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Magnetic Λ-quenching and Grand Activity Minima

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Abstract. The nonlinear interaction between dynamo-induced magnetic fields and differential rotation in stellar convection zones has been found to significantly control the long-term activity behavior of solar-type stars. The magnetic back reaction is threefold: Lorentz force, α-quenching and Λ-quenching. The first feedback may provide an interesting non-periodic behavior which, however, disappears when α quenching is included. Yet if a strong Λ-quenching is allowed to modify the differential rotation, the dynamo resumes its exotic performance with periodically changing cycle periods and fluctuating magnetic parities. The magnetic Prandtl number must be smaller than unity.

1. Introduction

The period of the solar cycle and its amplitude vary considerably around their averages. Sunspots were even almost entirely absent during the Maunder minimum between 1666 and 1715 (Spörer 1889). Similar lulls of activity, which we will henceforth call grand minima, were detected after the Maunder minimum and in earlier medieval times. The activity cycle of the Sun is not exceptional: The observation of chromospheric Ca-emission of solar-type stars yielded activity periods between 3 and 20 yr (Noyes et al. 1984, Baliunas & Vaughan 1985, Saar & Baliunas 1993). A few of these stars do not show any significant activity. This suggests that even the existence of grand minima is a typical property of cool main-sequence stars like the Sun.

2. The Dynamo and Differential Rotation Model

The model is based on magnetic feedback to the internal solar rotation (Weiss et al. 1984, Jennings & Weiss 1991). Kitchatinov et al. (1994) and Tobias (1996, 1997) even introduced the conservation law of angular momentum in the turbulent convection zone (CZ) including magnetic feedback in order to produce grand minima of the dynamo cycle. In the present paper, we present a 2D mean-field theory in spherical polar coordinates based on a solar overshoot dynamo model (Rüdiger & Brandenburg 1995) together with a theory of differential rotation based on the Λ-effect concept (Küker et al. 1993).

We assume axial symmetry of the hydro-magnetic state of the star and ignore meridional flows. The field equations for the CZ include the effects of diffusion, α-effect, toroidal field production by differential rotation, and the Lorentz force. The dimensionless form of the equations used is

$$\begin{aligned}\frac{\partial\Omega}{\partial t} &= \frac{\mathrm{Pm}}{r^4}\frac{\partial}{\partial r}\left(r^3\left(r\,\frac{\partial\Omega}{\partial r}-V^{(0)}\Omega\right)\right)+\frac{\mathrm{Pm}}{r^2\sin^3\theta}\frac{\partial}{\partial\theta}\left(\sin^3\theta\frac{\partial\Omega}{\partial\theta}\right)+\\ &+\frac{\mathrm{E}}{r^2\sin\theta}\left(\frac{1}{r}\frac{\partial A}{\partial\theta}\frac{\partial(rB)}{\partial r}-\frac{1}{\sin\theta}\frac{\partial A}{\partial r}\frac{\partial}{\partial\theta}(\sin\theta B)\right),\end{aligned} \tag{1}$$

$$\frac{\partial A}{\partial t}=\frac{\partial^2 A}{\partial r^2}+\frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial A}{\partial\theta}\right)+r\sin\theta\,C_\alpha\,\alpha B, \tag{2}$$

$$\begin{aligned}\frac{\partial B}{\partial t} &= \frac{1}{r}\frac{\partial^2}{\partial r^2}(rB)+\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta B)\right)+\frac{C_\Omega}{r}\frac{\partial\Omega}{\partial r}\frac{\partial A}{\partial\theta}\\ &+\frac{C_\Omega}{r}\frac{\partial\Omega}{\theta}\frac{\partial A}{\partial r}-\frac{C_\alpha}{r\sin\theta}\frac{\partial}{\partial r}\left(\alpha\frac{\partial A}{\partial r}\right)-\frac{C_\alpha}{r^3}\frac{\partial}{\partial\theta}\left(\frac{\alpha}{\sin\theta}\frac{\partial A}{\partial\theta}\right),\end{aligned} \tag{3}$$

