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Cycle times and magnetic amplitudes in nonlinear 1D $\alpha^2\Omega$-dynamos

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Abstract. A nonlinear, 1D-slab, $\alpha^2\Omega$-dynamo is analysed for magnetic field amplitudes and for the relation between the cycle time and the dynamo number. If the only nonlinearity is the conventional α-quenching, the magnetic field strongly grows with the dynamo number, while the dependence of the cycle time is only rather weak. The opposite is true if the nonlinear feedback is more consistently included: the *complete* effect of the turbulent EMF tensor is deformed and suppressed by the induced large-scale magnetic field. In particular, this involves η-quenching where the eddy diffusivity becomes a tensor whose components are different functions of the magnetic field. Thus, the magnetic field amplitude only scales with the small value $|m'| \lesssim 0.2$ while the cycle oscillation frequency depends much more strongly on the dynamo number ($n' \simeq 0.5$). The latter seems to be consistent with the results of the Mt. Wilson HK-project for stellar activity cycles, although our dynamo model only forms a rather rough approximation for stellar configurations.

Key words: Turbulence – Stars: activity – Stars: magnetic fields – Galaxies: magnetic fields

1. Introduction

A basic question in stellar physics is to find the influence of stellar parameters on the amplitude and the duration of magnetic activity cycles.

Observations of stellar activity demonstrate a broad variety of findings; well-known relations for magnetic field $\bar{B}$ and cycle frequency $\omega_{\rm cyc}$ are

$$\bar{B}_{\rm max} \propto \Omega^{*m}, \qquad \omega_{\rm cyc} \propto \Omega^{*n}, \qquad (1)$$

with m and n of different strength. Saar (1996) derives very small m (his Fig. 3) while Baliunas et al. (1996) find $n \simeq 0.47$ for young stars and $n \simeq 1.97$ for old stars. Here Ω^* is the Coriolis number,

$$\Omega^* = 2\tau_{\rm corr}\,\Omega, \qquad (2)$$

with $\tau_{\rm corr}$ being the characteristic value of the turnover time at the base of the stellar convection zone. It represents an important part of the dynamo number which combines the main ingredients of the basic dynamo, i.e. differential rotation and the turbulent α-effect. The remaining dimensionless number which defines the system is geometrical in nature, i.e. the thickness, H, of the convection zone.

If the dynamo number is denoted by $\mathcal{D}$ then the relations (1) can be reformulated as

$$B_{\rm max} = B_{\rm eq}\mathcal{D}^{m'}, \qquad \omega_{\rm cyc} = \mathcal{D}^{n'}/\tau_{\rm diff}, \qquad (3)$$

with the 'equipartition field' and the diffusion time, resp.,

$$B_{\rm eq} = \sqrt{4\pi\rho\langle \boldsymbol{u}'^2\rangle}, \quad \tau_{\rm diff} = H^2/\eta_0. \qquad (4)$$

Instead of the normalisation used in $(3)_2$ Soon et al. (1993) proposed the use of the basic rotation rate Ω; a procedure we do not follow here to retain consistency with conventional definitions of the dynamo number.

Here $\langle \boldsymbol{u}'^2\rangle$ is the turbulent intensity of the fluid, ρ its density and η_0 is the reference value of the eddy diffusivity which can be estimated by

$$\eta_0 = c_\eta \langle \boldsymbol{u}'^2\rangle \tau_{\rm corr}, \qquad (5)$$

where the influence of both the basic rotation as well as the mean magnetic field is neglected, by definition. The dimensionless factor c_η is always assumed to be smaller than unity. Rüdiger & Brandenburg (1995) demonstrated how c_η must be tuned in order to fit the cycle time of the solar overshoot dynamo to the observed value of 11 yr, even in the nonlinear system.

