LINE FORMATION IN INHOMOGENEOUS ATMOSPHERES AND THE MAGNETIC STRUCTURE OF THE INTERNETWORK

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ABSTRACT

A stochastic approach for polarized line formation in inhomogeneous atmospheres with small-scale magnetic structures is presented and applied to spectro-polarimetric observations from internetwork regions. The small-scale and fluctuating nature of the magnetic field structures in the quiet solar photosphere requires an appropriate model to adequately describe the line forming process in these regions. On the basis of a Markov process, a stochastic transport equation for polarized light is derived which accounts for correlated atmospheric fluctuations in the horizontal direction as well as in the direction along the ray path. The characteristic asymmetry of the Stokes profiles and in particular the net-circular-polarization (NCP) of the Stokes V signal is a direct function of the new statistical scattering term of the stochastic transport equation. This allows us to determine the degree of fluctuation within the atmosphere and to estimate the characteristic length scales of the magnetic structures by analyzing the Stokes line profiles. This approach is applied to observed spectro-polarimetric data from internetwork regions which indicates that the non-magnetic as well as the magnetic structures have finite correlation lengths. Moreover the correlation length or coherency of the magnetic structures show a distinct dependence on the spatial region, whether they are located in granular or intergranular regions.

Key words: Sun: atmosphere; radiative transfer; spectro-polarimetry; magnetic fields.

1. INTRODUCTION

The solar surface magnetism is organized on a wide range of different scales from magnetically driven surface features such as sunspots, pores to active region phenomena such as faculae and plages. But the field is not only concentrated in active regions, even the so-called quiet magnetic Sun reveals a magnetic activity in form of small-scale flux concentrations. As the spatial resolution and sensitivity of magnetograms and spectro-polarimetric observations has improved, it rapidly became clear that even the internetwork regions harbour a substantial amount of magnetic flux. The significance of the internetwork field comes from the fact that the internetwork regions cover a large fraction of the solar surface and may account for most of the unsigned magnetic flux and energy existing on the solar surface (Sanchez Almeida, 2004).

A reliable diagnostic of these very low flux structures is not straightforward. How can we describe and model the line formation in these magnetic structures? Do these small-scale structures represent the weak and small-scale end of the basic magnetic building blocks of the solar magnetism in form of thin magnetic flux-tubes (Stenflo, 1994)? This concept was applied with great success to magnetic structures in network regions and significantly contributed to our present knowledge of the solar surface magnetism (Solanki, 1993). Or is this field the expression of a small-scale and turbulent magnetic field which can be characterized by a micro-structured magnetic atmosphere (MISMA) as proposed by Sanchez Almeida et al. (1996)?

The interpretation of Stokes profiles in the context of the thin flux-tube model relies on the basic picture of an embedded cylindrical magnetic structure surrounded by a quasi field-free medium. Based on that assumption a so called 1.5-dimensional radiative transfer is applied where a number of rays piercing through the underlying 2- or 3-dimensional structure of the model atmosphere to obtain a quasi realistic spectral ‘signature’ of the solar photosphere and the underlying magnetic structures (e.g. Solanki, 1993). Numerical inversions in the recent years have lead to an ever increasing level of sophisticated flux-tube modelling (Bellot Rubio et al., 2000; Frutiger & Solanki, 2001). But if the underlying structure is much more dynamic, disrupted and intermittent, the conventional static flux-tube model allows only a poor representation. Even a multi-component flux-tube model with a large number of possible free parameters provides only a schematic and static picture with its inherent inflexibility and a-priori fixed geometric structure. In this sense the flux-tube modelling provides a rather macroscopic treatment of the problem – in the 1.5 dimensional
sense – since the averaging process for each ray and line-of-sight (LOS) is performed after the integration process of the transfer equation.

The other extreme, in contrast to the macroscopic view, is the MISMA approximation. Here the assumption is that the photons along a particular LOS undergo a random and rapid fluctuation of the atmospheric environment. The fluctuation of the atmospheric parameters occurs on very short scales, such that a micro-structured or micro-turbulent approach is justified. This allows an averaging over all atmospheric parameters at each location before the transfer equation is integrated. But it is apparent that the micro-structured approach has a very limited range of validity, as soon as only one of the ensemble structures exceeds the micro-scale criteria or simply the geometry of the underlying model atmosphere changes, the MISMA approximation will break down.

