

Eigenständigkeitserklärung

Ich versichere, dass ich, Avital Sievers, die vorliegende Arbeit selbstständig und nur mit den angegebenen Quellen und Hilfsmitteln angefertigt habe. Alle Stellen der Arbeit, die ich aus diesen Quellen und Hilfsmitteln dem Wortlaut oder dem Sinne nach entnommen habe, sind kenntlich gemacht und im Quellen- und Literaturverzeichnis aufgeführt. Weiterhin versichere ich, dass weder ich noch andere diese Arbeit weder in der vorliegenden noch in einer mehr oder weniger abgewandelten Form als Leistungsnachweis bereits verwendet haben oder noch verwenden werden. Ich versichere, dass ich keine KI-Werkzeuge für die Erstellung meiner Arbeit verwendet habe.

Hiermit versichere ich außerdem, dass ich die „Richtlinie zur Sicherung guter wissenschaftlicher Praxis für Studierende an der Universität Potsdam (Plagiatsrichtlinie) - Vom 20. Oktober 2010“ beachte (https://www.uni-potsdam.de/fileadmin/projects/ambek/Amtliche_Bekanntmachungen/2011/ambek-2011-01-037_039.pdf).

Es handelt sich bei dieser Arbeit um meinen ersten Versuch.



Potsdam, 9.4.2026

Avital Sievers

Comparing the Dynamical Mass Profile of Line-of-sight and Proper Motions for Draco

Master Thesis

9.4.2026

Universität Potsdam
Institut für Physik und Astronomie

Zusammenfassung

Dass es eine Diskrepanz zwischen beobachteter und vorhergesagter Masse im Universum gibt, ist bereits seit 1930 bekannt, woraus sich die Theorie der Dunklen Materie entwickelte. Zwerggalaxien sind besonders interessant für die Studie der Dunklen Materie, da dies die Objekte im Universum sind, die den größten Anteil von Dunkler Materie haben.

Eine Art, das tatsächliche Gravitationspotential eines Systems, hier der Zwerggalaxien, zu bestimmen, ist mithilfe der Bewegungen der Sterne in dem System. Durch die kollisionslose Boltzmann-Gleichung und die Poisson-Gleichung kann durch die Geschwindigkeiten das Potential berechnet werden. Das Lösen der Boltzmann-Gleichung ist in Praxis nicht direkt so möglich, und muss anders gelöst werden. Der Code GravSphere, den ich in meiner Arbeit benutze, macht dies durch Jeans-Modellierung.

Die Geschwindigkeitsdaten der Sterne stellen einen weiteren Punkt der Diskussion dar. Zum einen gibt es die Radialgeschwindigkeit, die Komponente, die in Sichtlinie des Beobachters liegt und mithilfe der Dopplereffekts gemessen werden kann. Zum anderen gibt es auch die Eigenbewegung der Sterne, die zeitintensiver und technisch anspruchsvoller zu messen ist, allerdings auch eine bessere Darstellung der tatsächlichen Bewegung der Sterne ist.

Da nun erstmals akkurate Daten der Eigenbewegung der Sterne in der Zwerggalaxie Draco vorhanden sind, prüfe ich in meiner Arbeit, wie diese sich auf die Regression des Massenprofil der Galaxie auswirken.

Die Anzahl der Sterne mit Radialgeschwindigkeitsdaten und Eigenbewegungsdaten ist ähnlich, die Verteilungen allerdings nicht. Die Sterne mit Radialgeschwindigkeitsdaten sind gleichmäßig über die Galaxie verteilt, während die Sterne mit Eigenbewegungsdaten sich auf zwei Felder nahe des Zentrums der Galaxie befinden.

Mithilfe von GravSphere schließe ich auf das Massenprofil von Draco auf Basis verschiedener kinematischer Datensets. Ich vergleiche Sets der Radialgeschwindigkeit mit verschiedenen Sternenmengen und Positionen als Basis für das Verständnis des Einflusses dieser Eigenschaften. Anschließend analysiere ich den Einfluss von der Eigenbewegung, wenn diese zu der Radialgeschwindigkeit hinzugefügt werden. Hierzu habe ich entsprechende Codefragmente geschrieben, die die Daten für GravSphere nutzbar machen.

Alle Ergebnisse sind miteinander kompatibel. Die Massenprofile für das gesamte Datenset der Radialgeschwindigkeiten und das Set in Kombination mit allen Eigengeschwindigkeiten liefern ein nahezu identisches Massenprofil. Für die Sterne mit Radialgeschwindigkeiten, die sich in den gleichen Feldern wie die mit Eigenbewegungen befinden, verbessert sich die Regression nur in diesen Entfernungen. Die physikalischen Parameter liegen in den Fehlerumgebungen voneinander und denen der Literatur. Die Dichteprofile verhalten sich für kleine Radien gleich. Das bedeutet, dass das Hinzufügen von Eigenbewegungen, die zur Zeit verfügbar sind, zur Zeit keinen positiven Einfluss auf die Analyse der Dunklen Materie mithilfe von Geschwindigkeitsdaten in Zwerggalaxien hat.

Abstract

The mass discrepancy observed in the Universe is a well established theory. Still, after nearly a hundred years, little is known about the proposed solution to this discrepancy, dark matter. In this thesis, I analyse the dark matter mass profiles of the satellite galaxy Draco calculated via dynamical mass estimation using the code GravSphere, and look at the impact of including proper motions data in addition to line-of-sight velocities and whether this inclusion results in a better fit. I also analyse the density profiles to see if proper motions help bridge the gap in the core-cusp problem. All results agree well with each other and the literature values with $M_{200}^{Draco} = (3.9^{+7.3}_{-1.8}) \cdot 10^9 M_{\odot}$ for line-of-sight only and $M_{200}^{PM+Draco} = (5.0^{+14.0}_{-2.5}) \cdot 10^9 M_{\odot}$ including proper motions. The mass profiles and density profiles do not show any improvements when including proper motions.

Contents

Zusammenfassung	i
Abstract	iii
Contents	v
List of Figures	vi
List of Tables	xi
1 Introduction	1
1.1 Dark Matter	1
1.1.1 Historical Background	1
1.1.2 Standard Model of Cosmology: Λ CDM	5
1.1.3 Properties	6
1.1.4 Tracing Dark Matter	8
1.1.5 Alternative Frameworks	9
1.2 Dwarf Galaxies	9
1.2.1 Small-Scale Challenges	11
1.2.2 Draco	12
1.3 Velocity Measurements	13
1.4 Research Question	14
1.5 Outline	14
2 Methods	15
2.1 GravSphere	15
2.1.1 NFW Profile	17
2.2 Modification	17
3 Data	18
3.1 Proper Motions	18
3.2 Line-of-Sight	19
3.3 Calculating the Surface Density	21

4	Results and Discussion	24
4.1	Priors	24
4.2	Line-of-Sight	24
4.2.1	Draco Line-of-Sight Data	24
4.2.2	PM Fields LOS Data	27
4.2.3	Comparison Field	31
4.3	Proper Motions	34
4.3.1	PM+Draco	34
4.3.2	PM+Fields	35
4.3.3	Comparison with Other Results	35
4.4	Density Profiles	38
4.5	Summary	41
5	Conclusion	44
5.1	Future Work	45
	References	45
	Appendices:	50
A	Data Sets Additional Information	51
B	Code	72

List of Figures

1.1.1	Rotation curve of the Andromeda galaxy from [36].	3
1.1.2	The mass peaks resulting from gravitational lensing (green outlines) are closer to the stellar mass peaks (blue) than the X-ray gas peaks (red) ([9]).	5
1.1.3	Energy-matter content of the universe ([7])	5
1.1.4	End result of a dark matter only cosmological simulation, showing the dark matter density in a Λ CDM universe. As a result from hierarchical structure formation, there are big structures in the Universe surrounded by smaller structures. Source: VIRGO/Millennium Simulation Project, Max Plank Institut für Astrophysik.	7
1.1.5	M/L ratios within the half-light radius for ultra-faint Milky Way satellite galaxies including Draco ([38]).	8
1.2.1	A variety of dwarf galaxies of different sizes and masses ([5]).	10
1.2.2	The Vast Polar Structure of the Milky Way. Dwarf galaxies are oriented along a plane around the Milky Way plane (black line) ([22]).	12
1.2.3	The cusp (red) vs. core (blue) behaviour at low radii. The density profile of the observed halos follows a constant-density core profile while the Λ CDM profile has much higher density ([28]).	12
3.1.1	Fields F1 and F2 from which proper motions were observed. The black ellipse showcases Draco's half-number radius ([43]).	19
3.2.1	Locations of data observation are showcased by the colourful points. The black ellipse has a semimajor axis of $2R_{half}/\sqrt{1-\epsilon}$, with R_{half} the projected half-light radius and ϵ the observed ellipticity([46]).	21
3.3.1	Positions of the stars of the different datasets.	22
3.3.2	Surface density as a function of radius. The blue vertical line indicates the half-light radius.	22

4.2.1	Comparison of the mass profiles for Line-of-sight Data and Draco Line-of-sight Data.	25
4.2.2	Mass profile for Draco Line-of-sight Data showing the dark matter and stellar fit with the 1σ and 2σ -environments represented by the darker and lighter filled-in area respectively. The blue line indicates the half-light radius.	26
4.2.3	M_{200} and c values for Draco Line-of-sight Data. The blue line corresponds to the median value and the green line to the mean value. Errors are the 1σ -environment.	27
4.2.4	Mass profile for PM Fields LOS Data showing the dark matter and stellar fit with the 1σ and 2σ -environments represented by the darker and lighter filled-in area respectively. The blue line indicates the half-light radius.	28
4.2.5	M_{200} and c values for PM Fields LOS Data. The blue line corresponds to the median value and the green line to the mean value. Errors are the 1σ -environment.	29
4.2.6	Comparison of the dark matter mass profile for Draco Line-of-sight (red) and PM Fields LOS (green) data. The vertical line indicates the half-light radius and the dashed lines the 1σ -environments. The stellar positions are displayed in corresponding colour as dots at corresponding radius at arbitrary height.	30
4.2.7	Comparison of the dark matter mass profile for Inner Field LOS (red) and PM Fields LOS (green) data. The vertical line indicates the half-light radius and the dashed lines the 1σ -environments. The stellar positions are displayed in corresponding colour as dots at corresponding radius at arbitrary height.	30
4.2.8	Mass profile for Comparison Field LOS Data showing the dark matter and stellar fit with the 1σ and 2σ -environments represented by the darker and lighter filled-in area respectively. The blue line indicates the half-light radius.	31
4.2.9	M_{200} and c values for Comparison Field LOS Data. The blue line corresponds to the median value and the green line to the mean value. Errors are the 1σ -environment.	32
4.2.10	Comparison of the dark matter mass profile for Comparison Field LOS (green) and PM Fields LOS (red) data. The vertical line indicates the half-light radius and the dashed lines the 1σ -environments. The stellar positions are displayed in corresponding colour as dots at corresponding radius at arbitrary height.	33

4.2.11	Comparison of the dark matter mass profile for Comparison Field LOS (green) and Draco Line-of-sight (red) data. The vertical line indicates the half-light radius and the dashed lines the 1σ -environments. The stellar positions are displayed in corresponding colour as dots at corresponding radius at arbitrary height.	34
4.3.1	Mass profile for PM+Draco Data showing the dark matter and stellar fit with the 1σ and 2σ -environments represented by the darker and lighter filled-in area respectively. The blue line indicates the half-light radius.	35
4.3.2	M_{200} and c values for PM+Draco Data. The blue line corresponds to the median value and the green line to the mean value. Errors are the 1σ -environment.	36
4.3.3	Comparison of the dark matter mass profile for PM+Fields (green) and PM+Draco (red) data. The vertical line indicates the half-light radius and the dashed lines the 1σ -environments. The stellar positions are displayed as dots at corresponding radius at arbitrary height.	37
4.3.4	Mass profile for PM+Fields Data showing the dark matter and stellar fit with the 1σ and 2σ -environments represented by the darker and lighter filled-in area respectively. The blue line indicates the half-light radius.	38
4.3.5	M_{200} and c values for PM+Fields Data. The blue line corresponds to the median value and the green line to the mean value. Errors are the 1σ -environment.	39
4.3.6	Comparison of the dark matter mass profile for PM+Draco (green) and Draco Line-of-sight (red) data. The vertical line indicates the half-light radius and the dashed lines the 1σ -environments. The stellar positions are displayed in corresponding colour as dots at corresponding radius at arbitrary height.	40
4.3.7	Comparison of the dark matter mass profile for PM+Fields (green) and PM Fields LOS (red) data. The vertical line indicates the half-light radius and the dashed lines the 1σ -environments. The stellar positions are displayed in corresponding colour as dots at corresponding radius at arbitrary height.	40