It is necessary to briefly summarise the present results of dynamo theory concerning cycle times. The main issues are already formulated by Noyes et al. (1984). Their

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argumentation concerns a zero-order $\alpha\Omega$-dynamo model for which the coefficient n in the scaling $\omega_{\rm cyc} \propto \mathcal{D}^{n/2}$ is determined. The linear case yields $n = 4/3$ for the most unstable mode (cf. Tuominen et al. 1988). For pure α-quenching as the nonlinearity $n = 0$ is found, while for models with flux loss as the nonlinearity (rather than α-quenching) $n = 1$ results, or $n = 2/3$, – if only the toroidal field is quenched by magnetic buoyancy. Importantly, note the complete absence of such a relation for simple α-quenching (see, however, Meunier et al. 1996). If the theory is correct, and the observations are following relations such as (1), then the nonlinear dynamo can never work with α-quenching as the basic nonlinearity.

Much more complex dynamo models confirmed the findings of Noyes et al. (1984). Moss et al. (1990), for large dynamo number, also derived $n = 1$ for the case that the (spherical) dynamo is saturated by magnetic buoyancy. Schmitt & Schüssler (1989) in their Fig. 6 also showed for a 1D model, that there is practically no dependence of the cycle frequency with the (large) dynamo number for α-quenching but they find a strong scaling ($n \simeq 2$) for their 'flux-loss models'. Also for a 2D model in spherical symmetry, the dependence of the cycle frequency on the dynamo number proved to be extremely weak, $n \lesssim 0.1$ (Rüdiger et al. 1994, Fig. 4, dashed line).

The latter application however, requires further consideration. One finds a finite value for n, if the magnetic feedback is not only considered acting upon the α-effect, but also for the eddy diffusivity (tensor). What we assume here is that the magnetic field always suppresses and deforms the turbulence field, and this has consequences for both the α-effect and the eddy diffusivity. The theory for a second-order correlation-approximation is given in Kitchatinov et al. (1994), and first applications are summarised in Rüdiger et al. (1994).

In principle, the η-quenching concept is described with negative λ in Noyes et al. (1984). As the magnetic fields become super-equipartitioned however, a series expansion such as used in Noyes et al. (1984) cannot be adopted.

It is shown in the following that a special 1D slab dynamo, as a generalisation of Parker's zero-dimensional wave dynamo (Parker 1979), without η-quenching has a very low n while it exceeds unity if the η-quenching is included. Buoyancy effects are therefore not the only possibility addition to dynamo theory to explain observations as given in (1); the magnetic suppression of the turbulent magnetic diffusivity also yields an explanation of the observations.

2. The dynamo model

The dynamo equation includes the turbulent EMF $\boldsymbol{\mathcal{E}} = \langle \boldsymbol{u}' \times \boldsymbol{B}' \rangle$ as well as differential rotation,

$$\frac{\partial \bar{\boldsymbol{B}}}{\partial t} = \mathrm{rot}\left(\bar{\boldsymbol{u}} \times \bar{\boldsymbol{B}} + \boldsymbol{\mathcal{E}}\right). \tag{6}$$

As usual, the turbulent EMF is developed as a series expansion,

$$\mathcal{E}_i = \alpha_{ij}\ \bar{B}_j + \eta_{ijk}\ \bar{B}_{j,k}, \tag{7}$$

so that α_{ij} yields the α-effect and the η-tensor gives the eddy diffusivity. Here we are working not with the simplest possibility, i.e. with

$$\alpha_{ij} = \alpha\ \delta_{ij}, \qquad \eta_{ijk} = \ \eta_{\rm T}\ \ \epsilon_{ijk}. \tag{8}$$

In this *Letter* the full feedback of the induced magnetic field on the turbulent EMF is included, i.e. the magnetic suppression and deformation ('quenching') of both the tensors α and η. In particular, the influence of the magnetic field on the magnetic diffusivity is often ignored in dynamo computations and we shall demonstrate that remarkable differences in the gross properties of the solutions with and without η-quenching exist.