with the poloidal-field potential A, toroidal field B and angular velocity Ω. We used the normalizations $r=R\tilde{r}$, $t=\tilde{t}R^2/\eta_\mathrm{T}$, $\Omega=\Omega_0\tilde{\Omega}$, and $A=R^2B_\mathrm{eq}\tilde{A}$, $B=B_\mathrm{eq}\tilde{B}$ with the equipartition field $B_\mathrm{eq}=(\mu_0\rho\langle u'^2\rangle)^{1/2}$. For the quantities ν_T and η_T as the turbulent viscosity and magnetic diffusivity, only the ratio $\mathrm{Pm}=\nu_\mathrm{T}/\eta_\mathrm{T}$ is relevant. The one-point correlation tensor of the velocity fluctuations, Q_{ij}, is expressed by

$$Q_{r\phi}=\nu_\mathrm{T}\sin\theta\left(-r\frac{\partial\Omega}{\partial r}+V^{(0)}\Omega\right),\qquad Q_{\theta\phi}=-\nu_\mathrm{T}\sin\theta\frac{\partial\Omega}{\partial\theta}. \tag{4}$$

$V^{(0)}$ determines the rotation law without magnetic field. The computation domain consists of three layers: the top of the CZ with the Λ-effect, the bottom of the CZ with both Λ-effect and α-effect working, and the top of the core with 100 times lower viscosity and diffusivity. The boundary conditions are specified as $Q_{r\phi}=\partial A/\partial r=B=0$ at at $r=R$ and $Q_{r\phi}=A=B=0$ at the inner boundary.

The model is defined by five dimensionless numbers, namely the magnetic Reynolds numbers of differential rotation and α-effect, the magnetic Prandtl number, Elsasser number and the strength of the Λ-effect:

$$C_\Omega=\frac{\Omega_0R^2}{\eta_\mathrm{T}},\quad C_\alpha=\frac{\alpha_0R}{\eta_\mathrm{T}},\quad \mathrm{Pm}=\frac{\nu_\mathrm{T}}{\eta_\mathrm{T}},\quad \mathrm{E}=\frac{B_\mathrm{eq}^2}{\mu_0\rho\eta_\mathrm{T}\Omega_0},\quad \text{and } V^{(0)}. \tag{5}$$

With the eddy diffusivity, $\eta_\mathrm{T}=c_\eta\langle u'^2\rangle\,\tau_\mathrm{corr}$, and the Coriolis number $\Omega^*=2\tau_\mathrm{corr}\Omega_0$, the Elsasser number reads $\mathrm{E}=2/(c_\eta\Omega^*)$. In the α-effect, α_0 is the dynamo-alpha amplitude and in the spatial distribution, $\alpha=\alpha_0\cos\theta\sin^2\theta$, the factor $\sin^2\theta$ has been introduced to restrict magnetic activity to low latitudes and $\alpha_0\simeq l_\mathrm{corr}\,\Omega_0$, so that

$$|C_\alpha|\simeq\frac{\Omega_0\;R}{c_\eta\;u'}. \tag{6}$$

Similarly we find $C_\Omega/|C_\alpha|\simeq R/l_\mathrm{corr}$, whence C_Ω generally exceeds C_α ('$\alpha\Omega$ dynamo'); here we used $C_\alpha=-10$ and $C_\Omega=10^5$. $V^{(0)}$ is positive in order to produce the required super-rotation, its amplitude is 0.37.