4.3.8	The relative error on the enclosed mass profile for all datasets. The vertical line indicates the half-light radius and the dashed lines the 1σ -environments. The stellar positions are displayed in corresponding colour as dots at corresponding radius at arbitrary height. The positions of the stars in the proper motions sets are similar to those of PM Fields LOS.	41
4.4.1	Comparison of the density profile for PM+Draco (green) and Draco Line-of-sight (red) data. The vertical line indicates the half-light radius and the dashed lines the 1σ -environments. The stellar positions are displayed in corresponding colour as dots at corresponding radius at arbitrary height.	41
4.4.2	Comparison of the density profile for PM+Fields (green) and PM Fields LOS (red) data. The vertical line indicates the half-light radius and the dashed lines the 1σ -environments. The stellar positions are displayed in corresponding colour as dots at corresponding radius at arbitrary height.	42
4.4.3	Comparison of the density profile for PM+Draco (red) and PM+Fields (green) data. The vertical line indicates the half-light radius and the dashed lines the 1σ -environments. The stellar positions are displayed as dots at corresponding radius at arbitrary height.	42
A.0.1	All physical parameters of Draco Line-of-sight. In order: M_{200} : Virial mass, c : concentration, r_c : core radius, n : describes shape of the dark matter density profile, r_t : tidal radius, M_{star} : stellar mass	52
A.0.2	The stellar projected velocity dispersion σ_{LOS} profile for Draco Line-of-sight.	53
A.0.3	The Virial shape parameter v_{S1} parameter for Draco Line-of-sight (more detail in Read and Steger(2017) ([34]), Equation 21).	53
A.0.4	The Virial shape parameter v_{S2} parameter for Draco Line-of-sight (more detail in Read and Steger(2017) ([34]), Equation 23).	54
A.0.5	All physical parameters of PM Fields LOS.	55
A.0.6	σ_{LOS} profile for PM Fields LOS.	56
A.0.7	v_{S1} parameter for PM Fields LOS.	56
A.0.8	v_{S2} parameter for PM Fields LOS.	57
A.0.9	Mass profile of Inner Field LOS.	57
A.0.10	All physical parameters of Inner Field LOS.	58
A.0.11	σ_{LOS} profile for Inner Field LOS.	59

A.0.12 v_{S1} parameter for Inner Field LOS.	59
A.0.13 v_{S2} parameter for Inner Field LOS.	60
A.0.14 All physical parameters of Comparison Field LOS.	61
A.0.15 σ_{LOS} profile for Comparison Field LOS.	62
A.0.16 v_{S1} parameter for Comparison Field LOS.	62
A.0.17 v_{S2} parameter for Comparison Field LOS.	63
A.0.18 All physical parameters of PM+Draco.	64
A.0.19 σ_{LOS} profile for PM+Draco.	65
A.0.20 The stellar proper motions dispersion in radial direction σ_{PMr} profile for PM+Draco.	65
A.0.21 The stellar proper motions dispersion in tangential direction σ_{PMt} profile for PM+Draco.	66
A.0.22 v_{S1} parameter for PM+Draco.	66
A.0.23 v_{S2} parameter for PM+Draco.	67
A.0.24 All physical parameters of PM+Fields.	68
A.0.25 σ_{LOS} profile for PM+Fields.	69
A.0.26 σ_{PMr} profile for PM+Fields.	69
A.0.27 σ_{PMt} profile for PM+Fields.	70
A.0.28 v_{S1} parameter for PM+Fields.	70
A.0.29 v_{S2} parameter for PM+Fields.	71

List of Tables

1.2.1 Values for selected dwarf galaxies([29] at https://github.com/apace7/local_volume_database)	10
3.3.1 Datasets used in this project.	23
4.1.1 M_{200} : Virial mass, c : concentration, r_c : core radius in kpc, n : parameter describing the shape of the dark matter density profile, r_t, δ : tidal radius [kpc] where ρ steepens at rate $\propto r^{-\delta}$, β : anisotropy profile	24
4.3.1 All values of M_{200} with unit of $10^9 M_{\odot}$	37
4.3.2 Values for radius of outermost star of Draco Line-of-sight $R_{max} =$ 0.85 kpc and the dark matter mass at this radius $M \cdot 10^8$	38
4.5.1 Summary of physical properties as produced by GravSphere for all relevant datasets.	43
A.0.1 Values for radius of outermost star R_{max} and the dark matter mass at this radius M times the multiplier.	51

Chapter 1

Introduction

1.1 Dark Matter

Dark matter is a theorized type of non-baryonic matter. While little is known about it, it is by definition non-luminous and interacts (only) gravitationally.

1.1.1 Historical Background

Dark matter as a concept has been around for nearly 100 years. It is based on the observations that there is a matter discrepancy between visible matter and the amount necessary to create the dynamical behaviour of galaxies, galaxy clusters, and structures we observe in the Universe. Over time, different methods have come to be important in demonstrating the need for dark matter.

1.1.1.1 Velocity Dispersion in the Coma Cluster

The first time the term "dark matter" has been coined was by Zwicky(1933) ([48]) in his paper "Die Rotverschiebung von Extragalaktischen Nebeln", a study on extragalactic nebulae, now understood to be galaxies. As part of the data set, various galaxy clusters were included. In particular, the velocity dispersion of the Coma cluster, the most well-studied cluster at the time, was found to be very high.

There are velocity differences between 1500 to 2000 km/s among the galaxies of the cluster. Using this result and an estimation on the total visible mass of the system, Zwicky observed the following:

When assuming the cluster to be in a stationary state, the Virial theorem applies:

$$\overline{E_k} = -\frac{1}{2}\overline{E_p}, \quad (1.1)$$

where $\overline{E_k}$ and $\overline{E_p}$ are the mean kinetic and potential energy respectively.

The total mass M can be estimated by taking the individual masses of the galaxies and their distribution into account. From that, the total potential energy Ω can be calculated. By averaging over the mass, $\overline{E_p}$ is calculated:

$$\overline{E_p} = \frac{\Omega}{M} = -\frac{3}{5}\Gamma\frac{M}{R}, \quad (1.2)$$

where Γ is the gravitational constant.

Via (1.1) and

$$\overline{E_k} = \frac{\overline{v^2}}{2} \quad (1.3)$$

he arrives at an expected velocity of $(\overline{v^2})^{1/2} = 80 \text{ km/s}$ if the total mass of the cluster is equal to the visible matter. For the velocities derived from the observed Doppler effects, the average density and thus the total mass of the cluster needs to be at least 400 times bigger. The observed velocity dispersion simply cannot result from a stable system of the visible mass of the Coma cluster.

This might imply that the underlying assumption of stability could be incorrect. While this not only would lead to a very short lived structure, it also only reduces the average density needed by a factor of 2. Here, energy conservation applies and thus the relation of these energies must be $\overline{E_k} = -\overline{E_p}$, instead of the Virial theorem. These relations differ by a factor of $\frac{1}{2}$, leading to the before mentioned factor of 2.

It is to be mentioned that at that time, dark matter was simply considered to be regular matter not yet observable due to technical constraints like black holes, low-luminosity stars, and free-floating planets. This idea is not completely dismissed – MaCHOs or Massive Compact Halo Objects, which that matter would fall into, are still considered an option – but cannot make up all of dark matter ([8], Section 1.1.3). Nowadays, dark matter most commonly is thought to be a particle.

1.1.1.2 Rotation Curves

Rotation curves are the relation between the rotational velocity of stars in a spiral galaxy and their distance from the center. Rotation curves of spiral galaxies have been observed since the 1940s ([24]) and gained traction after the discovery of the 21 cm emission line in the 50s ([31], [12]). However, only in the 1970s were rotation curves discovered to be flat at large radii. The work by Rubin and Ford(1970) ([36]) in particular looks at the rotation of the Andromeda galaxy. They calculated the velocity from emission lines to find a rotation curve for distances from the galactic center of $R \geq 1.6 \text{ kpc}$ (see Fig. 1.1.1). At large distances, the velocity is not decreasing, as would be expected. When only looking at the visible matter, the rotation curve is expected to follow a Keplerian decline, as the luminous mass decreases with distance from the galactic center. However, at this time Rubin and Ford were unwilling to infer anything about the

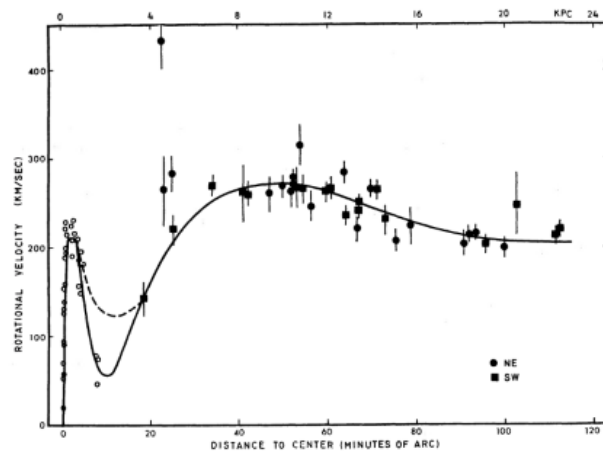


Figure 1.1.1: Rotation curve of the Andromeda galaxy from [36].

mass beyond $R = 24 \text{ kpc}$ as they considered the data to be too unreliable.

In Rubin et al.(1980) ([37]), when looking at the rotational properties of 21 SC galaxies as defined by the Hubble classification where they again found flat rotation curves they concluded: "The conclusion is inescapable that non-luminous matter exists beyond the optical galaxy". In the time between those two papers, the problems of the velocity dispersion and rotational curves were brought together to describe one problem of missing matter. This, finally, made dark matter the main theory to solve the mass discrepancy in the universe.

1.1.1.3 Gravitational Lensing

Compared to both velocity dispersion and rotational curves, gravitational lensing is a much newer method to find evidence for dark matter. While there are various types of lensing such as strong or weak, and the effect is used for different purposes on different objects, the possibly most impactful gravitational lensing study is the weak lensing result from the Bullet Cluster – two galaxy clusters undergoing an active merger. The work from Clowe et al.(2006) ([9]) shows not only a clear case for a matter discrepancy but also a case against modified gravity (1.1.5) as an alternative to dark matter.

Gravitational lensing relies on the basis that general relativity is the correct way to describe gravity but does not require any assumptions on the dynamical state of the object like equilibrium.

The light of background objects is impacted by the gravitational potential of a foreground object. Instead of traveling in a straight line it gets deflected by an angle. Measuring this angle gives insight on the gravitational potential and consequently the mass distribution of the foreground object.

In the case of weak lensing, the important thing to measure are the shapes of the background galaxies. The small signal of weak lensing induces ellipticities of an order of 1%. The signal is on its own negligible compared to the intrinsic shape of galaxies and therefore has to be measured statistically using the shear which gives insight on the induced ellipticities. When assuming that galaxies are randomly oriented, the ellipticities can be used to infer the strength of the gravitational field ([16]).

There are two distinct mass peaks for both stellar matter and X-ray gas in the case of the Bullet Cluster. The majority of visible matter is in the gas component. As such, the mass peaks resulting from the lensing should correspond to the gas mass peaks if there is no dark matter. Instead, they are closer to the stellar mass peaks (see Figure 1.1.2). This, for one, gives insight on the behaviour of dark matter: it behaves collisionlessly under a merger. More importantly, this result at present cannot be explained by modified gravity theories.

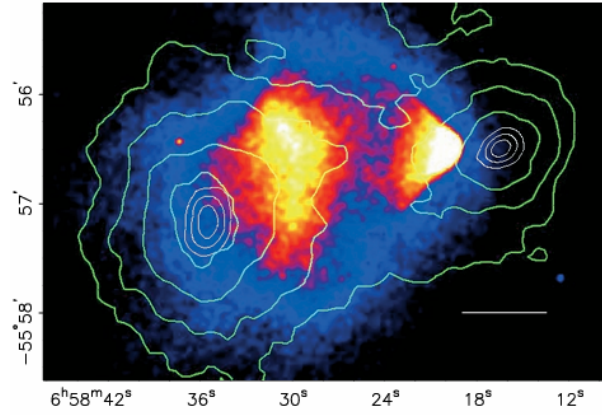


Figure 1.1.2: The mass peaks resulting from gravitational lensing (green outlines) are closer to the stellar mass peaks (blue) than the X-ray gas peaks (red) ([9]).

1.1.2 Standard Model of Cosmology: Λ CDM

The standard model of cosmology is based on general relativity. The Λ stands for the dark energy component, accounting for roughly 70% of the matter-energy component in the Universe (see Figure 1.1.3). CDM stands for "cold dark matter" and makes up five times the amount of baryonic matter. The only constraint this puts on the candidates for dark matter is that they should be cold, meaning non-relativistic.

