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Letter to the Editor

The existence of the Λ effect in the solar convection zone indicated by SDO observations

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ABSTRACT

The empirical finding with data from the Solar Dynamics Observatory (SDO) of positive (negative) horizontal Reynolds stress at the northern (southern) hemisphere for solar giant cells (Hathaway et al. 2013) is discussed for its consequences for the theory of the solar/stellar differential rotation. Solving the nonlinear Reynolds equation for the angular velocity under neglect of the meridional circulation we show that the horizontal Reynolds stress of the northern hemisphere is always negative at the surface but it is positive in the bulk of the solar convection zone by the action of the Λ effect. The Λ effect, which describes the angular momentum transport of rigidly rotating anisotropic turbulence and which avoids a rigid-body rotation of the convection zones, is in horizontal direction of cubic power in Ω and it is always equatorward directed. Theories without Λ effect which may also provide the observed solar rotation law only by the action of a meridional circulation lead to a horizontal Reynolds stress with the opposite sign as observed.

Key words. convection – Sun: rotation – turbulence

1. Introduction

The nonrigid rotation of the stellar surfaces which currently has been observed for many thousands of stars (Reinhold et al. 2013) demonstrates the existence of an effective angular momentum transport in rotating convection zones. There are various transporters of angular momentum in a rotating turbulence field. The turbulence-induced Reynolds stress transports by its components $\langle u_r u_\phi \rangle$ and $\langle u_\theta u_\phi \rangle$ angular momentum in radial and in latitudinal directions. Boussinesq (1897) and Taylor (1915) connected the one-point correlation tensor $Q_{ij} = \langle u_i(\boldsymbol{x}, t) u_j(\boldsymbol{x}, t)\rangle$, which is symmetric by definition in its indices i and j, with the shear of a large-scale flow $\boldsymbol{U}$ so that $Q_{ij} = ... - \nu_{\rm T}(U_{i,j} + U_{j,i})$ results with the positive-definite eddy viscosity $\nu_{\rm T}$. Here the notation $\boldsymbol{U} + \boldsymbol{u}$ as the fluctuating velocity field with the background flow $\boldsymbol{U}$ has been used. This ansatz which does not reflect the anisotropies in the turbulence field immediately yields

$$Q_{r\phi} = -\nu_{\rm T} r \sin\theta \frac{\partial \Omega}{\partial r}, \qquad Q_{\theta\phi} = -\nu_{\rm T} \sin\theta \frac{\partial \Omega}{\partial \theta} \tag{1}$$

for the mentioned cross-correlations. As they vanish for rigid rotation they cannot serve to maintain nonuniform rotation $\Omega = \Omega(r, \theta)$. The turbulence in a convection zone, however, is subject to a distinct radial anisotropy by the central gravity which questions the validity of the relations (1). Kippenhahn (1963) with the concept of an anisotropic viscosity tensor derived a generalized linear relation for the radial transport, i.e.

$$Q_{r\phi} = \nu_{\rm T} \left(-r \frac{\partial \Omega}{\partial r} + V(r)\Omega \right) \sin\theta, \tag{2}$$

where the second term in (1), however, remains unchanged[1].

[1] One finds more details to the history of 'anisotropic viscosity' and the related references in Rüdiger (1989)

Stationary solutions for the angular velocity Ω have to fulfill the Reynolds equation

$$\frac{1}{r^2}\frac{\partial \rho r^3 Q_{r\phi}}{\partial r} + \frac{1}{\sin^2\theta}\frac{\partial \rho \sin^2\theta Q_{\theta\phi}}{\partial \theta} = 0, \tag{3}$$

which ensures the conservation of the angular momentum in the rotating turbulence under neglect of the mean meridional circulation and the molecular viscosity. Here ρ is the density. Inserting $(1)_2$ and (2) into (3) one finds Ω dependent on r but independent of θ. For positive radial shear in the rotation law, on the other hand, a meridional flow *towards the equator* is induced at the surface which accelerates the equator by transporting angular momentum. Such a meridional flow, however, has not been observed so far. In contrast, the observed slow meridional flow rises at the equator and flows in polar direction along the surface (Gizon & Rempel 2008; Schad et al. 2012).