The main consequence of the inclusion of η-quenching is the appearance of a nonlinear 'magnetic velocity' in $\boldsymbol{\mathcal{E}}$,

$$\boldsymbol{\mathcal{E}} = \cdots\ + \boldsymbol{U}^{\rm mag} \times \bar{\boldsymbol{B}}, \tag{9}$$

with

$$\boldsymbol{U}^{\rm mag} = \hat{\eta}(\bar{B})\nabla\log\bar{B} + \eta_z(\bar{B})\ \frac{\mathrm{rot}\,\bar{\boldsymbol{B}} \times \bar{\boldsymbol{B}}}{\bar{B}^2}, \tag{10}$$

(Kitchatinov et al. 1994). Where

$$\eta_{\rm T} = \eta_0\varphi, \qquad \eta_z = \eta_0\varphi_z, \qquad \hat{\eta} = \eta_0\hat{\varphi}. \tag{11}$$

The coefficient functions $\varphi(\beta)$ are

$$\varphi\ =\ \frac{3}{2\beta^2}\left(-\frac{1}{1+\beta^2} + \frac{1}{\beta}\arctan\beta\right), \tag{12}$$

$$\varphi_z\ =\ \frac{3}{8\beta^2}\left(1 + \frac{2}{1+\beta^2} + \frac{\beta^2-3}{\beta}\arctan\beta\right), \tag{13}$$

$$\hat{\varphi}\ =\ \frac{3}{8\beta^2}\left(-\frac{5\beta^2+3}{(1+\beta^2)^2} + \frac{3}{\beta}\arctan\beta\right), \tag{14}$$

with $\beta = |\bar{B}|/B_{\rm eq}$. The α-tensor which we adopt is given in Rüdiger & Schultz (1996). It includes the components $\alpha_{\phi\phi}$ and $\alpha_{\varpi\varpi}$ (which are equal in our approximation), i.e.

$$\alpha_{\phi\phi} = -\frac{2}{5}\Omega^*\ \frac{d\log\rho}{dz}\ \langle u_0^2\rangle\ \tau_{\rm corr}\ \psi, \tag{15}$$

with ψ as the α-quenching function

$$\psi = \frac{15}{32\beta^4}\left(1 - \frac{4\beta^2}{3(1+\beta^2)^2} - \frac{1-\beta^2}{\beta}\arctan\beta\right). \tag{16}$$

The α-effect distribution and the diffusivity quenching functions are approximated by $\psi(\beta) \simeq 1 - \frac{12}{7}\beta^2, \varphi(\beta) \simeq 1 - \frac{6}{5}\beta^2, \varphi_z(\beta) \simeq \frac{2}{5}\beta^2$ and $\hat{\varphi}(\beta) \simeq \frac{3}{5}\beta^2$, for weak magnetic fields. The only coordinate about which our quantities are allowed to vary is the direction of the rotation axis, i.e.

z. For simplicity all the numbers in (15) are summarised in the form $\alpha = -\alpha_0 \ \sin 2z$, where the lower and upper boundary shall be located at $z = 0$ and $z = \pi$

A 1D, α^2-Ω model is used to simulate a magnetic dynamo in an infinite disk. The plane has infinite extent in the x and y direction, and it is restricted by boundaries in the z-direction. We assume that the magnetic field component in azimuthal direction, B, and the radial component $\partial A/\partial z$ depend on z only. The normalised dynamo equations are derived in the same way as in Rüdiger et al. (1994):

$$\begin{aligned} \frac{\partial A}{\partial t} &= C_\alpha \hat{\alpha}(z)\, \psi(B_{\rm tot}) B + \varphi(B_{\rm tot}) \frac{\partial^2 A}{\partial z^2} - w \frac{\partial A}{\partial z}, \\ \frac{\partial B}{\partial t} &= C_\alpha \frac{\partial}{\partial z}\Big(\hat{\alpha}(z)\, \psi(B_{\rm tot}) \frac{\partial A}{\partial z}\Big) - C_\Omega \frac{\partial A}{\partial z} + \\ &\quad + \frac{\partial}{\partial z}\left(\varphi(B_{\rm tot}) \frac{\partial B}{\partial z}\right) - \frac{\partial}{\partial z}\Big(wB\Big), \end{aligned} \tag{17}$$

with

$$w = \frac{2\hat{\varphi}(B_{\rm tot}) - \varphi_z(B_{\rm tot})}{B_{\rm tot}} \frac{\partial B_{\rm tot}}{\partial z}, \tag{18}$$

and $B_{\rm tot}^2 = B^2 + (\partial A/\partial z)^2$. The boundary conditions are $B(z=0) = B(z=\pi) = 0$ and $\partial A/\partial z|_{z=0} = \partial A/\partial z|_{z=\pi} = 0$ which limits the magnetic field to the disk. The influence of the α-effect is controlled by C_α, fixed here to the value 5. Positive C_α describes a positive α in the upper half of the layer and vice versa. C_Ω represents the strength of the differential rotation and is varied in the present paper. Positive C_Ω represent positive shear.