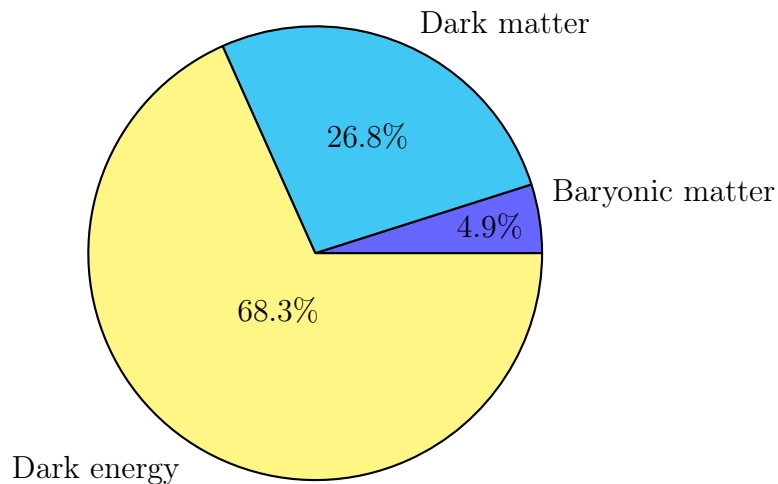


Figure 1.1.3: Energy-matter content of the universe ([7])

1.1.2.1 Hierarchical Structure Formation

In the early stages of the universe, matter – both visible and dark – was distributed in a nearly homogeneous manner. Still, there were small over- and underdensities of matter. With time, these overdensities merge as they are attracted to each other gravitationally. This means that smaller structures form first and merge into bigger ones later on. This is called hierarchical structure formation. As there is much more dark matter than visible matter and it is not suppressed during recombination, this is the dominating component of this behaviour. Without it, structures could not form on the timescale necessary for them to be as developed as we see them today (see Figure 1.1.4).

1.1.3 Properties

As seen in Section 1.1, dark matter clearly is very abundant in the universe. This makes understanding it vital. While there are few things we know about it, there are properties we have information or constraints on.

Limit on Baryonic Matter

In the Big Bang nucleosynthesis only light elements (Hydrogen, Helium, and Lithium) are created. The abundances of Deuterium, Helium 3 and 4, and Lithium 7 depend on the density of baryonic matter as it impacts the reaction speed and efficiency of their formation. Their relative abundances constrain $\Omega_b \approx 0.4 \Omega_{crit}$ ([8]). Since Ω_M , the total matter density, is much larger, not all matter can be baryonic. There is a limit on baryonic matter in the Universe. Therefore, baryonic candidates for dark matter like some types of MaCHOs cannot cover the entirety of the mass discrepancy observed.

Interaction

The standard model of particle physics introduces four interactions: weak, strong, electromagnetic, and gravitation. The idea behind the simplest definition of the modern concept of dark matter is that it only interacts gravitationally. It thus cannot be detected via any other interaction.

However, there are theories that allow for other types of interactions as well. A very popular dark matter candidate is called the WIMP – weakly interacting massive particle. As the name implies, this particle could also interact via the weak force. By introducing this option for a weak interaction, the particle could be detected via other means, including a direct detection in a scattering event at detectors ([11]).

Another candidate for dark matter that allows more types of interaction is the so-called self-interacting dark matter: SIDM. This idea simply adds a new type of interaction acting only between dark matter particles.

Distribution

When simulating dark matter, one starts with a near homogeneous distribution in the early universe and lets gravity act on it over time. The result is a multitude of dark matter halos with varying masses structured similarly to the visual structure of the universe (Figure 1.1.4).



Figure 1.1.4: End result of a dark matter only cosmological simulation, showing the dark matter density in a Λ CDM universe. As a result from hierarchical structure formation, there are big structures in the Universe surrounded by smaller structures. Source: VIRGO/Millennium Simulation Project, Max Plank Institut für Astrophysik.

These halos can be described by the NFW density profile:

$$\rho_{NFW}(r) = \frac{\rho_s}{r/r_s(1 + r/r_s)^2}, \quad (1.4)$$

where r_s is the characteristic radius and ρ_s the scale density ([27]).

Mass

Generally, different dark matter candidates cover a very large range of masses. From $10 M_\odot$ for primordial black holes to 10^{-22} eV for ultra-light candidates and everything in between.

There is, however, a limit on just how light a potential dark matter candidate can be. This is because of the structure of our universe.

When running gravitational simulations with dark matter of varying masses: heavier for cold dark matter, lighter for hot dark matter and something in between for warm, the final outcome looks drastically different. Hot dark matter

suppresses structures on small scales, forming only large scale structures. Given what we know about structure formation with general relativity, dark matter cannot be relativistic and has a limit on just how light it can be (see 1.1.5).

In regards to dark matter halos, the mass of the halo is related to the radius as follows ([27]):

$$M_{NFW}(r) = 4\pi\rho_s r_s^3 [\ln(1 + r/r_s) - \frac{r/r_s}{1 + r/r_s}]. \quad (1.5)$$

1.1.4 Tracing Dark Matter

Dark matter and visible matter evolve together. Since dark matter dominates gravitation, visible matter is attracted to it. This means that galaxies trace the underlying distribution of dark matter. The most interesting place to look at in regards to this are dwarf galaxies, as they are the most dark matter dominated objects in the universe. Their luminosities are smaller than their dynamical masses by at least a factor of 100 (see Figure 1.1.5), which is much more than a stellar mass-to-light ratio of $2M_\odot/L_\odot$. This means that the dynamical masses of dwarf galaxies have to be dominated by dark matter instead ([38]).

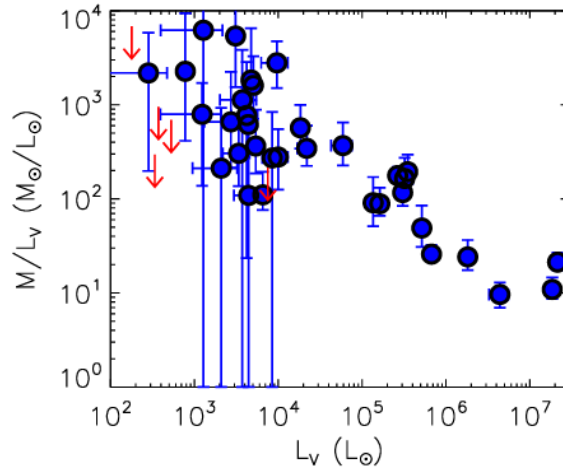


Figure 1.1.5: M/L ratios within the half-light radius for ultra-faint Milky Way satellite galaxies including Draco ([38]).

1.1.5 Alternative Frameworks

While Λ CDM is the standard model of cosmology, there is no shortage of alternative frameworks and ideas – in particular regarding the CDM component.

Warm Dark Matter

As mentioned previously there has to be a lower limit to the mass of the dark matter particle. Mass limits are not very well defined. Depending of the specific research question and type of analysis, different limits, in particular lower limits, are set.

While simulations favour cold dark matter to create the structures we see in the universe, warm dark matter (WDM) pushed the lower limit down and now the challenge is to find a balance between this limit and observations.

In Decant et al.(2025) ([13]), a high-end mass for WDM (and therefore a lower limit on the CDM mass) of 30 keV produces equivalent results to CDM and the lower limit is chosen to be 3 keV. However, Viel et al.(2005) ([41]) set lower limits on the WDM mass of 550 eV and 2 keV, depending on the method used.

Modified Newtonian Dynamics

There are a plethora of theories and frameworks that move away from the idea of the existence of dark matter entirely. There are issues with our current understanding of gravity (see Section 1.2.1, the lack of quantized gravity ([6])), and as such, there are alternative frameworks looking at gravity as opposed to the dark matter candidates to solve these issues; one that comes up in the discussion around dark matter consistently is modified Newtonian dynamics (MOND), first introduced in ([26]). Generally, MOND introduces a low acceleration regime which differs from Newtonian dynamics and transitions to Newtonian dynamics for high acceleration via a fitting function which has multiple variations. It was initially conceptualized to explain the observed flat rotation curves (Figure 1.1.1), which it has historically fitted well ([4]). As of now, there is no full relativistic version of this theory though attempts have been made for a while.

1.2 Dwarf Galaxies

As the name implies, dwarf galaxies are small galaxies. However, how many stars constitute a "small" galaxy is not properly defined. Generally, the Large Magellanic Cloud is considered the upper limit to the size of a dwarf galaxy at $\log_{10} 9.4 M_{\odot}$ ([29]). While on average a typical dwarf galaxy contains some billion stars, they can get as small as only some tens of stars (see Figure 1.2.1). As these galaxies are faint and small objects, it makes them hard to detect. While

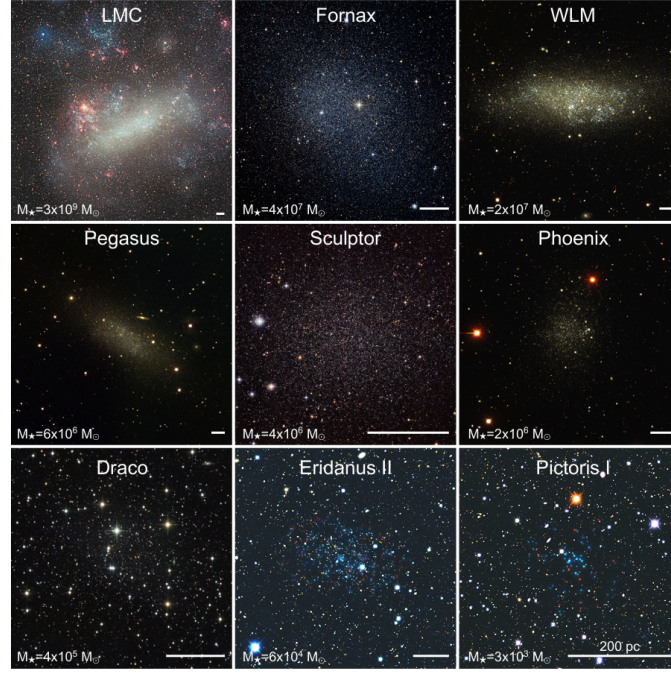


Figure 1.2.1: A variety of dwarf galaxies of different sizes and masses ([5]).

dwarf galaxy	half-light radius in arcmin	stellar mass in $\log_{10} M_\odot$
LMC	192.9	9.4
Fornax I	19.9	7.5
Draco I	9.6	5.7
Pisces II	1.3	3.9

Table 1.2.1: Values for selected dwarf galaxies([29] at https://github.com/apace7/local_volume_database).

predicted to be abundant throughout the universe, the best known dwarfs are located in the Local Group. Given their faintness, it is easiest to detect them close by. The dwarf galaxies that are gravitationally bound to a bigger galaxy are called satellite galaxies.

Dwarf galaxies are an interesting object to study in regards to dark matter as they are the most dark matter dominated objects in the universe, meaning they have the highest mass-to-light ratio or M/L in comparison to other objects like regular sized galaxies and star clusters. This makes them ideal laboratories for dark matter experiments.

1.2.1 Small-Scale Challenges

Λ CDM works very well for large scale observations of the Universe. However, at small scales (defined as $M \leq 10^{11} M_{\odot}$) there are a couple of challenges where model predictions seem to be in tension with observations. This scale includes dwarf galaxies ([5]).

1.2.1.1 Missing Satellites Problem

When comparing the number of observed satellites for a galaxy of a certain size to that of a comparably sized galaxy in simulations, it becomes apparent that there seem to be many more satellites in the simulations. It is possible to match the various simulated dark matter halos to the observed dwarf galaxies, correlating the biggest halos to the biggest satellites. This process is called abundance matching. When doing so, many small halos are without an observed counterpart. However, it is highly unlikely that those halos are able to form stars to begin with. By including baryonic physics, the over-prediction is reduced significantly ([3]).

1.2.1.2 Too-Big-To-Fail Problem

This form of abundance matching is not perfect. Comparing the biggest satellites and the biggest dark matter halos, the halos are much too centrally dense to host the satellites. In 1.2.1.1, the problem could be solved due to small halos not forming stars in the first place. These big halos, however, have to have formed stars. They are too big to fail at that ([21]).

1.2.1.3 Satellite Planes

When looking at the satellites of both the Milky Way and Andromeda, two things are apparent: most satellites are located in a plane around the host galaxy, and seem to be co-rotating. The likelihood for this to happen in a simulation is very low, close to 0% for both these galaxies ([30]).

1.2.1.4 Core-Cusp Problem

In CDM-only simulations, the dark matter halos have a density profile that is both dense and cuspy at small radii. Those density profiles follow $\rho(r) \propto r^{-\gamma}$ with $\gamma \approx 0.8 - 1.4$. However, many observed rotation curves instead show a core-like behaviour at those radii and are also less dense, following $\gamma \approx 0 - 0.5$ ([5]). This means that Λ CDM halos are too cuspy and too dense (Figure 1.2.3).

