[z](1+z)^{-3}$ and $o[z] = O[z](1+z)^{-2}$. the formula between the comoving quantities $v[z]$ and $o[z]$ is not spoiled by evolution effects. However, we observe $v[z]$ by counting, but only $O[z](1+z)^2 = o[z](1+z)^4$ by apparent luminosity. The pure effect of curvature is second order in redshift.

The intrinsic connection between distance in space and distance in time makes it impossible to determine the curvature of space independent of the evolution of the universe in first order of the redshift z. To first order, any measurement is a measurement of the deceleration parameter,

$$q = -\frac{R\ddot{R}}{\dot{R}^2}, \qquad q_0 = -\lambda_0 + \frac{1}{2}\Omega_0. \tag{14}$$

Quantities, in which the cosmological evolution is cancelled, the influence of the curvature of space is cancelled also (for example the surface brightness of an object of fixed physical size and absolute magnitude,

$$S[z] = S_0(1+z)^{-4}. \tag{15}$$

4 The quasar absorption forests

The forest of narrow absorption lines shortward of the redshifted Lyman-alpha emission in quasars has to be interpreted as indication of a population of intervening absorbers not connected with the quasar, but uniformly distributed in space. This hypothesis of uniform distribution makes these absorbers a possible standard for geometric evaluation. We want to show the interdependence of cosmological evolution (geometry of space-time), physical evolution (effective size of the absorbers), and geometrical configuration (dimension of the distribution).

We assume some homogeneous configuration of absorbers between the observer and the quasar. If this configuration shows no peculiar evolution in structure and comoving size, resp. cross-section σ, the distribution of absorption lines is independent of the comoving distance χ. The mean number of lines in an interval of comoving distance measures just this interval,

$$\mathrm{d}N \propto \sigma \mathrm{d}\chi, \tag{16}$$

written as density in redshift[1]

$$N[z]\mathrm{d}z = n_0 \sigma \frac{\mathrm{d}z}{h[z]}. \tag{17}$$

The number N represents the density of absorbers in comoving coordinates, and is assumed to be constant. It may vary in a model which supposes merging to be essential. We have now to account for the evolution of the absorbing objects. If they are of fixed physical size, and isolated in space, their comoving cross-section goes as

$$\sigma \propto L^2[z] = L^2_{\mathrm{phys}}(1+z)^2. \tag{18}$$

We relate the evolution of physical size to the actual physical size,

$$L^2[z] = L_0^2 l^2[z](1+z)^2 \tag{19}$$

and we get the known formula

$$N[z]\mathrm{d}z = N[0]\frac{\mathrm{d}z}{h[z]} l^2[z](1+z)^2 \tag{20}$$

In addition, we expect an column density proportional to the total absorbing mass, which might evolve,

$$M[z] = M_0 m[z], \tag{21}$$

and inversely proportional to the cross-section, which acts as a dilution of the material, and inversely proportional to the the surface at the given comoving distance, which dilutes the absorbers.

$$S[z] = S_0 \frac{m[z]}{l^2[z]}. \tag{22}$$

[1] Throughout this section, functions of z denoted by small letters are evolution factors with the value 1 at $z = 0$.

If the configuration if higherdimensional, if the absorbers form filaments and sheets, the effect of evolution in size dependes on the dimension in question. In case of filaments (dimension $d = 1$), the size enters linearily,

$$N[z]\mathrm{d}z = N\frac{\mathrm{d}z}{h[z]}l[z](1+z), \tag{23}$$

$$S[z] = S_0\frac{m[z]}{l[z]}(1+z). \tag{24}$$

In case of sheets [8] or bubble walls ([11],[17],[18]) one gets

$$N[z]\mathrm{d}z = N\frac{\mathrm{d}z}{h[z]} \tag{25}$$

$$S[z] = S_0 m[z](1+z)^2. \tag{26}$$

In general, the evaluation of the absorption spectra yields a combined evaluation of the evolution of the universe, $h[z]$, of the evolution of the absorbing mass density $m[z]$ and the size of the absorbers $l[z]$, and the dimension d (which might evolve also, if understood as fractal):

$$n[z] = h^{-1}[z](l[z](1+z))^{2-d} \tag{27}$$

$$s[z] = \frac{m[z]}{l^2[z]}(l[z](1+z))^d. \tag{28}$$

These formulas only illustrate the general situation, that no clear statement about the curvature (or evolution of the universe) can be made without assumptions about the evolution of the observed objects and their configuration.