After $(1)_2$ an accelerated equator provides a negative (positive) horizontal Reynolds stress at the northern (southern) hemisphere. Recently, Hathaway et al. (2013) from the data of the NASA Solar Dynamics Observatory (SDO) empirically isolated a giant cell pattern at the solar surface where the proper motions form a horizontal cross-correlation $Q_{\theta\phi}$ which is – as a fulfilled necessary condition – antisymmetric with respect to the equator. The correlation is positive (negative) at northern (southern) mid-latitudes and its amplitude of $2 \cdot 10^5$ cm^2/s^2 suggests rms velocities of the cells of (say) 10 m/s (Fig. 1). Cells with faster rotation tend to move equatorward and vice versa.

Because of the symmetry conditions for the horizontal cross correlation (vanishing at poles and equator by definition) it makes sense to introduce the dimensionless factor W by means of

$$Q_{\theta\phi} = \nu_{\rm T} \Omega_0 \cos\theta \sin^2\theta W. \tag{4}$$

The product $\nu_{\rm T}\Omega_0$ (Ω_0 the characteristic angular velocity) is the scalar with the correct dimension which reflects the fact that only rotating turbulence possesses finite horizontal cross correlations $Q_{\theta\phi}$. After Hathaway et al. (2013) we have $W \simeq 0.3/\nu_{12}$ with $\nu_{12} = \nu_{\rm T}/(10^{12}{\rm cm}^2/{\rm s}^2)$. Previous investigations with magnetic tracers led to values for W higher by two orders of magnitudes (Ward 1965; Gilman & Howard 1984; Balthasar et al. 1986) while the statistical analysis of coronal bright points yielded smaller numbers (Vrsnak et al. 2004). Note that for solitary spots Nesme-Ribes et al. (1993) even find small but negative correlations.

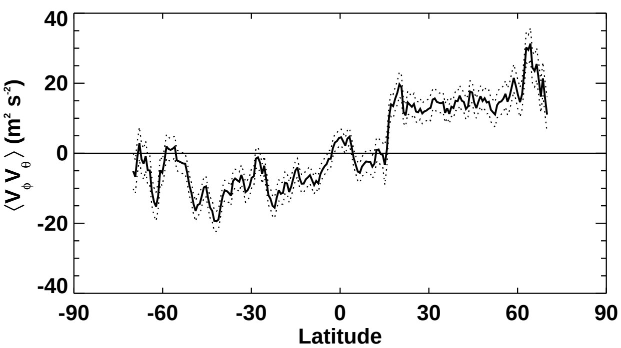

Fig. 1. The horizontal Reynolds stress observed with HMI on the NASA Solar Dynamics Observatory by Hathaway et. al. (2013). Permission by D.H. Hathaway.

The empirical results obviously indicate the invalidity of the simple Boussinesq relation $(1)_2$ for rotating convection. The observed equatorial acceleration – if it is a deep-seated phenomenon – would always provide negative (positive) cross-correlation values for the northern (southern) hemisphere. As this is not observed it should be natural to replace the expression $(1)_2$ by

$$Q_{\theta\phi} = \nu_{\rm T}\left(-\sin\theta\frac{\partial\Omega}{\partial\theta} + \sin^2\theta\cos\theta H(\frac{\Omega}{\Omega_0})^2\Omega\right), \qquad (5)$$

where the second term on the RHS is of cubic order in Ω. The counterpart of (5) with respect to the radial transport of angular momentum is

$$Q_{r\phi} = \nu_{\rm T}\left(-r\sin\theta\frac{\partial\Omega}{\partial r} + \sin\theta V\Omega - \sin\theta\cos^2\theta H(\frac{\Omega}{\Omega_0})^2\Omega\right) \qquad (6)$$