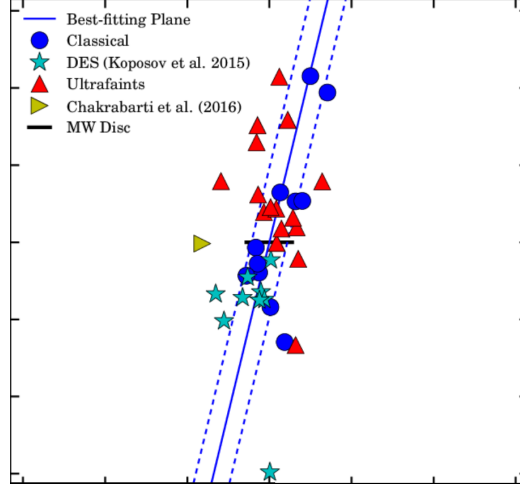


Figure 1.2.2: The Vast Polar Structure of the Milky Way. Dwarf galaxies are oriented along a plane around the Milky Way plane (black line) ([22]).

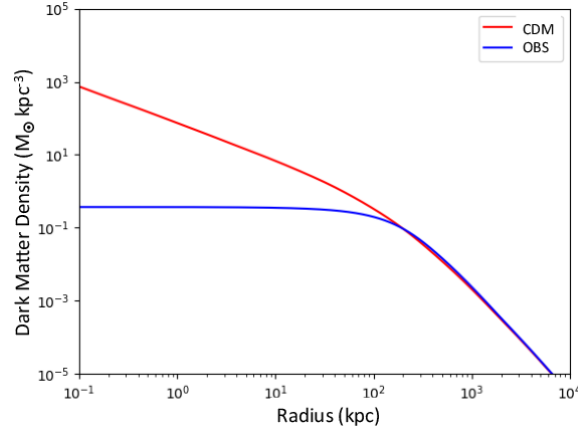


Figure 1.2.3: The cusp (red) vs. core (blue) behaviour at low radii. The density profile of the observed halos follows a constant-density core profile while the Λ CDM profile has much higher density ([28]).

1.2.2 Draco

Draco is a dwarf galaxy in the Local Group. It was discovered by Wilson(1955) ([47]) and examined by Baade et al.(1961) ([2]). They studied its Colour-Magnitude Diagram (CMD) and variable stars, as well as measured a linear distance of 68 kpc. Now, that value is at $d = 80 \pm 7$ kpc ([1]). Hodge(1964) ([18]) used this result to determine a cutoff radius of 0.51 ± 0.04 kpc. The half-light radius is $9.6'$ ([29]), which at a distance of 68 kpc corresponds to 0.19 kpc.

1.3 Velocity Measurements

Observing the velocity and motion of an object like a star can be done using different approaches.

Line-of-sight

It is well known that the spectral lines of stars get red- or blue-shifted, depending on the direction and velocity of the star with respect to the observer: the Doppler effect. This shift can be used to determine this velocity, which is then called the line-of-sight velocity, as it is only the velocity component along the line of sight of the observer.

This effect in stellar spectra was first observed by Huggins(1868) ([19]) for the F line in the spectrum of Sirius. However, it was not possible to get good measurements for the velocity even nearly 20 years later, given the poor resolution of the spectrographs of the time ([23]).

Proper motions

The proper motions of a star is measured by observing changes in its angular position over a long time frame. This, in principle, can be observed without high-resolution telescopes for the brightest stars.

Proper motions were first discovered by Halley(1718) ([17],[32]) when comparing current positions with those recorded in ancient Greece. In order to know the velocity from these changes in angular position, it is necessary to know the distance to the star. Therefore, the early measurements of proper motions go hand in hand with improvements in distance measurements ([32]).

A popular resource for proper motions data is the Hipparcos catalogue. This catalogue initially recorded stellar positions. Combining these with current positions allows for accurate proper motions data ([32]).

Currently, there are the Gaia data releases by ESA. Gaia EDR3 in 2020 provided proper motions for $\sim 1.5 \cdot 10^9$ stars, and there is a Gaia DR4 release upcoming. Gaia is a spacecraft including an astrometric instrument to measure proper motions. It consists of 62 CCDs in the focal plane and has high angular resolution. To measure line-of-sight velocities, it also carries a integral-field spectrograph as a radial velocity spectrometer in the near infrared with medium resolution ([14]).

1.4 Research Question

Given that proper motions velocity data is less convenient to gather than line-of-sight velocity data, there are many objects for which only line-of-sight data is available. For Draco, in 2024 sufficiently accurate proper motions data to resolve the internal motions of stars was published ([42],[43]), allowing for comparisons between the results of calculations reliant on velocity data of those two types. In this thesis I aim to answer the question of whether proper motions data has 1) better constraints on the dark matter profile calculated by the code GravSphere and 2) helps bridge the gap of the differences in the core-cusp problem.

1.5 Outline

In chapter 2, GravSphere – the code used in this project – will be introduced. In chapter 3, I will discuss the data sets used. Finally, in chapter 4, the results will be explained and discussed, and summarized in chapter 5.

Chapter 2

Methods

In theory, the way to derive the mass distribution of a stellar system using its velocity data is not complicated. Stars in galaxies obey the collisionless Boltzmann equation

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \nabla_x f \cdot \vec{v} - \nabla_v f \cdot \nabla_x \Phi = 0, \quad (2.1)$$

with f the distribution function of the stars in phase space, $\vec{x}$ and $\vec{v}$ the position and velocity.

Assuming a steady state, $\frac{\partial f}{\partial t}$ is negligible, allowing to directly solve (2.1) for the gravitational potential Φ to then arrive at the matter distribution ρ including all matter, dark and baryonic, via the Poisson equation

$$\nabla_x^2 \Phi = 4\pi G \rho \quad (2.2)$$

This is hard in practice given that f is 6-dimensional and poorly sampled. (2.1) includes its derivatives, aggravating the issue. To solve the Boltzmann equation, strategies are needed ([34]).

2.1 GravSphere

GravSphere is a code designed by Read and Steger(2017) ([34]) to break the density-anisotropy degeneracy using Jeans modelling. In order to solve (2.1), it employs the moment method that was first proposed in Jeans(1922) ([20]). For this, (2.1) gets integrated over each velocity component to arrive at a set of three moment equations. The integration over v_r and the assumption of no rotation leads to the spherical Jeans equation.

In order to arrive at the mass profile for both the line-of-sight velocities for Draco ([44]) and the proper motions ([42], [43], [46]), GravSphere is used. The code is described in detail in Read and Steger(2017) ([34]), Read et al.(2018)([35]) and updated to Python in Genina et al.(2020) ([15]). The current version¹ is discussed in Collins et al.(2021) ([10]). Therefore, here I will give only a brief introduction.

Dwarf galaxies can be assumed to be non-rotating and near-spherical stellar systems. Integrating the collisionless Boltzmann equation (2.1) over the radial velocity component,

$$\int dv_r \left(\frac{\partial f}{\partial t} + \nabla_x f \cdot \vec{v} - \nabla_v f \cdot \nabla_x \Phi \right) \quad (2.3)$$

results in the spherical Jeans equation

$$\frac{1}{\nu} \frac{\partial}{\partial r} (\nu \sigma_r^2) + \frac{1\beta(r)\sigma_r^2}{r} = -\frac{GM(< r)}{r^2}. \quad (2.4)$$

Here, M is the cumulative mass function of the system including all tracers with mass, $\sigma_r^2 = \langle v_r^2 \rangle - \langle v_r \rangle^2$, $\langle v_r^n \rangle = \int d^3\vec{v} (v_r^n f)$ is the radial velocity dispersion, $\beta \equiv 1 - \sigma_t^2/\sigma_r^2$ the velocity anisotropy describing the stellar orbital structure, and $\nu = \int d^3\vec{v} f$ the density of massless tracers moving within the gravitational potential of M .

(2.4) can be further integrated over r to arrive at an equation for $\sigma_r(r)$:

$$\sigma_r^2(r) = \frac{1}{\nu(r)g(r)} \int_r^\infty dr' \frac{GM(r')\nu(r')}{r'^2} g(r'), \quad (2.5)$$

with $g(r) = \exp(2 \int dr \beta(r)/r)$ and R a projected radius.

For the observed line-of-sight velocities, (2.5) needs to be projected along the line-of-sight to arrive at an expression for the line-of-sight dispersion:

$$\sigma_{LOS}^2(R) = \frac{2}{\Sigma(R)} \int_R^\infty dr (1 - \beta(r) \frac{R^2}{r^2}) \nu(r) \sigma_r^2 \frac{r}{\sqrt{r^2 - R^2}}, \quad (2.6)$$

with Σ the tracer surface mass density. The issue here is that there is a degeneracy of β with σ_r and consequently with M and ρ . While there are many suggestions on how to solve this degeneracy, GravSphere recovers β and ρ needing only the assumptions of spherical symmetry and a steady-state distribution

¹<https://github.com/justinread/gravsphere>

function.

The method of using the spherical Jeans equation to solve (2.1) has a number of advantages and disadvantages. For one, the form of f does not need to be specified and the calculation is fast. On the other hand, there is no guarantee of a physical solution. GravSphere also requires binned data, necessarily causing a loss of information.

The results of the code include the mass profile $M(< r)$, the density profile $\rho(r)$, and the velocity anisotropy profile $\beta(r)$.

2.1.1 NFW Profile

GravSphere initially runs using the NFW profile to fit the dark matter profile. The version I am using fits a coreNFW_{tides} profile in particular, which is described by

$$\rho_{cNFWt}(r) = \begin{cases} \rho_{cNFW} & r < r_t \\ \rho_{cNFW}(r_t) \left(\frac{r}{r_t}\right)^{-\delta} & r > r_t \end{cases} \quad (2.7)$$

where $\rho_{cNFW} = f^n \rho_{NFW} + \frac{nf^{n-1}(1-f^2)}{4\pi r^2 r_c} M_{NFW}$, $f^n = [\tanh(\frac{r}{r_c})]^n$, r_c the core size parameter, r_t the radius at which mass is tidally stripped from the galaxy, δ setting the density slope beyond r_t , and

$$M_{cNFWt}(< r) = \begin{cases} M_{cNFW}(< r) & r < r_t \\ M_{cNFW}(r_t) + 4\pi \rho_{cNFW}(r_t) \frac{r_t^3}{3-\delta} \left[\left(\frac{r}{r_t}\right)^{3-\delta} - 1\right] & r > r_t \end{cases} \quad (2.8)$$

where $M_{cNFW}(< r) = M_{NFW} f^n$ ([35]). As mentioned previously, the NFW profile is described by (1.5) This change from NFW to coreNFW_{tides} is to 1) force a core-like center for the mass profile, and 2) to account for dwarf galaxies having had the outer dark matter halo profile impacted by tidal stripping.

2.2 Modification

The current GravSphere code and data does not include any proper motion data for Draco. As such, the current API does not include any handling of it. I have written an extension of the GravSphere Draco API to account for this.

This will be useful in the future when more proper motion data will be available and can easily be analysed using this API (see Appendix B).

Chapter 3

Data

3.1 Proper Motions

Given the distance to Draco, there are only three telescopes available to measure proper motions: Gaia, HST, and JWST. Gaia's proper motion uncertainties are too large for precise measurements and JWST has not been in service for long enough to have produced the epochs necessary. In order to measure the individual proper motions of the stars at least two epochs need to be combined. The HST is stable, has high spatial resolution, and has been in service for long enough to be the currently only viable telescope to measure precise internal proper motions nearby when combining at least two epochs of data ([43]).

The dataset for Draco's proper motions ([42], [43]) was obtained via 4 epochs of Hubble Space Telescope (HST) imaging over 18 years, primarily within the HSTPROMO (HST Proper Motions) Collaboration. It includes two fields that had three and four epochs of imaging over the past two decades.

Initially, the fields F1 and F2 were observed (Figure 3.1.1) by Sohn et al.(2017) ([39]).

F1, located at a right ascension of 17:21:01,34 and a declination of +57:58:38,50, had been observed for three epochs in 2004, 2006, and 2013.

F2, located at a right ascension of 17:21:51,69 and a declination of +58:01:41,00, had been observed for two epochs in 2004, and 2012.

With the work of Vitral et al.(2024) ([43]), an additional epoch was observed in 2022 with the same filter (F606W) and telescope positioning.