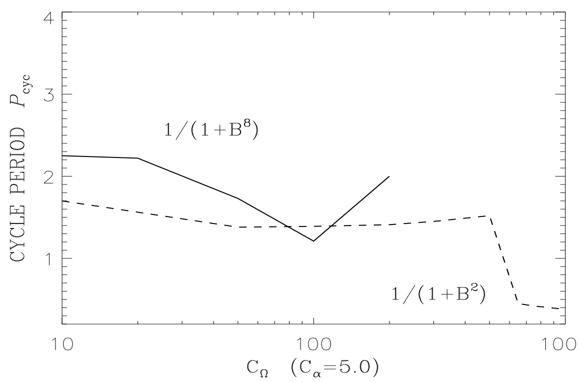

Fig. 1. The cycle period of oscillatory solutions of the $\alpha^2\Omega$-dynamo versus C_Ω with different α-quenching functions.

3. Discussion

All induced magnetic fields in our nonlinear dynamo model are symmetric with respect to the equatorial plane. Both the α^2-dynamo (small C_Ω) as well as the $\alpha\Omega$-dynamo

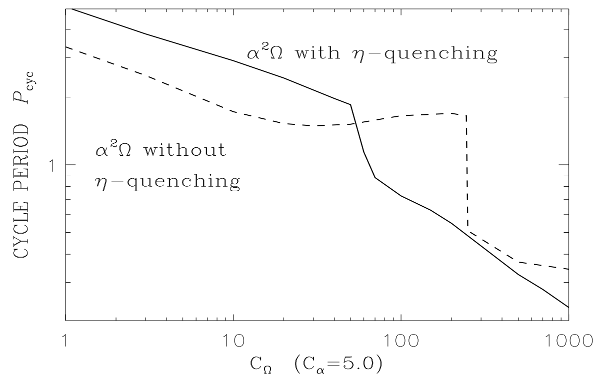

Fig. 2. The cycle period of oscillatory solutions of the $\alpha^2\Omega$-dynamo versus C_Ω with (solid) and without (dashed) η-quenching.

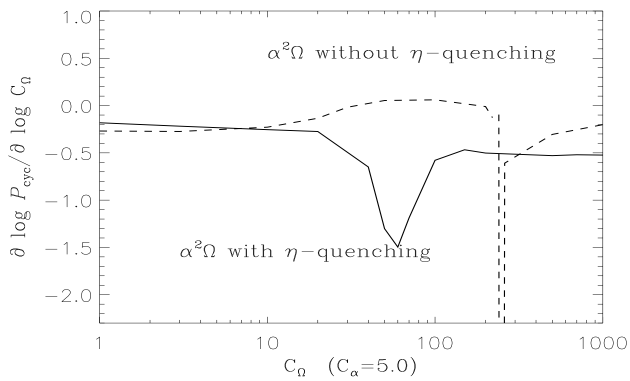

Fig. 3. The negative exponent n' in (3) with (solid) and without (dashed) η-quenching. n' proves to be always positive.

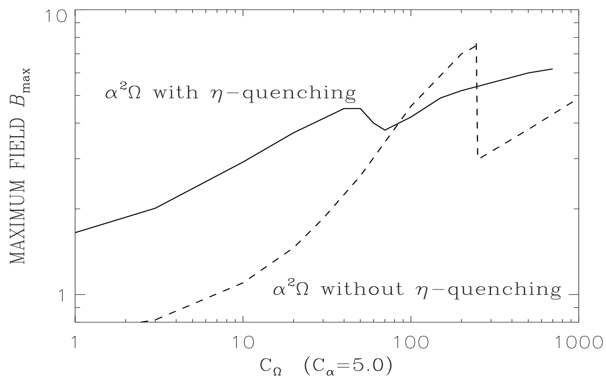

Fig. 4. Maximum fields of the oscillatory solutions of the $\alpha^2\Omega$-dynamo versus C_Ω with (solid) and without (dashed) η-quenching.