Magneto-convective simulations suggest that neither a predefined static macro-structuring nor a pure micro-structuring in the solar photosphere is an appropriate starting point for the interpretation of polarized spectra, the possible structuring comprises more likely a broad range from small to large scales, indicating a lesser degree of organization with more fragmented and incoherent structures in low flux regions (Vögler et al., 2005; Stein & Nordlund, 2002).

This paper is organized as follows: In Sect. 2 we summarize the basic concept of the line formation in stochastic media and present the polarized stochastic transfer equation. In Sect. 3 we show how the statistical scattering and absorption terms of the stochastic transfer equation have a major impact on the area and amplitude asymmetry of the resulting Stokes V profiles. Sect. 4 emphasizes how a simple imbalance of the correlation lengths among the different atmospheric structures can reduce the measured apparent magnetic flux density. In Sect. 5 we give an overlook of a first preliminary investigation of spectro-polarimetric observations from internetwork regions with the presented stochastic approach, and in Sect. 6 we summarize our presentation.

2. THE STOCHASTIC TRANSFER EQUATION FOR POLARIZED LIGHT

Any radiative transfer modelling makes assumptions about the underlying atmospheric structure. How can we cope with an atmospheric structure which has an inherent high degree of intermittency, in both, the horizontal and vertical directions? The two modelling approaches based on the macro- or micro-structured descriptions of the atmosphere can not deal with an a-priori unknown structuring on finite scales along the ray path. In order to allow for an arbitrary fluctuation of atmospheric components perpendicular and along the line-of-sight (LOS) we pursue a rigorous probabilistic formulation to characterize the fluctuating atmosphere. The basic assumption made here, is that the a-priori unknown atmospheric region or volume of interest consists of random structures with different but finite spatial extensions, so that correlations between these structures can no longer be neglected. We will not derive here the entire stochastic model for the atmosphere nor we derive the stochastic transport equation for polarized light, this is done in detail by Carroll & Staude (2004, 2005). In this contribution we will only summarize the basic concept of this approach.

The atmospheric volume of interest is characterized by a fluctuation of different magnetic and non-magnetic components which all possess a finite spatial extension along an arbitrary ray path. The degree of coherency or correlation of the individual atmospheric structures is subject to a Markov process. This allows us to break down all higher spatial correlation effects to the first order. If we introduce a random atmospheric vector \( \vec{B} \) which comprises all atmospheric parameters such as temperature, pressure, velocity, magnetic field strength, magnetic field inclination, etc., which are necessary to characterize a particular atmospheric regime or component, we can then specify a suitable discontinuous Markov process with the following Poisson-like Kubo-Anderson process (Frisch & Frisch, 1976):

\[
p(B_{s+\Delta s} | B_s) = e^{\text{e}^{-s}} \delta(B_s - B_{s+\Delta s}) + (1 - e^{-s}) p(B_{s+\Delta s}).
\]

This conditional probability density describes the probability of being in the state or atmospheric structure \( B_{s+\Delta s} \) at the spatial position \( s + \Delta s \) after having moved a short distance \( \Delta s \) from the known atmospheric state \( B_s \) at \( s \). The probability for staying in the same regime like in the initial regime \( B_s \), expressed by the Dirac delta function, decays exponentially with \( \Delta s \) while a sudden jump into another regime \( B_{s+\Delta s} \) has an exponentially increasing probability and is weighted by the overall probability density of the final state \( B_{s+\Delta s} \). This conditional probability density describes the correlation between the two spatial points \( s \) and \( s + \Delta s \) along the ray path. The degree of correlation is controlled by the parameter \( \lambda_s \), the correlation length, which describes the mean path length (extension of the particular atmospheric structure). This generic stochastic model of the atmospheric structure allows us to formulate a differential equation for the depth-dependent atmospheric probability density function (PDF) from which we can derive a stochastic transport equation for the polarized light (Carroll & Staude, 2003, 2004, 2005). The stochastic transport equation can be expressed by means of a continuous Markov model which describes a smooth transition between the atmospheric structures and which leads to a Fokker-Planck-like transport equation, (Carroll & Staude, 2005), or a discontinuous model like the Kubo-Anderson process (2) which allows us to derive a master-like transport equation for the mean conditional Stokes vector,