where here also a term V linear in Ω appears. All the nondiffusive terms in the expressions for the turbulent angular momentum transport can be written in the tensorial form $Q_{ij} = ... + \Lambda_{ijk}\Omega_k$ with the Λ effect tensor Λ_{ijk} symmetric in its first two indices which is even in Ω by definition. This is why all terms in (5) and (6) are odd in Ω. A tensor of $3^{\rm rd}$ rank even in Ω can only be constructed in turbulent fluids if the turbulence is anisotropic by itself and/or inhomogeneous as a consequence of the density stratification. Both conditions are fulfilled in stellar convection zones.

We have shown earlier by means of quasilinear turbulence theory that the function V is negative and exists mainly in the supergranulation layer while H is positive and exists mainly in the bulk of the convection zone. Numerical simulations provide very similar results (Chan 2001; Käpylä et al. 2011; for more details see Rüdiger et al. 2013). We also know that the positive quantity H in (5) and (6) both contribute to the equatorial acceleration so that it is by far not trivial to find the sign of (4) as a function of depth.

Solving the equation (3) with the expressions (5) and (6) leads for a simple model with weak shear and uniform density, uniform V and H and for stress-free boundaries ($Q_{r\phi} = 0$) to the the estimates

$$\frac{\delta\Omega}{\Omega_0} \simeq \frac{1+d}{2}\,H, \qquad W \simeq -dH \qquad (7)$$

(thin shells) for the normalized equator-pole-difference of Ω and the amplitude of W both taken at the surface. Both observable values do not depend on the viscosity value. Here $d \simeq 0.3$ is the normalized thickness of the convection zone. The negative sign of W means that the latitudinal shear at the surface (due to the action of H) always exceeds the value of H. However, the data of Hathaway et al. (2013) concern the giant cell pattern which exists deep in the convection zone rather than at its surface. The shear there is reduced by the lower boundary condition (tachocline!) so that W becomes positive. The analytical relation for W at the bottom of the convection zone reads $W \simeq dH > 0$ because of the reduction of the latitudinal shear. The observed positivity of the quantity $\cos\theta Q_{\theta\phi}$ appears to be compatible with the existence of the solar tachocline which rapidly reduces the equator-pole difference to zero (probably by Maxwell stress).

2. Mean-field models

Let us solve the mean-field equation (3) in the northern hemisphere in the domain $0.6 \le x \le 1$ with the natural boundary conditions $\partial\Omega/\partial\theta = 0$ at the polar and the equatorial axis and the stress-free condition $Q_{r\phi} = 0$ at the surface. In order to model a fast tachocline-like transition at the bottom of the domain rigid-body rotation is there prescribed. The first-order term V is assumed to exist only in the outermost layers while the third-order effect H exists down to $x = x_{\rm in} = 0.7$. This simple but nonlinear model which only ignores the transport of angular momentum by the meridional flow yields the numerical results given in the Figs. 2– 4 which demonstrate the characteristic behavior of the horizontal stress function W as the solution of the Reynolds equation (3).

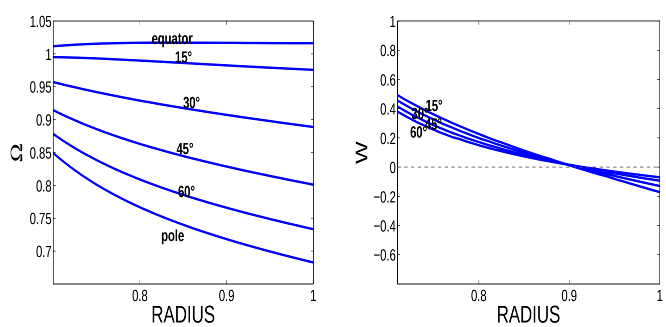

Fig. 2. The rotation law (left) and the horizontal Reynolds stress function W (right) for a mean-field model with $V = 0$ and $H = 1$ in the entire convection zone ($x_{\rm in} \le x \le 1$). Here x is the fractional radius of the star. Density is assumed as uniform.