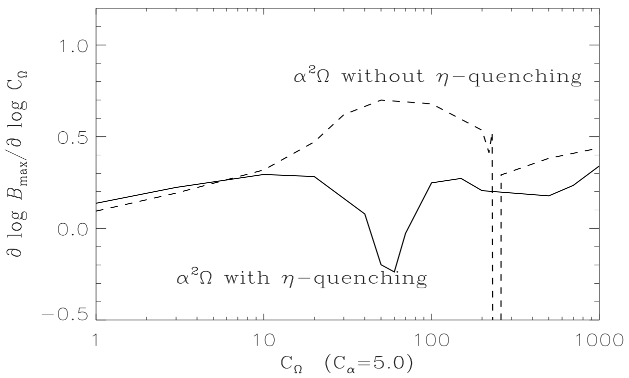

Fig. 5. The exponent m' in (3) with (solid) and without (dashed) η-quenching. m' has no characteristic sign.

(large C_Ω) are oscillating. The transition between both regimes occurs at $C_\Omega \simeq 50$. The cycle periods are always constant over time. Fig. 1 shows the resulting cycle frequencies for the conventional α-quenching expressions. The cycle period is strikingly independent of the dynamo number (see also Noyes et al. 1984, Schmitt & Schüssler 1989, Jennings & Weiss 1991). On the other hand, Fig. 2 gives the resulting cycle frequencies of the oscillatory solutions for the $\alpha^2\Omega$-dynamo with our quenching concept, the functions ψ and φ. The cycle periods are given for models, with and without, η-quenching. The exponent n' is variable and positive for the model with η-quenching, whereas it is much more complex for the model without η-quenching.

For large dynamo number we note from Fig. 3 the result $n' \simeq 0.5$ in a quite paradoxical similarity to the behaviour of linear dynamos. In order to compare this value with the observed value of n in Eq. $(1)_2$ we have to introduce the scaling of the physical quantities with the Coriolis number Ω^*. From Kitchatinov et al. (1994) we adopt the scaling $\eta_0 \propto 1/\Omega^*$, and use the parameterisation

$$\frac{\partial \log \Omega}{\partial \log r} \propto \Omega^{*\kappa}. \tag{19}$$

Negative κ describe a *decrease* of the unknown normalised differential rotation in stellar convection zones with increasing basic rotation and vice versa. Negative κ are indeed produced by theoretical models of the differential rotation, but only under inclusion of the meridional flow (Kitchatinov & Rüdiger 1995). Positive κ's have been obtained for models without meridional flow (Küker et al. 1993). The dynamo number then scales as

$$\mathcal{D} \propto \Omega^{*\kappa+3}, \tag{20}$$

if the α-effect for fast rotation is assumed independent of Ω^* (Rüdiger & Kitchatinov 1993). Then we find from (1) and (3) the relation

$$n = (3+\kappa)n' - 1. \tag{21}$$

For large dynamo number we note from Fig. 3 that $n' \simeq 0.5$. Thus, $\kappa \simeq 2n - 1$. The key question for comparisons with stellar cycles is whether the n in Eq. $(1)_2$ exceeds 0.5 or not. If not, then $\kappa < 0$, i.e. the normalised radial differential rotation *decreases* with increasing basic stellar rotation. The $n \simeq 0.26$ derived for *all* stars by Baliunas et al. (1996) could easily point in this direction.

The magnetic amplitude behaviour is more subtle. After Fig. 5 the exponent m' fluctuates with $|m'| \lesssim 0.2$ around zero so that almost no conclusion can be drawn from the relation similar to (21), i.e.

$$m = (3+\kappa)m'. \tag{22}$$

The equipartition field strength is maintained, independent of the basic rotation. Even then, and with the small absolute values of m', it is hardly possible to reproduce the observed values of m of order unity. More detailed analysis of various stellar objects is needed for a better discussion but we feel that the *cycle time statistics forms the main ingredient for constructing a theory of the stellar dynamo.*

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