\[
\frac{\partial Y_B(s)}{\partial s} = -KY_B + \epsilon + \int_{B} \lambda_B^{-1} Y_B' p(B', s) dB'_s - \int_{B} \lambda_B B Y_B p(B', s) dB'_s, \tag{2}
\]
where the mean conditional Stokes vector is defined as
\[
Y_B(s) = \int_B I_B(s) p(B, s) dB .
\] (3)

The stochastic transport equation for the mean conditional Stokes vector has an intuitive physical interpretation. It is important to realize that \(Y_B\) is conditioned on the particular atmospheric regime \(B\) and the transport Equation (2) describes its statistical evolution through the atmosphere along the LOS. There are four physical processes that govern the transport of the mean conditional Stokes vector: the two processes of absorption and emission and two more processes that describe the statistical inflow and outflow of intensity to or from the regime \(Y_B\) under consideration. In detail, the third term on the r.h.s. of Equation (2) gives the amount of intensity or photons which may enter from all other atmospheric regimes into the regime \(Y_B\) under consideration and can therefore be considered as an additional (statistical) source or scattering vector, while the fourth term describes the (statistical) loss or absorption of intensity due to transitions of photons to all other possible atmospheric states or regimes. The degree of statistical scattering and absorption is controlled by the correlation length \(\lambda\) of the particular atmospheric structures. The observable of our problem – the expectation value of the Stokes vector – at the height \(s\) can easily be obtained from a final integration of the mean conditional Stokes vector over the entire atmospheric state space \(B\),
\[
< I(s) > = \int_B Y_B(s) p(B, s) dB .
\] (4)

3. STOKES PROFILE ASYMMETRIES

Stokes \(V\) line profile asymmetries can be attributed to the existence of gradients in velocity and magnetic fields (Landolfi & Degl’Innocenti, 1996; Lopez Ariste, 2002). The net-circular-polarization or area asymmetry is a good parameter which indicates the inhomogeneity of the atmosphere along the LOS. As can be shown (Carroll & Staude, 2005), in a fluctuating or random atmosphere, the first statistical scattering term in Equation (2) produces most efficiently the NCP or area asymmetry of the Stokes \(V\) line profile. To demonstrate this behavior we have performed some simple model calculations in a stochastic two-component scenario. Here we have assumed that the magnetic ensemble has a field strength of 500 G and is embedded in a field-free medium with a net downflow of 1.5 km/s. The area asymmetry \(\delta A\) and the amplitude asymmetry \(\delta a\) are defined as follows:
\[
\delta A = s \int_{-\infty}^{\infty} V(\lambda) d\lambda ,
\]
\[
\delta a = s \frac{\text{max}(V(\lambda)) + \text{min}(V(\lambda))}{\text{max}(V(\lambda)) - \text{min}(V(\lambda))} ,
\] (5)

where \(s\) is the sign of the blue lobe of the Stokes \(V\) profile. In the following Figures (1 and 2) the Stokes \(V\) profile area asymmetry and the amplitude asymmetry is plotted over the correlation length. The effect is demonstrated by using the iron line FeI \(\lambda 630.25\) nm. As can be seen from the the two Figures (1 and 2), the area and amplitude asymmetry are directly dependent on the correlation length (the fluctuation rate) of the atmosphere. The smaller the correlation length or higher the fluctuation rate the stronger the asymmetry is pronounced. Please note that only the correlation length is changed in that scenario. For very small correlation length the asymmetry asymptotically reaches the micro-turbulent limit, which is calculated under the MISMA approximation and is denoted by the dashed line in the Figures (1 and 2). For very large correlation length and a low fluctuation rate the asymmetry asymptotically goes to zero where all asymmetries vanish, which is the expected result because the magnetic structures were assumed to have no gradients in the magnetic field. This behavior is a direct

![Figure 1](image1.png)

**Figure 1.** The area asymmetry vs. the correlation length. The area asymmetry exhibits a clear functional dependence on the correlation length, no other atmospheric parameters in this scenario are changed. The dashed line indicates the area asymmetry which would originate from a micro-structured atmosphere, calculated under the MISMA approximation.