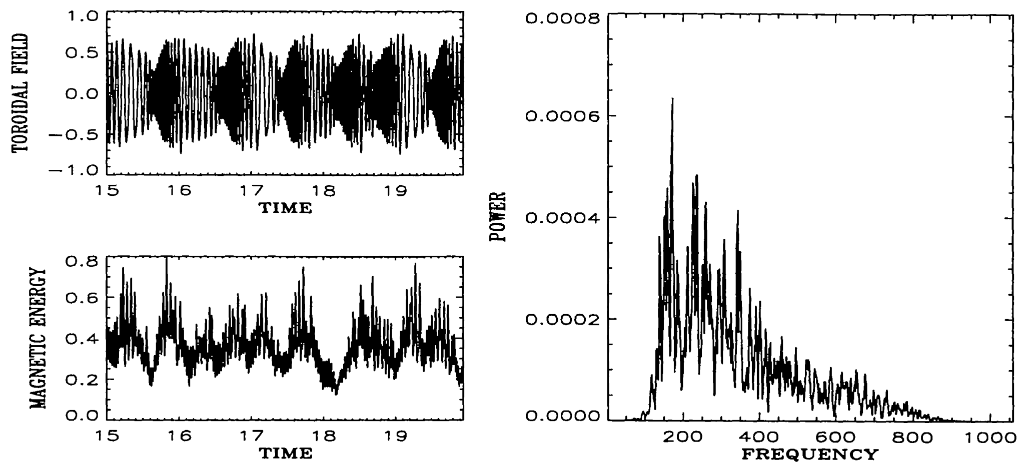

Figure 1. Time series (LEFT) and spectrum (RIGHT) of the full model with Λ-quenching ($\lambda = 25$). TOP-LEFT: Toroidal magnetic field, BOTTOM-LEFT: magnetic energy

3. Results and Discussion

When only considering large-scale Lorentz forces on differential rotation (Malkus & Proctor 1975), one gets irregular grand minima with strong variations in the cycle period (Tobias 1996). If the suppression of dynamo action by α-quenching, such as

$$\alpha \propto \frac{1}{1 + (B_{\rm tot}/B_{\rm eq})^2}, \tag{7}$$

is included, the dynamo returns to oscillations with one frequency. If a strong feedback of small-scale flows on the generation of Reynolds stress (Λ-quenching) as expressed by

$$V^{(0)} \propto \frac{1}{1 + \lambda (B_{\rm tot}/B_{\rm eq})^2}, \tag{8}$$

is added, grand minima occur at a reasonable rate between 10 and 20 cycle times if $\lambda \geq 20$. The cycle period varies by a factor of 6–8. Figure 1 gives the toroidal field at a fixed point, total magnetic energy and the power spectrum for $\lambda = 25$. All times and periods are given in units of a diffusion time $R^2/\eta_{\rm T}$. Field strengths are measured in units of $B_{\rm eq}$. The spectral graph of Figure 1 contains a set of frequencies with the highest peaks near the activity cycle frequency.

The correlation between the dipolar component $E_{\rm A}$ of the magnetic field, the quadrupolar component $E_{\rm S}$, and the differential rotation in terms of $(\partial\Omega/\partial r)^2$, averaged over the latitude θ, is shown in Figure 2. The dots in the interior of the diagram represent the actual time series; projections of the trajectory are given at the sides. The phase graph is similar to those given in Knobloch et al. (1998). The trajectory resides at strong differential rotation during normal cyclic activity and oscillates in a wide range of energies. Oscillation amplitudes diminish as the field starts to suppress the differential rotation. The actual grand minimum is found at very low dipolar energies; the quadrupolar component, however,

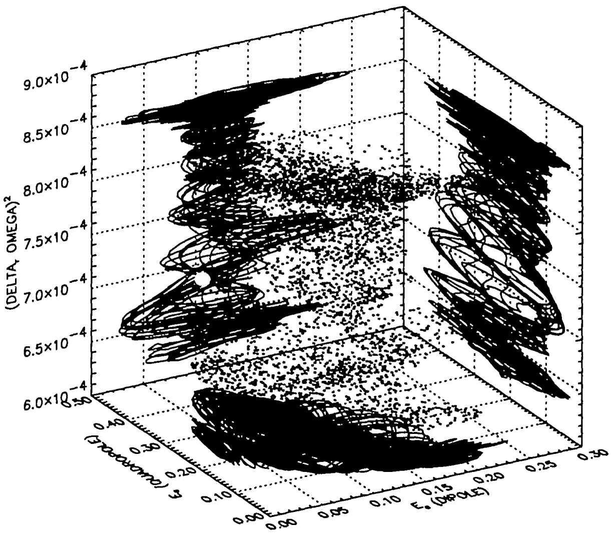

Figure 2. Correlations between magnetic field energies for both parities (dipolar and quadrupolar) and a measure of differential rotation

remains present throughout the minimum. The differential-rotation measure is already growing at that time.

The northern and southern hemispheres differ slightly in their temporal behavior. This is a general characteristic of mixed-modes dynamo explanations of grand minima. In all cases studied, the magnetic Prandtl number is Pm = 0.1. For Pm = 1.0, however, grand minima *do not appear* in models as Fig. 1. The magnetic Prandtl number controls the intermittency of the activity cycle; for smaller Pm, grand minima occur less and less frequent.

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