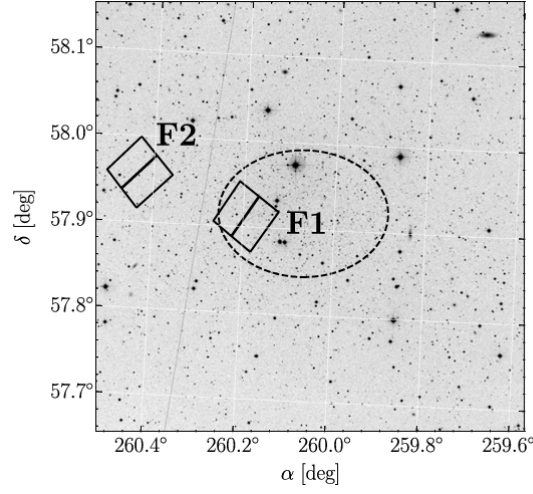


Figure 3.1.1: Fields F1 and F2 from which proper motions were observed. The black ellipse showcases Draco’s half-number radius ([43]).

3.2 Line-of-Sight

For the line-of-sight dataset, I have used data from Walker et al.(2023) ([46]). This spectrographic data (Figure 3.2.1) was taken by the Hectochelle spectrograph at the 6.5 m MMT telescope. In comparison to older data, this set provides higher spectrographic resolution given the high resolution of ~ 34000 ([40]), and multi-epoch temporal coverage from 2005 to 2022 with up to 13 measurements per source. The lack of multi-epoch temporal coverage leads to worse accuracy when inferring the intrinsic velocity dispersions of the stars, impacting the result for the dynamical mass of the system.

As the catalog is much larger than that of Vitral et al.(2024) ([42]) (3.3), I look at it in various subsets: stars in the same field as the proper motions, one of identical size to the proper motions, and a comparison set (see Table 3.3).

There are three line-of-sight sets of relevance:

Draco Line-of-sight Data

- confirmed members of Draco from the total set (Line-of-sight Data), resulting in 422 stars, which is about 4% of the total set
- full set includes all stars in direction of Draco, not just Draco members
- $N_{bin} = 30$, with N_{bin} being the number of stars per bin

- stars distributed across Draco
- this set has the most information of all line-of-sight sets and is the primary comparison to the proper motions results

PM (proper motions) Fields LOS (line-of-sight) Data

- stars in the same area as both proper motions fields. There are no overlapping stars in the same fields for line-of-sight and proper motions, meaning that there are no stars for which there are both proper motion and line-of-sight measurements
- 56 stars
- $N_{bin} = 28$
- this set is used to compare line-of-sight and proper motions for stars in the same limited distribution

Comparison Field LOS Data

- stars in one field roughly the same size as the proper motions fields combined with roughly the same amount of stars, positioned at about the same distance in the opposite direction
- 62 stars
- $N_{bin} = 28$
- this set is used to discuss the impact of set size in comparison with the previous sets

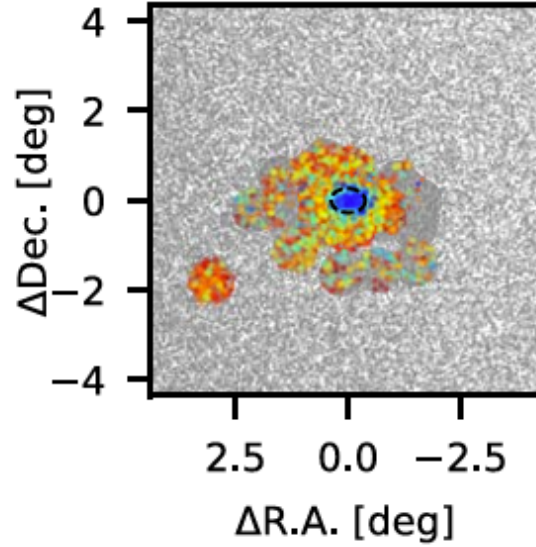


Figure 3.2.1: Locations of data observation are showcased by the colourful points. The black ellipse has a semimajor axis of $2R_{half}/\sqrt{1-\epsilon}$, with R_{half} the projected half-light radius and ϵ the observed ellipticity([46]).

3.3 Calculating the Surface Density

Since the data for the proper motions in Draco not only includes fewer stars than the data for the line-of-sight velocities, but those stars are also close to the center, when determining the surface density, I used the data for Draco ([44]) already included in the data GravSphere is set to run on to define the surface density of Draco.

This extrapolation is also applied for the line-of-sight data for comparison. All results have the same output for the surface density (Figure 3.3.2).

GravSphere is not made to run with one bin only and does not give reliable fits for the number of stars per bin N_{bin} a lot smaller than 30. The boundaries of the proper motions fields given by the stars lead to a sample of 56 stars in the line-of-sight survey. In order to have results of comparable reliability, I have relaxed the boundaries slightly to arrive at a sample size of approximately 60. Both Same Field Data and Comparison Data have been binned with $N_{bin} = 28$, everything else with $N_{bin} = 30$. The proper motions data is binned with $N_{bin} = 50$.



Figure 3.3.1: Positions of the stars of the different datasets.

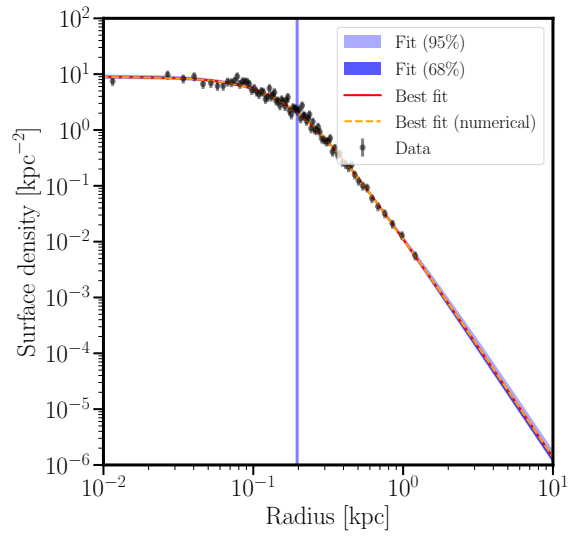


Figure 3.3.2: Surface density as a function of radius. The blue vertical line indicates the half-light radius.

Dataset	los/pm	#stars	source	selection criteria
Proper Motions Data	pm	364	[42],[43]	full set
Line-of-sight Data	los	10232	[46]	all stars in Draco field, including many back- and foreground stars
Draco Line-of-sight Data	los	422	[46],[43]	confirmed members of Draco
PM Fields LOS Data	los	56	[46]	stars in the same field as Proper Motions Data (4.2.2)
Comparison Field Data	los	62	[46]	comparison field of comparable size and number of stars to Same Field Data (4.2.3).

Table 3.3.1: Datasets used in this project.

Chapter 4

Results and Discussion

4.1 Priors

These are the priors used for all datasets: These are the unchanged priors used in Read et al.(2018) ([35]).

parameter	minimum value	maximum value
$\log M_{200}$	7.5	11.5
c	1.0	50.0
r_c	10^{-2}	10.0
n	0.0	1.0
r_t	1.0	20.0
δ	3.01	5.0
β_0	-0.01	0.01
β_∞	-0.1	1.0

Table 4.1.1: M_{200} : Virial mass, c : concentration, r_c : core radius in kpc, n : parameter describing the shape of the dark matter density profile, r_t, δ : tidal radius [kpc] where ρ steepens at rate $\propto r^{-\delta}$, β : anisotropy profile

4.2 Line-of-Sight

4.2.1 Draco Line-of-Sight Data

Using the entire set of stars in Draco’s direction (Line-of-sight Data), including foreground and background stars, the inferred halo mass would reach up to

$10^{10} M_{\odot}$ at maximum radius (Figure 4.2.1, green). This is much too high compared to the literature value of order $10^9 M_{\odot}$ ([33], discussed more in 4.2.1.1). Using only Draco Line-of-sight Data, the result is more realistic (Figure 4.2.2).

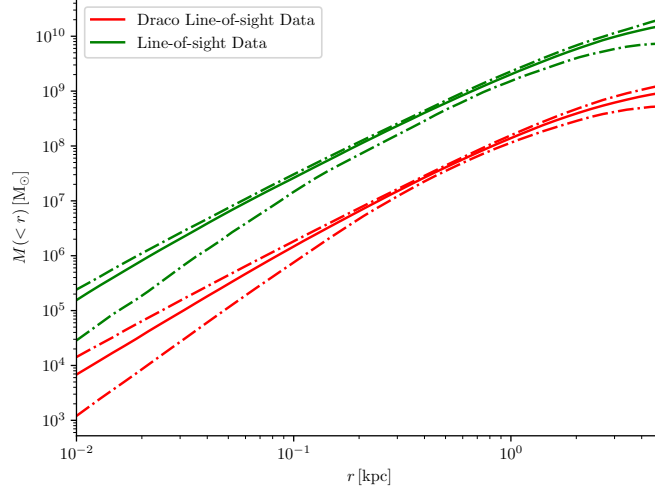


Figure 4.2.1: Comparison of the mass profiles for Line-of-sight Data and Draco Line-of-sight Data.

The maximum here is about $10^9 M_{\odot}$, which aligns much better with the literature. The dark matter fit is very accurate just past the half-light radius with small errors, at both minimum and maximum radius the errors are the biggest (see Figure 4.3.8).

This dataset results in the Virial mass $M_{200}^{Draco} = (3.9^{+7.3}_{-1.8}) \cdot 10^9 M_{\odot}$ and the concentrations $c_{Draco} = 16.61^{+19.11}_{-5.88}$ (see Figure 4.2.3). The full set of physical parameters is available in the appendix (A.0.1). There is some degeneracy between M_{200} and c ; for larger values of M_{200} , c is smaller, and for smaller values of M_{200} , c is larger. This degeneracy is well constrained.

4.2.1.1 Comparison with Literature

The paper by Read and Erkal(2019) ([33]) uses two different methods to determine M_{200} of Draco, resulting in M_{200}^{abund} for abundance matching, and M_{200}^{dyn} for dynamical estimation using data from McConnachie(2012) ([25]), Apericio et al.(2001) ([1]), and Walker et al.(2015) ([45]) with 468 stars at similar distribution.

For the method of abundance matching, Read and Erkal(2019) ([33]) took the

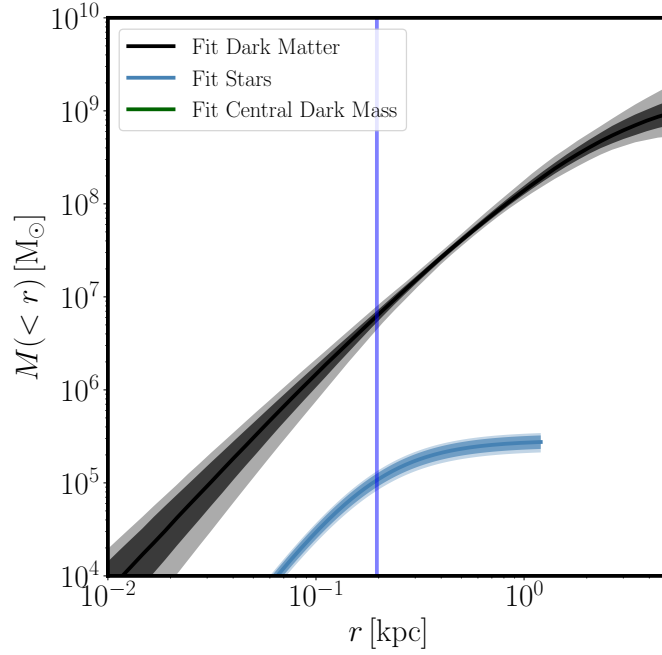


Figure 4.2.2: Mass profile for Draco Line-of-sight Data showing the dark matter and stellar fit with the 1σ and 2σ -environments represented by the darker and lighter filled-in area respectively. The blue line indicates the half-light radius.

cumulative number density function for the star formation rate $N_{<SFR>}$ and used it to map $<SFR>(N)$ to $M_{200}(N)$. With this, they arrived at a value of $M_{200}^{abund} = (1.8 \pm 0.7) \cdot 10^9 M_{\odot}$.

Alternatively, when determining M_{200} using GravSphere for the dynamic estimation of the pre-infall halo mass, their results are $M_{200}^{dyn} = (3.5^{+1.5}_{-1.0}) \cdot 10^9 M_{\odot}$. Their priors differ slightly as they study multiple dwarfs and thus need to account for lower boundaries. Here, $8.5 < \log M_{200} < 10.5$, which is a smaller range by 2, and $9 < c < 24$, which is a smaller range by 34 (see more in Read and Erkal(2019) ([33]), Section 4.2.2).