![Figure 2](image2.png)

**Figure 2.** The amplitude asymmetry vs. the correlation length. The area asymmetry is, similar to the area asymmetry, a function of the correlation length. The dashed line indicates the amplitude asymmetry (in percent) as calculated under the MISMA approximation.
consequence of the stochastic transport equation (2 where the statistical scattering and absorption is inverse proportional to the correlation length $\lambda$. Carroll & Staude (2005) show that the here presented stochastic transfer equation includes the micro-structured (MISMA) approximation as well as the macro-structured approach as limiting cases. The asymmetries of the circular polarization signal allows us therefore not only to estimate how the atmosphere is structured in the horizontal or perpendicular to the LOS, it also allows us to determine the structural length scale of the different atmospheric components.

4. APPARENT MAGNETIC FLUX REDUCTION

Another quite drastic consequence of a finite correlation length results from the possible imbalance of the mean structural length scale among the different atmospheric structures. Besides an imbalance of the conventional filling factor (which is the mean horizontal or surface filling fraction of the components) we consider here what happens if the 'vertical fill fraction', the correlation length is not equally distributed among the atmospheric structures. A simple scenario illustrates the drastic effect an imbalance among the atmospheric structures may have on the resulting polarization signal. In the following test scenario we calculated the polarized spectrum based on a stochastic 2 component ensemble, similar to the configuration in the preceding section (3). The magnetic field strength is again set to 500 G and the ambient non-magnetic structures have a net downflow of 1 km/s, the overall filling factor of the magnetic structures are 5%. The unsigned magnetic flux density is usually estimated from the degree of circular polarization (Stokes V). Under the assumption of weak magnetic fields the so called magnetograph formula can be used to approximate the longitudinal flux density $B_{FD}$ as a function of the measured polarization over a certain spectral bandwidth (Landi Degl’Innocenti, 1992, Socas-Navarro, Martinez Pillet & Lites, 2004),

$$
B_{FD} \simeq C \frac{\int V(\lambda) p(\lambda) d\lambda}{\int |dI(\lambda)/d\lambda| p(\lambda) d\lambda},
$$

where $C$ is a constant with $C = -1/(4.67 \times 10^{-13} \lambda^2 g)$ and $p(\lambda)$ is a step-like filter function which limits the bandwidth. Starting from equally sized non-magnetic and magnetic structures with a correlation length of 1000 km (almost macro-structured) the correlation length of the magnetic structures is gradually reduced. Please note that neither the filling factor nor other parameters are changed here. The decrease of the unsigned circular polarization in Figure (3) with the decreasing correlation length of the magnetic medium is exclusively driven by the reduction of the magnetic correlation length. The correlation length can be considered here as the filling factor along the LOS. It is clear that besides the conventional horizontal filling factor the correlation length determines the degree of circular polarization and therefore the measured magnetic flux density. The weak flux of the very dynamic quiet photosphere is not only a matter of the small-scale filling factor or of the frequent occurrence of mixed polarities in these regions, it also has to take into account that the structures have finite correlation lengths which can be considerable smaller than that of the ambient non-magnetic convection. In particular the absence of a measurable linear polarization in the internetwork (Lites, 2002), which would indicate horizontal fields, has its possible explanation in the fact that horizontal fields may have a much smaller correlation length and the resulting polarization signal is therefore drastically reduced.