Cloud of fixed temperature and mass, confined by the pressure of the hotter gas around, have to expand rapidly ($l^3 \propto (1+z)^{-5}$ if the surrounding gas cools by expansion, $l^3 \propto (1+z)^{-3}$ if the surrounding gas does not cool with expansion, because of the intergalactic radiation field). Cooling flows could produce an increasing mass [7], but in this model the increase is proportional to the surface of the clouds, not to their mass.

4.1 The Einstein-deSitter universe

If we assume an Einstein-deSitter (CDM) universe, we read the equations (27),(28) as determination of the evolution of effective size and effective absorbing mass. If we intend to assume a peculiar cosmological model, we write the equations (27), (28) in the form

$$l[z] = (n[z]h[z])^{\frac{1}{2-d}}(1+z)^{-1} \tag{29}$$

$$m[z] = s[z](1+z)^{-2}n[z]h[z] \tag{30}$$

These equation now determine the evolution factors $m[z]$ and $l[z]$ as functions of the observable factors $n[z]$ and $s[z]$, with an assumption about the evolution $h[z]$ of the expansion rate.

Usually, the evolution factor $n[z]$ is accepted in the form

$$n[z] = (1+z)^{\gamma} \tag{31}$$

while the exponent varies from $\gamma = 0.79$ [9] to 5.7 ± 1.9 [16] (1.81 ± 0.48 [43], $\gamma = 1.7$ [1], $\gamma = 2.7$ [22], [4], $\gamma = 2.1$ [41], 2.17 ± 0.36 [13], $\gamma = 2.09 \pm 0.48$ [10], 2.37 ± 0.26 [20]). Hoell and Priester [11] count approximately $\gamma = 0.25$. It is obvious to try likewise

$$s[z] = (1+z)^{\sigma}. \tag{32}$$

Combining the catalogues published in [1], [3], [2], [5], [12], [21], [23], [24], [25], [26], [27], [42], one has to correct for completeness in different ways, but mainly for the minimum equivalent width, which the spectra allow to find. The effect of this limit can be estimated if we suppose a particular (the exponential) distribution of the equivalent width at fixed redshift. We get the values

$$0.2 < \sigma < 1.6, \tag{33}$$

and

$$0.2 < \gamma < 0.9. \tag{34}$$

In Einstein-deSitter universes, the Lyα-effective mass seems to evolve fast

$$m[z] \approx (1+z)^{\mu}, \quad \mu \approx \sigma + \gamma - \frac{1}{2}, \tag{35}$$

independent of the dimension of the configuration. This is a strong decrease, presumably by heating the absorbing hydrogen into the ($n = 2$) state. This alone contrasts the model of slow condensation and cooling in general and the model of pressure-confined clouds in particular. In addition, we get for isolated clouds, $d = 0$, only a slow evolution of the physical size,

$$l[z] \approx (1+z)^{\varepsilon}, \quad \varepsilon \approx \frac{\gamma}{2} - \frac{1}{4}. \tag{36}$$

For $\gamma > 0.5$, this is a contraction. For filaments ($d = 1$), the size has to decrease in time still faster,

$$\varepsilon \approx \gamma + \frac{1}{2}. \tag{37}$$

For sheets, we get the condition

$$\gamma + \frac{3}{2} = 0, \tag{38}$$

which is far from being observed. At this stage, the evolution of equivalent width seems to rule out the combination of the Einstein-deSitter (cold dark matter) model of the universe with pressure-confined absorbers of constant effective mass.