We start with the homogeneous turbulence effects $V = 0$ and $H = 1$ in the entire convection zone. It is easy to see from (6) that the outer boundary condition requires $\partial\Omega/\partial r = 0$ at the equator and negative radial shear in higher latitudes. The immediate consequence is a uniform rotation beneath the equator and a very strong subrotation along the polar axis. The equator-pole difference of Ω is always positive and it grows outwards (Fig. 2). It is so strong at the surface that the W becomes negative there while it takes large positive values in the bulk of the convection zone. Note, that the amplitudes of the normalized differential rotation at the surface and the W at the bottom of the convection zone are numerically rather similar. Even the analytical estimates (7) are approximately fulfilled although they are obtained with a highly simplified linear theory.

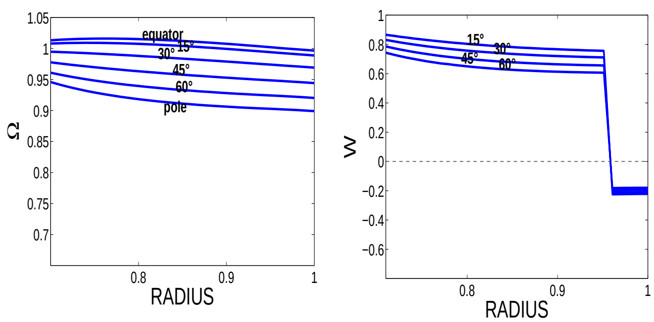

Fig. 3. The same as in Fig. 2 but with $H = 1$ only in the bulk of the convection zone ($x_{\rm in} \leq x \leq 0.95$) and $V = -0.1$ for $x > 0.95$.

The rotation law in Fig. 2, however, fails to deliver the characteristic superrotation of the solar equator at the bottom of the convection zone which has been helioseismologically derived and which is the source of the 'dynamo dilemma' (as it prevents the reproduction of the butterfly diagram of the sunspot locations within the solar cycle). It is also known, however, that the simple model used for Fig. 2 does not correctly describe the radial profiles of the turbulent Λ terms. The linear-in-Ω term V mainly exists in the outer part of the convection zone while the Ω^3 term H only exists in the inner part of the convection zone (see Rüdiger et al. 2013). This has consequences for the outer boundary condition which now tends to produce negative gradients of Ω at all latitudes. The result for a model with $V = -0.1$ (for $x > 0.95$) and with $H = 1$ for $x_{\rm in} \leq x \leq 0.95$ is given in Fig. 3. Indeed, a weak equatorial superrotation is now indicated at the bottom of the convection zone. The latitudinal shear, however, is reduced so that the W after (4) becomes larger than in Fig. 2. It is again negative in the surface region where the function H is small or even zero.

The equatorial acceleration is reduced in Fig. 3 in comparison to the results of the 'uniform' model of Fig. 2. This is due to the absence of the function H in the outer layers. If the weight of these layers is reduced by a strong negative density gradient then the surface rotation law recovers and the observed value of the equator-pole difference of Ω appears again (Fig. 4). The used density profile is simply written as $\rho \propto \exp(-G(x - x_{\rm in}))$ which has been applied in the Reynolds equation (3) with $G = 20$.

All the calculations lead to the result that

– the equatorial acceleration is of the observed amount,
– $W < 0$ at the surface and $W > 0$ in the bulk of the convection zone,
– W is positive if averaged over the radius.