The value of M_{200}^{Draco} agrees very well with M_{200}^{dyn} . These values differ by $0.4 \cdot 10^9 M_{\odot}$ and are easily within 1σ of each other. They were determined using the same method with similar priors and a slightly larger (by 46) dataset, and so it is a good sign for the results to be consistent. However, there is no agreement with M_{200}^{abund} , not for M_{200}^{dyn} and just barely for M_{200}^{Draco} .

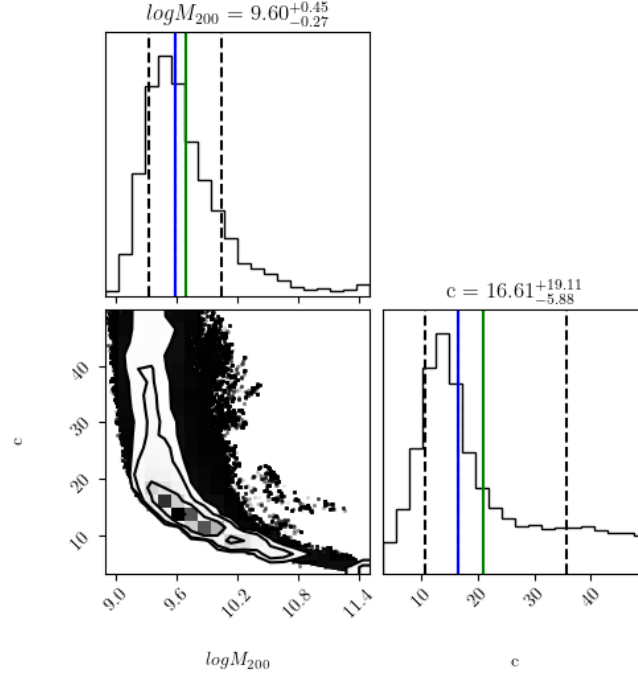


Figure 4.2.3: M_{200} and c values for Draco Line-of-sight Data. The blue line corresponds to the median value and the green line to the mean value. Errors are the 1σ -environment.

4.2.2 PM Fields LOS Data

To later compare with the proper motions results, I have also taken stars from the Draco Line-of-sight data that are roughly in the same area as the proper motion fields. They cannot match each other perfectly; the stars are distributed differently (there are no stars with both proper motion and line-of-sight data) and taking the same boundaries would result in a set of 30 stars. GravSphere is not reliable with bins containing significantly fewer stars than 30, and needs at least two bins to run. Because of this, I have relaxed the boundaries of the fields to end up with a dataset of 56 stars and 28 stars per bin. This results in two fields in a similar position (see Figure 3.3.1) with $260.10 < \text{ra} < 260.25$, $57.85 < \text{dec} < 57.97$ and $260.36 < \text{ra} < 260.49$, $57.90 < \text{dec} < 58.04$ defining the boundaries. The resulting mass profile is shown in Figure 4.2.4. Similarly to Draco Line-of-sight, the fit is best just outside the half-light radius and worst at maximum and minimum radius (see more in Section 4.2.2.1). The fields of stars are located close to the half-light radius (see Figure 4.2.6). Therefore, the fit at maximum and minimum radii need to be inferred via extrapolation and have higher uncertainty

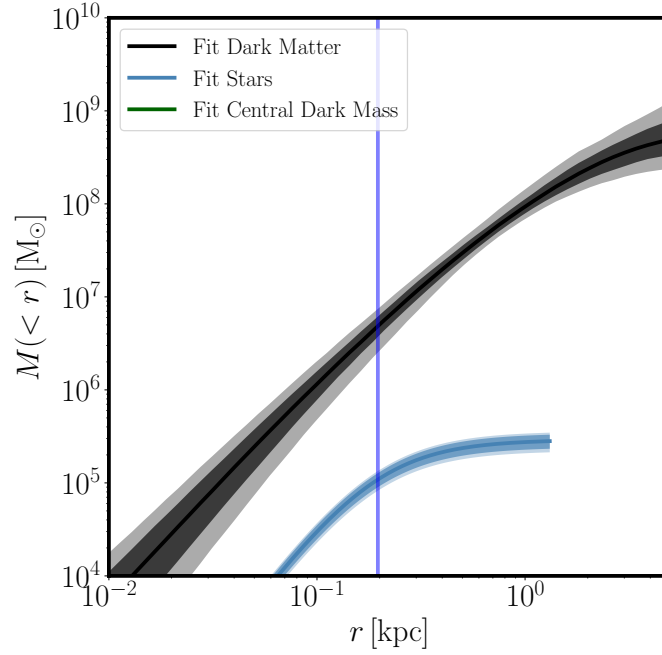


Figure 4.2.4: Mass profile for PM Fields LOS Data showing the dark matter and stellar fit with the 1σ and 2σ -environments represented by the darker and lighter filled-in area respectively. The blue line indicates the half-light radius.

(see Figure 4.3.8).

Here, the result for $M_{200}^{FieldsLOS} = (1.6_{-0.8}^{+4.6}) \cdot 10^9 M_{\odot}$ and $c = 18.49_{-8.60}^{+15.91}$ (see Figure 4.2.5). The relation between M_{200} and c , while still present, is less confined.

4.2.2.1 Comparison with other results

$M_{200}^{FieldsLOS}$ differs from M_{200}^{abund} by $0.2 \cdot 10^9 M_{\odot}$; those two results agree well with each other. However, unlike M_{200}^{abund} , thanks to its high upper limit, it does fall into the 1σ -environment of M_{200}^{dyn} and M_{200}^{Draco} .

The PM Fields LOS dataset has only 13.3% the amount of stars as Draco Line-of-sight. It is likely that the larger error is due to the much smaller amount of stars (see Section 4.2.3). Comparing the two datasets (Figure 4.2.6), they agree closely until the half-light radius. Most of the stars in the PM Fields LOS set are at a closer radius than the half-light radius, which explains the better fit. Beyond the half-light radius, the two fits start to diverge more but stay within

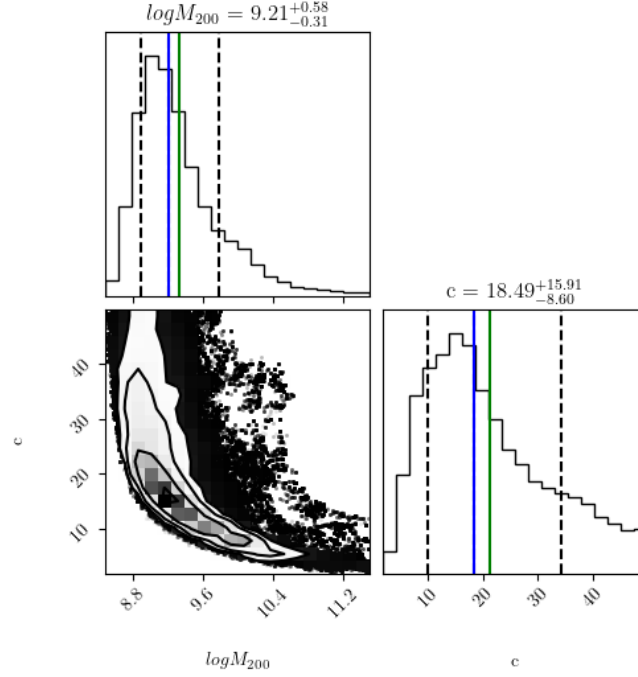


Figure 4.2.5: M_{200} and c values for PM Fields LOS Data. The blue line corresponds to the median value and the green line to the mean value. Errors are the 1σ -environment.

each others error environments, though it is close at about 1 kpc.

4.2.2.2 Only One Field

There are two proper motion fields and so PM Fields LOS, as is apparent from the boundaries and Figure 3.3.1 also has two fields: a big one close to the center of Draco, and a smaller one further out containing 9 stars. Given that $N_{bin} = 28 \gg 9$, and there only being 2 bins, the greater distance of just 9 of them might skew the distance per bin to larger values. To test the impact of this, I have also run GravSphere with only the inner field at a size of 46 stars to make two full bins at $N_{bin} = 23$. Details on this can be found in the appendix (A). For the Inner Field LOS dataset, $M_{200}^{Inner} = (1.1^{+3.0}_{-0.6}) \cdot 10^9 M_{\odot}$. This value deviates from $M_{200}^{FieldsLOS}$ by only $0.5 \cdot 10^9 M_{\odot}$, and is slightly better constrained. The errors are still big enough to fit within every other result for M_{200} . As can be seen in Figure 4.2.7, both fits agree well in general. For radii below the half-light radius, they are nearly identical. This is where the majority of stars are positioned for PM Fields LOS as well as all of the stars for Inner Field LOS. At larger radii,

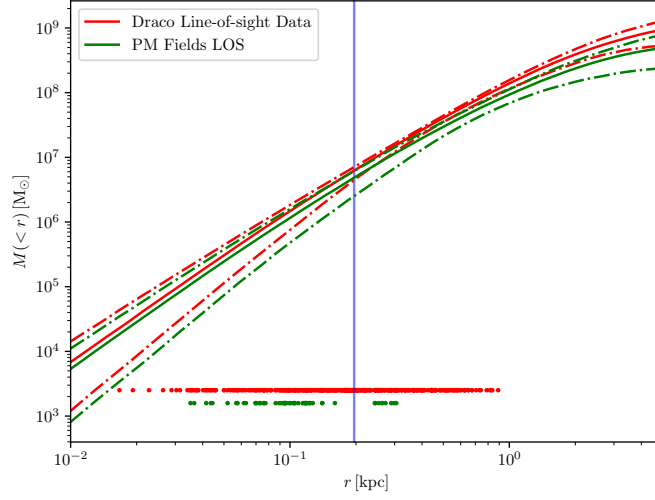


Figure 4.2.6: Comparison of the dark matter mass profile for Draco Line-of-sight (red) and PM Fields LOS (green) data. The vertical line indicates the half-light radius and the dashed lines the 1σ -environments. The stellar positions are displayed in corresponding colour as dots at corresponding radius at arbitrary height.

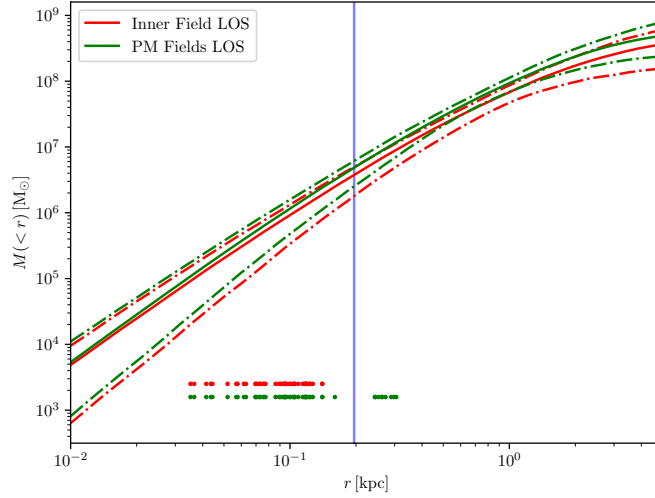


Figure 4.2.7: Comparison of the dark matter mass profile for Inner Field LOS (red) and PM Fields LOS (green) data. The vertical line indicates the half-light radius and the dashed lines the 1σ -environments. The stellar positions are displayed in corresponding colour as dots at corresponding radius at arbitrary height.

the two fits start to differ more, though are still in agreement with each other. At maximum radius, the Inner Field LOS fit has a lower value than PM Fields LOS.

In conclusion, the PM Fields LOS has a better fit at large radii than the Inner Field LOS. The stars in the smaller field do not interfere negatively with the final fit, but improve it.

4.2.3 Comparison Field

To see the impact of the much smaller sample size of PM Fields LOS dataset in comparison with Draco Line-of-sight, I have also run GravSphere with a dataset of comparable size to PM Fields LOS but at a different location. The boundaries of this field are defined by $259.85 < ra < 260.10$ and $57.90 < dec < 58.00$. It is at roughly the same distance from the center as the bigger field in PM Fields LOS. The mass profile shows the same behaviour as all previous ones (Figure 4.2.8). Here, the result for $M_{200}^{Comp} = (2.9^{+7.1}_{-1.7}) \cdot 10^9 M_{\odot}$ and $c = 19.22^{+15.77}_{-8.56}$ (see Figure

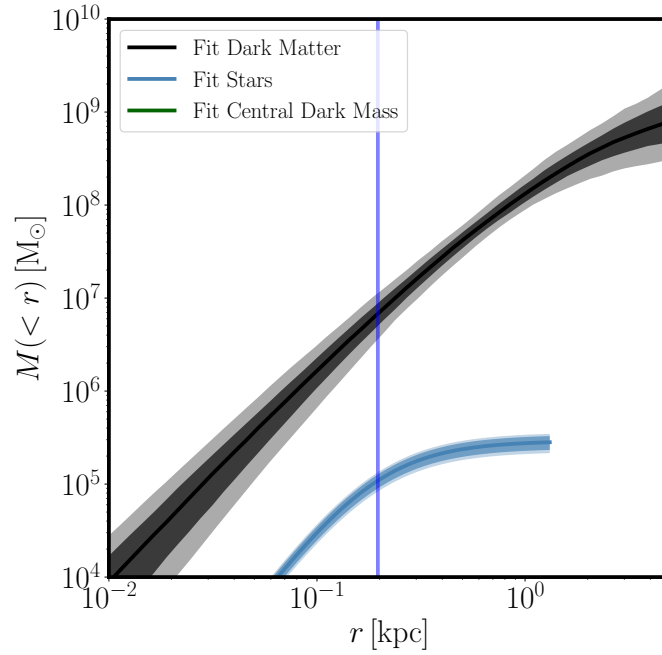


Figure 4.2.8: Mass profile for Comparison Field LOS Data showing the dark matter and stellar fit with the 1σ and 2σ -environments represented by the darker and lighter filled-in area respectively. The blue line indicates the half-light radius.