4.2 The universal bubble structure

Assuming the lines to be produced by the walls of a bubble structure like that in the CfA-survey [6], we suppose $d = 2$: This is the unique case, where evolution is absent in formula eq.(27), and where the density of lines may be directly translated into the history $h[z]$ of the expansion ([11],[17],[18]). This determines the universe to be of the Friedmann-Lemaître class with positive curvature ($R_0^2 \approx 10R_H^2$) and minimum expansion rate $h_{\min}^2 \approx 0.5$ at $z_{\min} \approx 3.5$. The size of the walls does not enter the equation for the column density, but the evolution in mass is determined to be

$$\mu \approx \sigma - 2, \tag{39}$$

which is a slow increase in time, indicating faster cooling by metal contamination than heating by the intergalactic radiation field. This seems to be an appropriate feature of the model.

Two additional facts may speek for the bubble-wall absorption: First, the number density of lines seems to have a maximum at $z \approx 3.5$, which could reject the simple models we considered for the Einstein-deSitter case, which contains powers of $(1+z)$ only. Second, we can infer the actual size of the bubbles assumed to produce the absorption lines by their walls. It is about the size of the bubbles in the CfA survey, and this is remarkable, because it connects features observed with very different methods, and for different classes of objects.

4.3 Filaments

If we interpret the time problem in Jeans-type gravitational condensation schemes as indication of the fact, that it is necessary to take kinematic effects like caustics of the primordial velocity field into account, we make a strong point in favour of a filamentary configuration ($d = 1$). Other models with filamentary structures can be found in [15]. As long as the size parameter

$$l[z] \approx (1 + z)^{\varepsilon} \tag{40}$$

increases fast enough with time ($\varepsilon < -\frac{1}{2}$), the history $h[z]$ yields a positive curvature again. Only in the case where we have to suppose a slower increase or decrease, this cannot be affirmed, and the qualitative picture (positive cosmological constant, positive curvature, small positive mass parameter) changes essentially.

4.4 Warning

The evaluations of the line catalogues are condensed here into the two numerical exponents in the equations (29) and (30). These exponents might be criticized from the points of parameter estimation and of identification of lines.

With respect to parameter estimation, one can try to correct all values by supposing appropriate distributions, as we already did with the equivalent width. The main question to be solved remains the identification of lines. There are different points of view on this problem, indicated by the following three different options:

- All lines between Lyman alpha and Lyman beta indicate a Lyman-alpha absorption. This point is backed by the limitation of the absorption forest at the Lyman-alpha emission line and the more or less equal uniform aspect of the lines in high-resolution spectra.

- Some line are metal lines and have to be eliminated from the list of Lyman-alpha lines. Here, we have to keep in mind, that too many lines may be affected, because of the direction of effort is identification, and because of the blending with hydrogen lines.

- Virtually all narrow lines are metal, we only have not the means to identify them all. This point of view again produces a distribution of redshift values of intervening objects, the statistics of which are similiar to that of the hydrogen-based point of view, the redshifts may only be smaller than in the first interpretation.

We analysed the published data from the second point of view, which is the most conservative one. and wait for the redshift list of the third point of view to repeat the procedure.

5 Concluding remark

Taken as isolated statement, the distribution of quasar absorption lines cannot really decide between the bubble-wall interpretation connected with curved space and small matter content and the interpretation by isolated clouds with a CDM Einstein-deSitter model. If we add, however, our knowlegde or our hypotheses about the behaviour of the absorbing masses, the indications from other types of determination of the parameters of the cosmological model and the configuration of galaxies and clusters of galaxies in our mapped neighbourhood, the configuration of the absorbers becomes a considerable geometrical tool to measure the geometry of the universe far outside the region, where we can map the position of the galaxies today.

The present paper is part of the program to model the configuration and evolution of the Lyman-alpha absorbers in the frame of a Friedmann-Lemaître universe and to determine its parameters, initiated by J.Hoell and W.Priester. I would like to acknowledge the discussion with them, likewise the dispute with A.G.Dorozhkevich, S.Gottlöber, J.P.Mücket and V.Müller, not to forget the interaction with H.-J.Treder [19].

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