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A Shear-Flow Dynamo as a Proxy for the Ap Star Magnetism

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Abstract. Slow rotation is one of the essential characteristics of magnetic A-stars. The anharmonic variations seen in many B_{eff} curves are incompatible with simple dipolar geometry. In some Ap stars a significant quadrupolar component is indicated. We interpret these facts as being a random realisation of a nonlinear spherical MHD-flow dynamo with differential rotation. The global structure of a self-excited magnetic field arising in such flows has been simulated in a sphere by a full 3D nonlinear approach. The rotation profile is prescribed by a function of the cylinder radius yielding non-uniform rotation in the outer regions but nearly rigid rotation close to the axis. Starting from arbitrary small perturbations, the magnetic and kinetic energies quickly grow by several orders. The dynamo-generated magnetic field is highly non-axisymmetric. The outer surface is penetrated by magnetic field lines in spot-like regions, which are located mainly in the equatorial plane. If we adopt stellar parameters, the field amplitude is of the (observed) order of more than a few kGauss.

1. Introduction

Fifteen years ago it seemed to be very simple to explain the Ap-star magnetism. Compared to the Sun the A stars are rotating fast. The α-tensor of the dynamo theory of turbulent media thus becomes highly anisotropic. As a consequence the simple α^2-dynamo yields a highly non-axisymmetric solution with $m = 1$ rather than a simple axial dipole with $m = 0$.

This solution survives surprisingly a rather strong differential rotation as well as the nonlinear feedback of the magnetic field to the α-effect (Rüdiger & Elstner 1994; Moss & Brandenburg 1995). Indeed, recent empirical results by Landstreet & Mathys confirm the concept of the oblique rotator (Krause & Oetken 1976). But two very basic questions remain: *(i) How strong is the differential rotation? (ii) Is the turbulence due to convection or other instabilities?*

The observational results show a high degree of randomness. Field geometry and amplitude, however, apart from the obliquity, differ from star to star. But there may be one rule: the Ap stars are slow rotators – and slow rotators

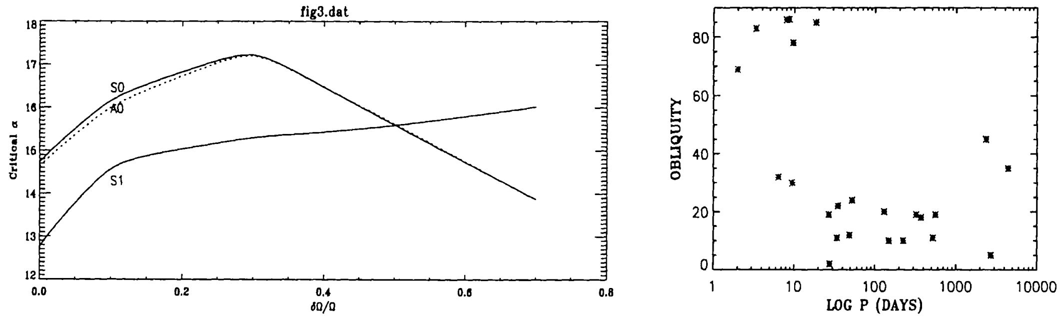

Figure 1. LEFT: For an anisotropic α-effect and not too strong differential rotation the equatorial ("oblique") dipole ($m = 1$, dashed) is more easily excited then the axisymmetric one ($m = 0$, solid) for both signs of α (Rüdiger & Elstner 1994). RIGHT: The magnetic obliquity of Ap star magnetic fields after Landstreet & Mathys (2000) depends on the rotation rate.

are Ap stars (Abt & Willmarth 2000). Stępień (2000) made the interesting suggestion to compare the situation within the A star group with that of the T Tauri stars (TTS). Among them there is also a subgroup with slow rotation, i.e. the CTTS which are systems including accretion disks. Magnetic star-disk interaction might easily spin down the central object which finally becomes the Ap star.

The question remains why slowly rotating A stars should become magnetic. The possibility which is favoured here is the formation of a differential rotation ($\Omega \propto s^{-q}$ with s the distance to the axis). Such a rotation law becomes unstable under the influence of a magnetic field.