5. ANALYSIS OF INTERNETWORK FIELDS

Based on the stochastic meso-structured approach we analyzed maps of full Stokes profiles of the pair of photospheric Fe I lines at 630 nm, taken at the High Altitude Observatory/National Solar Observatory Advanced Stokes Polarimeter (ASP). These quiet sun data were obtained by B. Lites on 1994 September 29 (Lites et al., 1996). In a first step we analyzed granular and intergranular regions in order to determine the convective characteristics of the non-magnetic components. We have utilized a simple least-square procedure to determine the convective velocity field for a number of positions in the map. For the granular and intergranular components we adopt the granular and intergranular model atmospheres from Borrero & Bellot Rubio (2002). Based on the temperature and pressure stratification of these models we assumed a simple 3-component stochastic velocity field whereby the inversion routine decides which of these components has an upstream (granular) or downstream (intergranular) characteristic. The free parameter of the fitting routine are the three single-valued velocities of each component, the probability values of the individual components (comparable to the conventional filling
Figure 4. Example of the fit of a granular Stokes $I$ profile of the Fe I 6301.5 nm line. The diamonds represent the observations, the solid line the fitted profile. A simple (stochastic) 3-component structure on meso-scales is sufficient to reproduce the Stokes $I$ profile, no micro-turbulent nor macro-turbulent broadening is used.

Figure 5. Fit of a granular Stokes $V$ profile of the Fe I 6301.5 nm line for the same pixel like the Stokes $I$ profile shown in Figure (4). The diamonds represent the observations, the solid line the fitted profile.

factor) and the correlation lengths of the individual components. For an upflow (granular) region a fit is shown in Figure (4). The remarkable result here is that there is no need to use nonphysical parameters like micro- or macro-turbulence to fit the profile. The convective velocity structures exhibits a clear meso-structured behavior on scales between 200 km and 400 km. The broadening of the profiles is not the result of micro-turbulence or macro-turbulence, the structures have a finite extent and fluctuate on a moderate scale. These results are in agreement with results of an earlier investigation of meso-turbulence by Gail & Sedlmayr (1974). We have analyzed a number of pixels that way, which possess a sufficient degree of polarization. Thereby we could place tight constraints on the ambient velocity field of the convection where the magnetic structures are embedded. The free parameter of the following determination of the magnetic structure are the velocity of the magnetic components the magnetic field strength of the individual magnetic components, and the correlation lengths. The Stokes $V$ signals as well as

the Stokes $I$ profile shown in the Figure(4), (5) and (6) were averaged with its nine neighboring pixels to reduce the noise. A two-component magnetic structure could reasonably well fit the Stokes $V$ profiles shown in Figure (5) and (6). In particular the asymmetries of the Stokes $V$ profiles are well reproduced. The structures for that granular magnetic field have a correlation length between 50 km and 125 km. These structures are clearly not a coherent entity in a macroscopic sense. The first results indicating the following picture of the internetwork magnetic structure: the typical mean length scale (correlation length) of the magnetic structures is larger in intergranular regions (150 km to 230 km) than in the granular regions (50 km to 125 km), the conventional filling factor is larger than a macro-structuring (single-valued flux-tube-like structure) would suggest. The stronger the field strength the larger the correlation length of the magnetic structures. The structural length scales found for the internetwork magnetic fields seem to be not consistent with a canopy structure (Stenflo, 1994), the typical canopy structure which a flux-tube in lateral pressure balance, hydrostatical equilibrium and with the constrain of flux conservation would imply a correlation length $> 500$ km.

6. SUMMARY

The intermittency of the magnetic fields in the quiet solar photosphere and the highly dynamic behaviour of the ambient plasma suggests that any line-of-sight through an unresolved atmospheric volume is subject to an inhomogeneous and/or discontinuous process. The here presented stochastic polarized radiative transfer equation offers an additional degree of freedom in the transport equation – the correlation length – to account for discontinuities and/or fluctuation of the atmospheric parameters along the LOS.

The NCP or area asymmetry of the Stokes $V$ profile strongly depends on the correlation length and allows
therefore, by means of the here presented stochastic approach, an estimation of the mean spatial extension (correlation length) of the individual atmospheric structures. In the light of our first investigation we can say that neither a micro-turbulent approach, such as the MISMA approximation (Sanchez Almeida et al., 1996), nor a typical macro-structured approach adequately model the line formation in small-scale intermittent magnetic and non-magnetic structures. A detailed derivation of the stochastic transfer equation for polarized light as well as the characterization of the magnetic atmosphere on meso-scales, in terms of a meso-structured magnetic atmosphere (MESMA), is given by Carroll & Staude (2005).

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