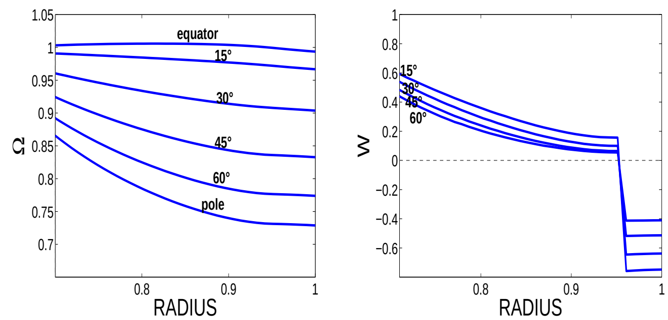

Fig. 4. The same as in Fig. 3 but with a density profile included which decreases outwards by more than six scale-heights ($G = 20$).

3. Two solar models

We have also computed two models for the solar differential rotation with the detailed physics described by Küker et al. (2011). One model includes the full Reynolds stress tensor while the other one only works with (1) without Λ (see Balbus et al. 2012). Both models are able to reproduce the surface rotation law (Fig. 5). A convection zone model computed with the MESA stellar evolution code (Paxton et al. 2011) has been used as a background model. The Reynolds stresses were computed with an average rotation period of 27 days and a value of 5/3 for the mixing length parameter. This choice sets the mixing length equal to the density scale height. The viscosity coefficient $\nu_{\rm T}$ of the models is of order 10^{13} cm^2/s with a maximum of $2 \cdot 10^{13}$ cm^2/s in a depth of $x \simeq 0.8$.

The model without Λ effect results in an internal rotation pattern with disk-shaped iso-contours (at the equator region). The meridional flow is directed towards the equator at the surface and towards the poles at the bottom of the convection zone ('clockwise'). The model which bases on the Λ effect provides a meridional circulation cell with the opposite flow direction ('counter-clockwise').

Figure 6 gives the horizontal Reynolds stress $Q_{\theta\phi}$ averaged over the radius in its dependence on the latitude. This quantity is antisymmetric with respect to the equator by definition and thus vanishes at the equator. If the Reynolds stress is purely viscous (dashed line) $Q_{\theta\phi}$ is negative (positive) in the northern (southern) hemisphere – in contrast to the empirical findings given in Fig. 1. For the model with the full Reynolds stress (solid line) the horizontal Λ effect exceeds the viscous part in the bulk of the convection zone where the total Reynolds stress is positive (negative) in the northern (southern) hemisphere. As it must, the horizontal stress $Q_{\theta\phi}$ vanishes both on the rotation axis and in the equatorial plane. It reaches its peak value at about $30°$ latitude. The amplitude of the horizontal cross correlation $Q_{\theta\phi}$ is of order 10^6 cm^2/s^2 which leads to $|W| \simeq 0.14$ well corresponding to the estimate $(7)_2$ as from the quasilinear turbulence theory $H \simeq 0.4$ results. The value in very good agreement with the numerical results for the cubic Reynolds equation given in Fig. 4.

We have to note that the numerical value of the positive maximum of 10^6 cm^2/s^2 exceeds the empirical results by a factor of five which might be a consequence of a too high viscosity value used in the simulations.