2. The simulations

Figure 2 gives the considered model for the presented global MHD instability. The model is non-stratified but its outer part is differentially rotating with $\Omega \propto s^{-q}$ (s distance to the axis). The innermost sphere is considered to be vacuum in order to simulate the very high magnetic diffusivity of convective cores. The sphere is embedded in vacuum and it is threaded by a large-scale magnetic field in vertical direction. The result of the stability diagram of the Reynolds number $C_\Omega = R^2\Omega_0/\eta$ versus Hartmann number $\mathrm{Ha} = B_0 R/\sqrt{\mu_0 \rho \eta \nu}$ is shown in Fig. 3. For a given Reynolds number of the rotation – exceeding a critical value – there are two limiting magnetic field amplitudes between which the differential rotation (represented by q) is not stable. Excitation of patterns of flow and field are the immediate consequence. Also flow and field modes of low mode numbers are excited, very often by the lowest Reynolds number. Only the nonlinear theory can provide (i) the whole spectrum of the flows and the fields and (ii) the meaning of the initial magnetic field. It is very clear that an initial field is necessary to start the instability, but if after a while the initial field is completely out of the memory, then one can speak of a dynamo. The

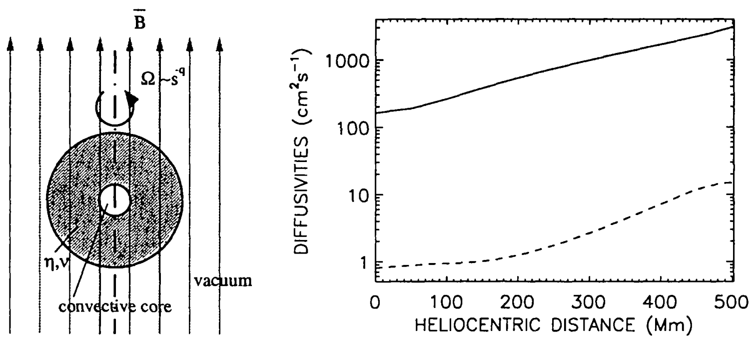

Figure 2. LEFT: Model of a non-stratified star with a differentially rotating radiative zone and a small convective core. RIGHT: Molecular magnetic diffusivity (solid) and viscosity (dashed) in the radiative interior of the Sun (Rüdiger & Kitchatinov 1996).

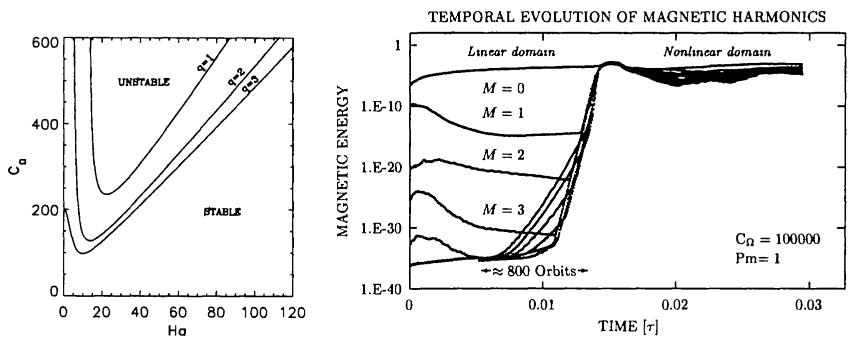

Figure 3. LEFT: MHD-instability of rotation laws with various exponents q for various magnetic Hartmann numbers (Kitchatinov & Rüdiger 1997). RIGHT: The instability rapidly saturates in the nonlinear regime after an exponential growth (Drecker, Rüdiger, & Hollerbach 2000).

nonlinear equations

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}\right) = -\nabla p + \nu\Delta\boldsymbol{u} + \boldsymbol{j}\times\boldsymbol{B}, \quad \frac{\partial \boldsymbol{B}}{\partial t} = \mathrm{rot}(\boldsymbol{u}\times\boldsymbol{B}) + \eta\Delta\boldsymbol{B} \quad (1)$$

with the additional conditions $\mathrm{div}\,\boldsymbol{u} = \mathrm{div}\,\boldsymbol{B} = 0$ are considered. The velocity consists of a basic rotation, $\Omega = \Omega_0\big/\sqrt{1+(s/s_0)^{2q}}$, and fluctuations. The equations are solved with the pseudo-spectral 3D code by Hollerbach (2000).