4.2.9). The relation between M_{200} and c , while still present, is less confined.

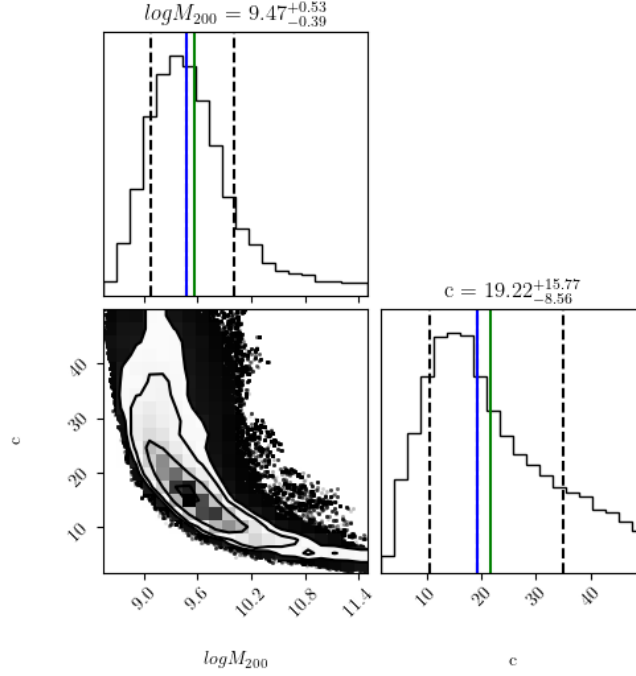


Figure 4.2.9: M_{200} and c values for Comparison Field LOS Data. The blue line corresponds to the median value and the green line to the mean value. Errors are the 1σ -environment.

4.2.3.1 Comparison with other results

While M_{200}^{Comp} agrees with all previous results on account of large errors, it lands in the middle in terms of value. It differs from M_{200}^{dyn} and M_{200}^{Draco} by $0.6 \cdot 10^9 M_{\odot}$ and $1 \cdot 10^9 M_{\odot}$ respectively at the top end and from $M_{200}^{FieldsLOS}$ by $1.3 \cdot 10^9 M_{\odot}$ at the bottom end.

Comparing the Comparison Field LOS and PM Fields LOS datasets (see Figure 4.2.10), both fits behave similarly with a constant mass offset between the two. The error-environments are of similar size (see Figure 4.3.8); the environment for PM Fields LOS briefly shrinks at around the half-light radius and expands back at 1 kpc. Since there are a few stars in the field at a larger distance, the error stays smaller than that of Comparison Field LOS by about 0.14. The number of stars in both sets is very similar (56 for PM Fields LOS and 62 for Comparison Field

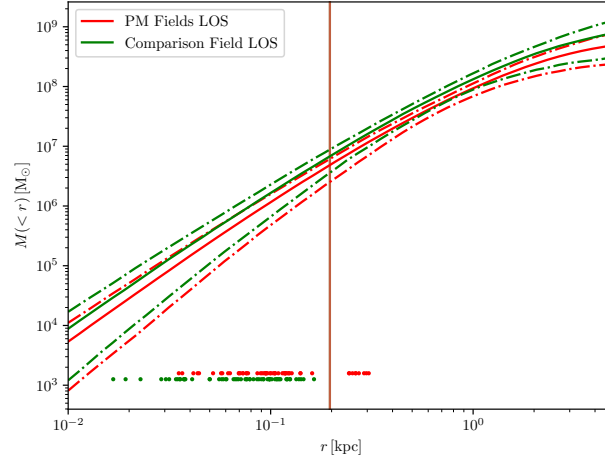


Figure 4.2.10: Comparison of the dark matter mass profile for Comparison Field LOS (green) and PM Fields LOS (red) data. The vertical line indicates the half-light radius and the dashed lines the 1σ -environments. The stellar positions are displayed in corresponding colour as dots at corresponding radius at arbitrary height.

LOS), however, at the same value for N_{bin} , there are three bins for Comparison Field LOS and 2 for PM Fields LOS.

Since both datasets have a similar amount of stars at similar positions it is to be expected that the mass profile would behave similarly. My results confirm that the offset can be explained by the very small datasets and thus introduce random scatter.

After looking at the comparison of the mass profiles for middle and lower end of M_{200} , the comparison of middle and upper end is shown in Figure 4.2.11. These fits are much closer together and show consistent behaviour. At lower radii, the Comparison Field LOS set infers a greater enclosed mass than Draco Line-of-sight, at large radii it is the opposite. They cross just beyond the half-light radius. While the error environments at small radii are similar - this is where all stars in the comparison field lie - the error is significantly smaller for Draco Line-of-sight at large radii. This is to be expected, given that this dataset covers much larger distances for stellar positions, allowing for a better fit.

Given that even at a radius at which there are still many Comparison Field LOS stars, the error for Draco Line-of-sight decreases faster which is most likely

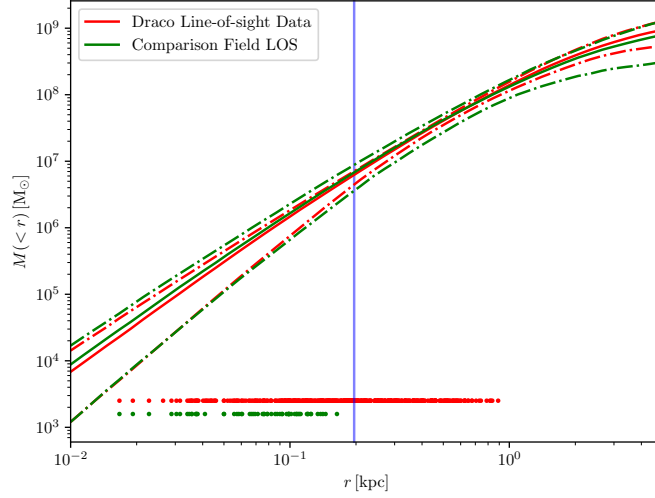


Figure 4.2.11: Comparison of the dark matter mass profile for Comparison Field LOS (green) and Draco Line-of-sight (red) data. The vertical line indicates the half-light radius and the dashed lines the 1σ -environments. The stellar positions are displayed in corresponding colour as dots at corresponding radius at arbitrary height.

due to the much greater amount of stars. While it is important to have a good distribution of stars to cover a large range of radii and limit extrapolation, having more stars also does improve the fit marginally.

4.3 Proper Motions

Running GravSphere with proper motions still requires kinematic data input. For maximum information, I have run GravSphere with proper motions ([42]) in combination with the Draco Line-of-sight kinematic input (PM+Draco), and for comparison, proper motions in combination with PM Fields LOS as the kinematic input (PM+Fields).

4.3.1 PM+Draco

As can be seen in Figure 4.3.1, the error environments for PM+Draco are very similar to those for line-of-sight only fits, in particular to Draco Line-of-sight (see more in Section 4.3.3). Looking at the values for $M_{200}^{PM+Draco}$ and c (Figure 4.3.2), the result for $c = 16.40^{+17.60}_{-7.32}$ does not fall out of the ordinary compared to previous results, as does $M_{200}^{PM+Draco} = (5.0^{+14.0}_{-2.5}) \cdot 10^9 M_{\odot}$. Both values have a high upper limit.

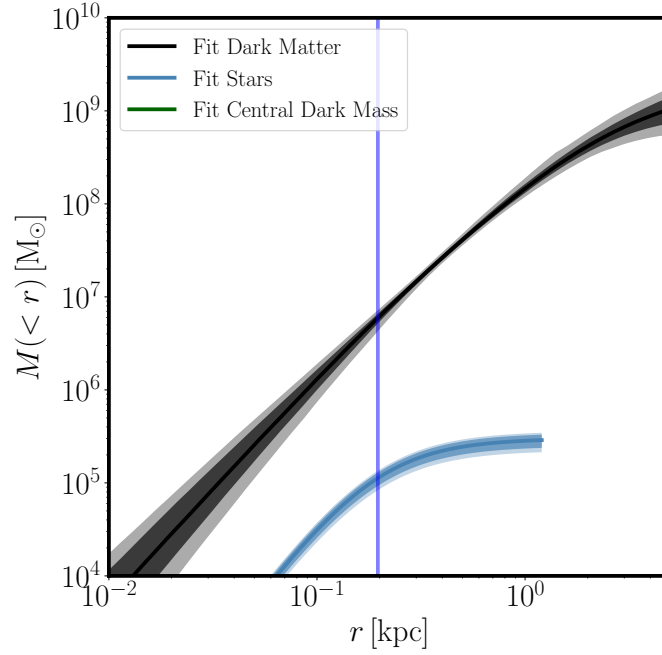


Figure 4.3.1: Mass profile for PM+Draco Data showing the dark matter and stellar fit with the 1σ and 2σ -environments represented by the darker and lighter filled-in area respectively. The blue line indicates the half-light radius.

4.3.2 PM+Fields

Taking only stars from the proper motions fields for the kinematic data results in a similar mass profile (Figure 4.3.4 and 4.3.3). At lower radii, PM+Fields infer a slightly greater enclosed mass, the fit itself being very similar to PM+Draco. It is at large radii where the two fits differ, still barely within each others error environment. PM+Fields falls below PM+Draco past the half-light radius and at maximum radius shows a lower enclosed mass which is also visible in the result of $M_{200}^{PMFields} = (1.3_{-0.6}^{+2.4}) \cdot 10^9 M_{\odot}$ and $c = 20.53_{-7.76}^{+17.93}$ (Figure 4.3.5). The behaviour of the fits in terms of slope is consistent.

4.3.3 Comparison with Other Results

4.3.3.1 Mass Profiles

Comparing line-of-sight and the corresponding proper motions mass profiles (Figure 4.3.6 and 4.3.7), the proper motions plus line-of-sight fits agree with their line-of-sight counterpart, especially at high radii. The lower $M(< r)$ for max-

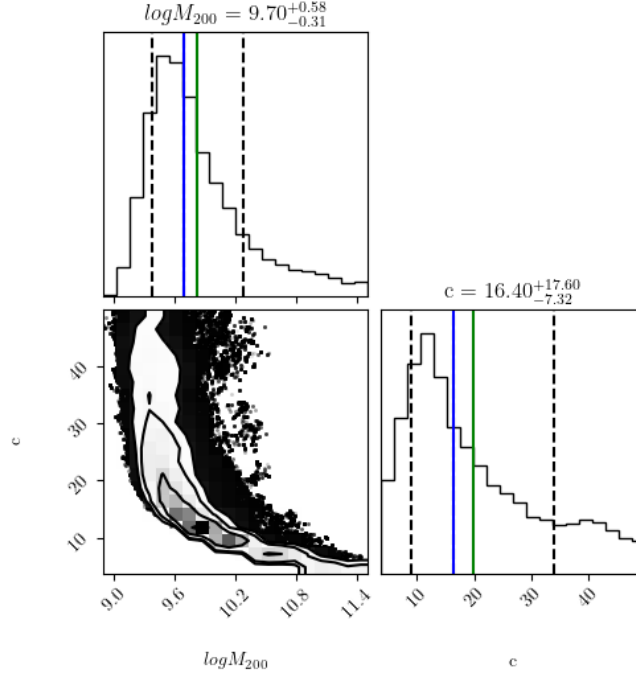


Figure 4.3.2: M_{200} and c values for PM+Draco Data. The blue line corresponds to the median value and the green line to the mean value. Errors are the 1σ -environment.

imum radius of PM+Fields is the same as PM Fields LOS, as is the case for the higher value for the other two sets. The fits all agree at low radii very well, including their error environments. There are no noteworthy differences for PM+Draco and Draco Line-of-sight, and the error at $r < 1$ kpc for PM Fields LOS is significantly larger than that of PM+Fields. Including the proper motions here improves the fit where there is proper motions data consistently up to around the half-light radius by up to 0.38 at $r = 0.24$ kpc (see Figure 4.3.8). A comparison of the relative error on the enclosed mass for all datasets can be seen in Figure 4.3.8.