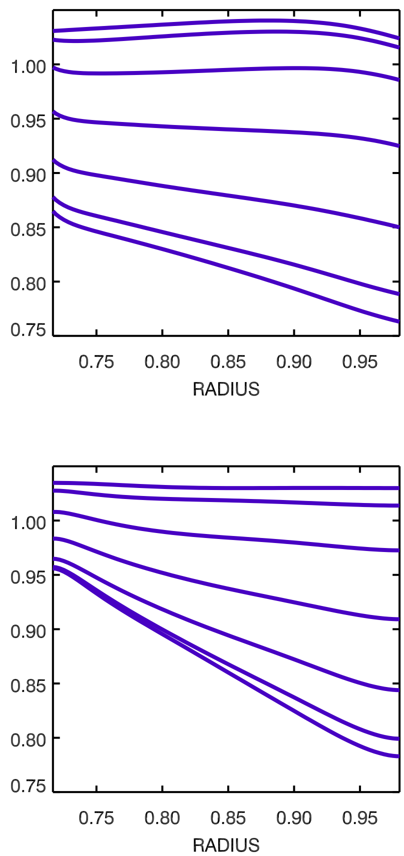

Fig. 5. The internal differential rotation for the full model with Λ effect included (top) and the rotation law driven only by baroclinic flows (bottom). The meridional flow at the surface goes poleward (top) and goes equatorward (bottom). The tachocline in the theoretical models is not modeled.

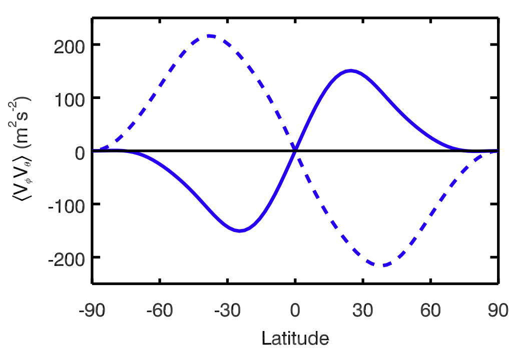

Fig. 6. The Reynolds stress $Q_{\theta\phi}$ from the simulations and averaged over the radius with Λ effect (solid) and without Λ effect (dashed). The Reynolds stress vanishes at the equator and changes the sign in the southern hemisphere (see Fig. 1).

4. Conclusions

It is hard to imagine that the mechanism of the solar dynamo could be understood without understanding of the maintenance of the solar rotation law. It is thus of relevance if one can show that the generation of the differential rotation of the solar convection zone bases on the existence of nondiffusive Λ terms in the Reynolds stress in a similar sense as the solar dynamo may base on the existence of nondiffusive α terms in the turbulent electromotive force. We argue that indeed the Λ effect in rotating anisotropic turbulence is *necessary* to explain the current observation of positive (negative) horizontal cross correlation $Q_{\theta\phi}$ at the northern (southern) hemisphere at the solar surface which Hathaway et al. indicated as a result of the proper motions of giant cells. All analytical and numerical studies lead to positive functions H which in the bulk of the convection zone are able to overcompensate the diffusive term which alone would lead to the opposite signs which are not observed. The overcompensation, however, is not trivial. It might be that in the outer supergranulation layer ($0.95 \lesssim x \lesssim 1$) the equatorial acceleration is too large and/or the H is too small. We have to mention in this respect that the H is a term which is of higher order in the Coriolis number $\Omega^* = 2\tau_{\rm corr}\Omega$ so that due to the short lifetimes of the granulation and supergranulation the $H \ll 1$ in the outer part of the convection zone and the turbulent medium behaves diffusive in the horizontal plane. Because of symmetry properties a term linear in Ω does not exist in the horizontal cross-correlation $Q_{\theta\phi}$ – in opposition to the radial tensor component $Q_{r\phi}$.

We have demonstrated also by means of a simplified model which ignores the meridional flow in the solution of the azimuthal Reynolds equation that $\cos\theta Q_{\theta\phi}$ – or which is the same – the quantity W is indeed negative in the surface layers. The negative sign, however, only describes a surface effect. In subsurface layers the equator-pole difference of Ω is reduced and the amplitude of the positive H grows. It is thus no surprise that in the deeper layers of the convection zone the sign of W becomes positive. This is a general result which does not depend on details of the eddy viscosity, of the density stratification, and/or influences of the meridional flow. Even our most complex Λ effect model which will reproduce the internal solar rotation law and also the observed pattern of the meridional flow exactly shows the described behavior. If the equatorial acceleration is *not* a consequence of the Λ effect but is due to a meridional flow (which *must* flow equatorward along the surface) then the function W would be negative-definite on the northern hemisphere through the entire convection zone .

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