A small number of azimuthal modes, i.e. a low azimuthal resolution leads to a decay of magnetic energy after a period of axisymmetric wind-up growth. A non-decaying magnetic energy was first found when we used 15 Fourier modes at a dynamo number of $C_\Omega = 10^5$, but the corresponding energy spectrum indicated that this simulation was still far from being numerically well resolved. When we doubled the number of harmonics the energy spectrum revealed a strong peak at $m = 15$ in this period of non-axisymmetric growth. The temporal evolution of the first 10 magnetic harmonics is shown in Fig. 3. During 800 orbits the

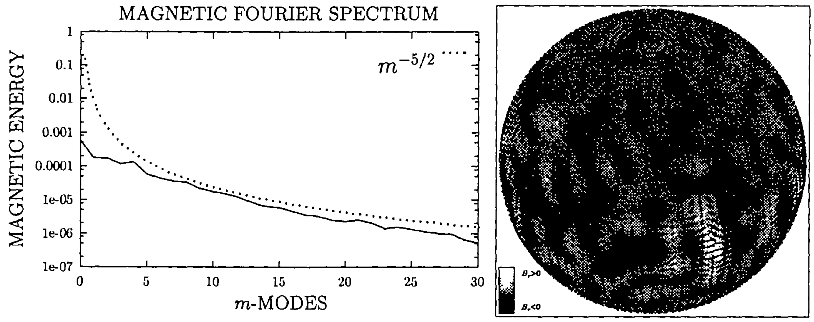

Figure 4. LEFT: The magnetic Fourier spectrum also includes global modes. RIGHT: At the surface local magnetic spots are concentrated at the equatorial plane.

higher harmonics grow exponentially at a rate of order $10^4 \ldots 3 \cdot 10^4 \sim O(C_\Omega)$, which makes this a very fast instability (Balbus 1995).

3. Results

The flow is almost axisymmetric while the higher modes, which show the typical Kolmogorov scaling are weaker by several orders of magnitude. Due to dissipation at small scales the energy of higher modes decreases and a turbulent cascade is established. A magnetic Fourier spectrum is shown in Fig. 4. The magnetic field, however, has strong non-axisymmetric contributions, where harmonics up to $m = 4$ are still comparable in power with $m = 0$. The magnetic Fourier spectrum for the small scales behaves like $E_{\rm mag}(m) \sim m^{-5/2}$. The dominance of the kinetic energy is only due to the very strong $m = 0$ contribution of the flow, whereas at small scales the magnetic modes dominate throughout.

The distribution of energy of the three lowest azimuthal modes of the magnetic field is shown in the left panel of Fig. 5. Non-axisymmetric modes form a considerable part of the field; the $m = 1$ mode even exceeds the axisymmetric part significantly for more than a hundred orbital revolutions. The spectra are averaged meridionally (over radius and latitude). The dominance of the $m = 1$ mode is typically limited to certain ranges in depth which vary with time.

One can ask whether a relation exists in the simulation between both the induced turbulent EMF and the large-scale magnetic field. In the right panel of Fig. 5 the quantities are plotted for various snapshots separated for both hemispheres. In the northern hemisphere EMF and the magnetic field are anticorrelated and in the southern hemisphere they are correlated. With other words, the alpha-effect is negative (positive) in the northern (southern) hemisphere.

An important test for the presented concept for the Ap star magnetism is given by the amplitude of the induced magnetic field. It is no problem to fulfil the Reynolds number constraints $\mathrm{Re} \approx R^2\Omega/\nu \approx 10^{15}$. The amplitude of the magnetic field is given by the condition $V_{\rm A} \lesssim V_{\rm rot}$ with $V_{\rm A}$ as the Alfvèn velocity of the magnetic field. Hence, $B \simeq \sqrt{\mu_0 \rho}\, V_{\rm rot} \hat{B}$ where the normalised amplitude, $\hat{B}$, in our simulations reaches values of order 10^{-3}. So we find

$$B \lesssim 100 \text{ kG},$$

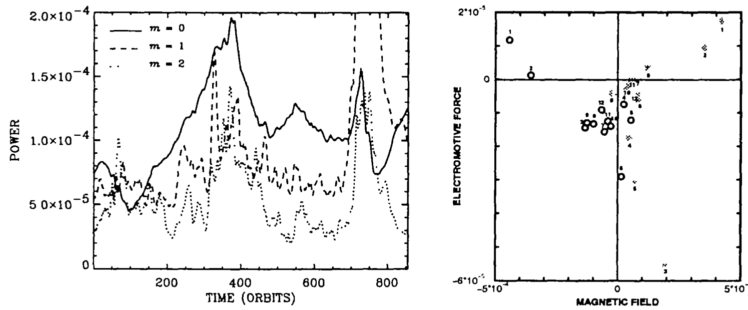

Figure 5. LEFT: Temporal behaviour of the azimuthal modes with largest wavelengths. RIGHT: The α-effect in the northern (asterisks) and southern (circles) hemispheres is negative and positive, respectively.

agreeing well with the observations.

A more complete theory of the Ap star magnetism requires the inclusion i) of the density stratification and ii) of density fluctuations in order to simulate buoyancy. Of course, in contrast to the present simulations the differential rotation must be considered as an *initial* condition. The resulting MHD instability lends to smooth the rotation profile and then, after the production of rigid rotation, the instability will die, but the fields remain frozen. So it can easily be that the Ap stars are now exhibiting the remnants of a *former* instability. If this is true then one of the main characteristics of the Ap star magnetism should be chaos – and this could easily be true.

References

Abt, H.A., & Willmarth, D.W. 2000, in Stellar Astrophysics, eds. K.S. Cheng et al. (Dordrecht: Kluwer), 175

Balbus, S.A. 1995, ApJ, 453, 380

Bouvier, J., Cabrit, S., Fernandez, M., Martin, E.L., & Matthews, J.M. 1993, A&A, 272, 176

Drecker, A., Rüdiger, G., & Hollerbach, R. 2000, MNRAS, 317, 45

Ghosh, P. 1995, MNRAS, 272, 763

Hollerbach, R. 2000, Int. J. Numer. Meth. Fluids, 32, 773

Hubrig, S., North, P., & Medici, A. 2000, A&A 359, 306

Kitchatinov, L.L., & Rüdiger, G. 1997, MNRAS, 286, 757

Krause, F., & Oetken, L. 1976, in IAU Coll. 32, Physics of Ap stars, eds. W.W. Weiss, H. Jenkner & H.J. Wood (Vienna: Universitätssternwarte), 29

Landstreet, J.D., & Mathys, G. 2000, A&A, 359, 213

Moss, D., & Brandenburg, A. 1995, GAFD, 80, 229

Rüdiger, G., & Elstner, D. 1994, A&A, 281, 46

Rüdiger, G., & Kitchatinov, L.L. 1996, ApJ, 466, 1078

Stępień, K. 2000, A&A, 353, 227

Discussion

REISENEGGER: How do you impose constant differential rotation in your models at all times, but at the same time include back-reaction of the magnetic field on the fluid?
RUEDIGER: That is a good point. Indeed, the magneto-rotational instability tends to destroy the non-uniform rotation. After a so far unknown time the star will exist in uniform rotation.
REISENEGGER: Do you have a feeling for the effect you expect from stable stratification on your dynamo model?
RUEDIGER: Yes, the dynamo will not be supported, the induced magnetic field shall be smaller, anisotropies become dominant.
HUBRIG: New studies of rotational periods of CTTS and WTTS show that rotational periods are of the same order of 4-5 days. In your model you require differential rotation at the ZAMS, i.e. you should observe an excess of young slowly rotating stars. In Hubrig, North, & Medici (2000), we show that such an excess is not observed.
RUEDIGER: I cannot comment the results of recent observations of TTS. My numbers are after Bouvier et al. (1993) and Ghosh (1995) plotted in Fig. 6. The presented concept indeed lives by the differences of star rotation with and without magnetic star-disk interaction.

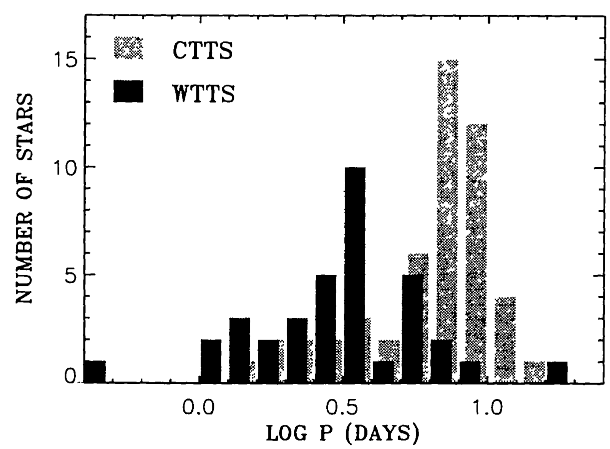

Figure 6. Rotation period differences for T Tauri stars with (CTTS) and without (WTTS) accretion disks after Bouvier et al. (1993).

TRUJILLO BUENO: How weak can be the initial magnetic field?
RUEDIGER: Probably, Ha=1 is a reasonable value, implying an order of 10^{-9} Gauss.