4.3.3.2 M_{200}

To easily compare all the results for M_{200} , they are compiled in Table 4.3.1. They all agree well with each other. PM+Draco is the largest value at $5.0 \cdot 10^9 M_{\odot}$, and LOS Inner Field the smallest at $1.1 \cdot 10^9 M_{\odot}$. PM+Draco, Draco Line-of-sight and LOS Comparison Field all have noticeably large upper errors. There are no significantly large lower errors.

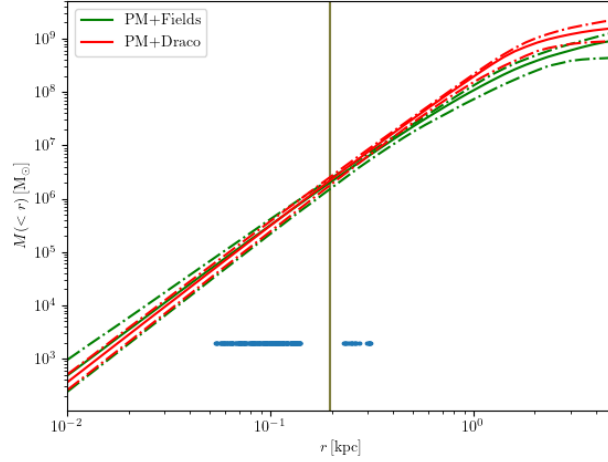


Figure 4.3.3: Comparison of the dark matter mass profile for PM+Fields (green) and PM+Draco (red) data. The vertical line indicates the half-light radius and the dashed lines the 1σ -environments. The stellar positions are displayed as dots at corresponding radius at arbitrary height.

	Dataset	M_{200}	M_{200}^{max}	M_{200}^{min}
Lit	Dyn	3.5	5.0	2.5
	Abund	1.8	2.5	1.1
LOS	Draco	3.9	11.2	2.1
	PM Fields	1.6	6.2	0.8
	Inner	1.1	4.1	0.5
	Comp	2.9	10.0	1.2
PM	+Draco	5.0	19.0	2.5
	+Fields	1.3	3.7	0.7

Table 4.3.1: All values of M_{200} with unit of $10^9 M_{\odot}$.

4.3.3.3 $M_{dark}^{R_{max}}$

I have also compared the results for $M_{dark}^{R_{max}}$. This is the dark matter mass at the radius of the maximum stellar position. Since all datasets with the exception of Draco Line-of-sight do not have stars at distances significantly past the half-light radius, this value would ordinarily be lower by at least an order of magnitude (Appendix A.0.1). For this reason, I have taken $R_{max} = 0.85$ kpc of Draco Line-of-sight for all datasets. The results are seen in Table 4.3.2. The result of Vitriol et al.(2024) ([43], Table 2 and 3) is of order $10^8 M_{\odot}$. This is consistent with my results.

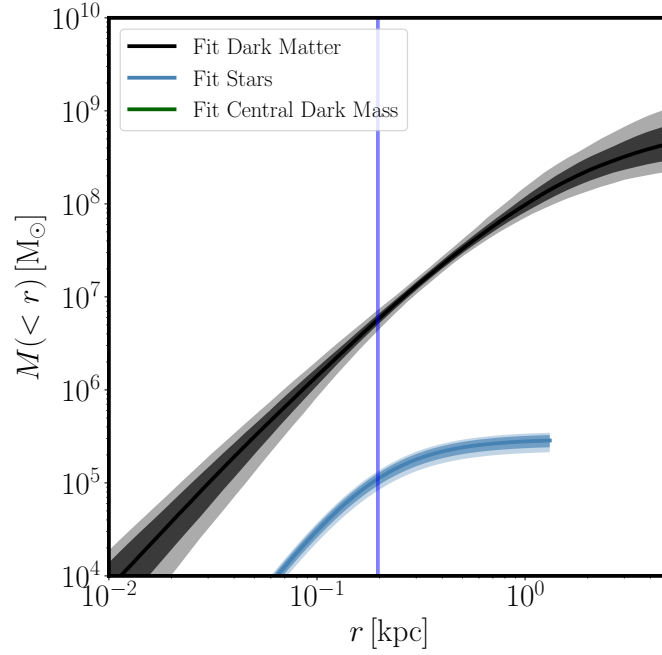


Figure 4.3.4: Mass profile for PM+Fields Data showing the dark matter and stellar fit with the 1σ and 2σ -environments represented by the darker and lighter filled-in area respectively. The blue line indicates the half-light radius.

	Data	$M(< R_{max})[M_{\odot}]$	$M_{max}[M_{\odot}]$	$M_{min}[M_{\odot}]$
LOS	Draco	1.05	1.17	0.89
	PM Fields	0.72	0.86	0.54
	Comp	1.02	1.26	0.70
PM	+Draco	1.10	1.22	0.91
	+Fields	0.77	0.90	0.58

Table 4.3.2: Values for radius of outermost star of Draco Line-of-sight $R_{max} = 0.85 \text{ kpc}$ and the dark matter mass at this radius $M \cdot 10^8$.

4.4 Density Profiles

To discuss the question whether including proper motions helps with solving the core-cusp problem, I compare the dark matter density profiles.

There is no significant difference made by including proper motions (Figure 4.4.1 and Figure 4.4.2). Similarly to the mass profile (Figure 4.3.7), PM Fields LOS

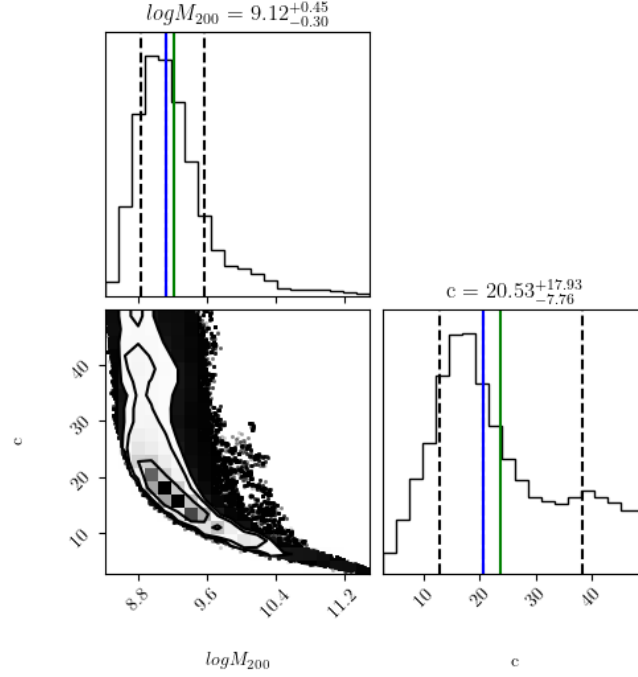


Figure 4.3.5: M_{200} and c values for PM+Fields Data. The blue line corresponds to the median value and the green line to the mean value. Errors are the 1σ -environment.

has a significantly larger error environment at radii below the half-light radius. Looking at the inner region in regards to the core-cusp problem, there is no improvement when considering proper motions. The density at minimum radius is slightly smaller when including proper motions for Draco Line-of-sight, and slightly larger for PM Fields LOS. Both proper motions density profiles agree with each other at low radii (Figure 4.4.3). They differ more at high radii with a steeper slope for PM+Fields between the half-light radius and 1 kpc before they have the same slope again, resulting in a lower density at maximum radius for PM+Fields (and consequently also for PM Fields LOS compared to Draco Line-of-sight) than for PM+Draco. The large errors at high radii for all sets are likely due to extrapolation since there are no stars in any dataset reaching past $r = 1$ kpc.

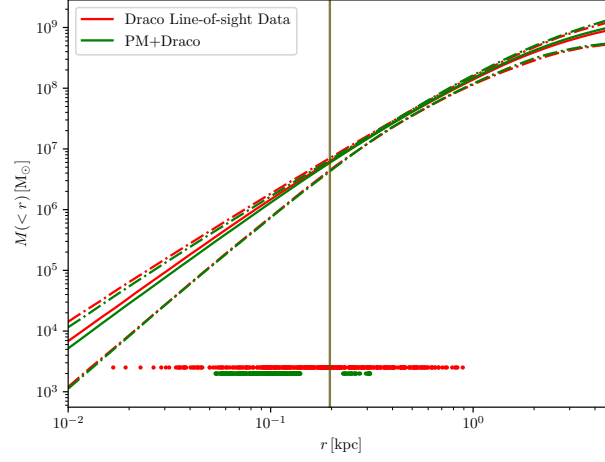


Figure 4.3.6: Comparison of the dark matter mass profile for PM+Draco (green) and Draco Line-of-sight (red) data. The vertical line indicates the half-light radius and the dashed lines the 1σ -environments. The stellar positions are displayed in corresponding colour as dots at corresponding radius at arbitrary height.

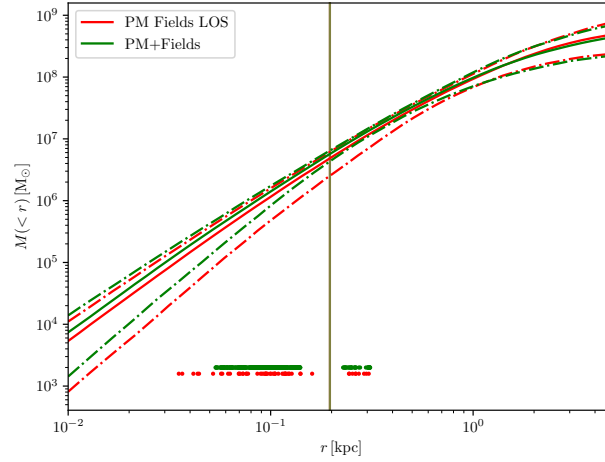


Figure 4.3.7: Comparison of the dark matter mass profile for PM+Fields (green) and PM Fields LOS (red) data. The vertical line indicates the half-light radius and the dashed lines the 1σ -environments. The stellar positions are displayed in corresponding colour as dots at corresponding radius at arbitrary height.

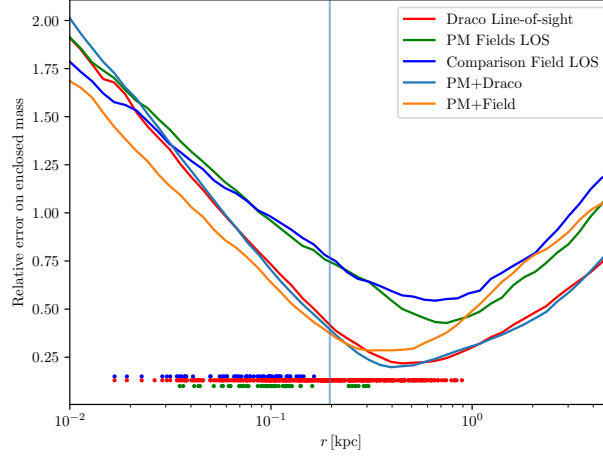


Figure 4.3.8: The relative error on the enclosed mass profile for all datasets. The vertical line indicates the half-light radius and the dashed lines the 1σ –environments. The stellar positions are displayed in corresponding colour as dots at corresponding radius at arbitrary height. The positions of the stars in the proper motions sets are similar to those of PM Fields LOS.

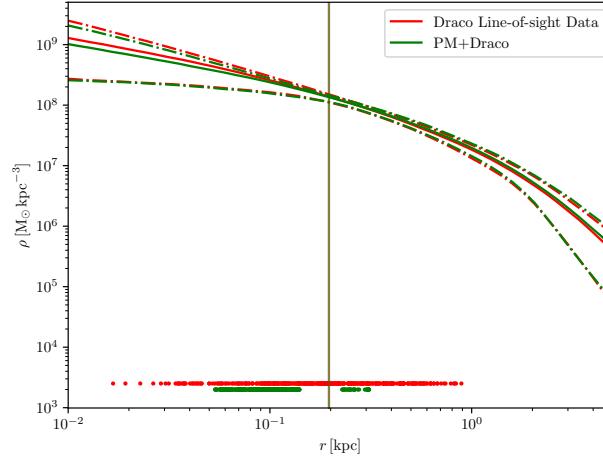


Figure 4.4.1: Comparison of the density profile for PM+Draco (green) and Draco Line-of-sight (red) data. The vertical line indicates the half-light radius and the dashed lines the 1σ –environments. The stellar positions are displayed in corresponding colour as dots at corresponding radius at arbitrary height.

4.5 Summary

The physical properties as produced by GravSphere are summarized in Table 4.5.1.

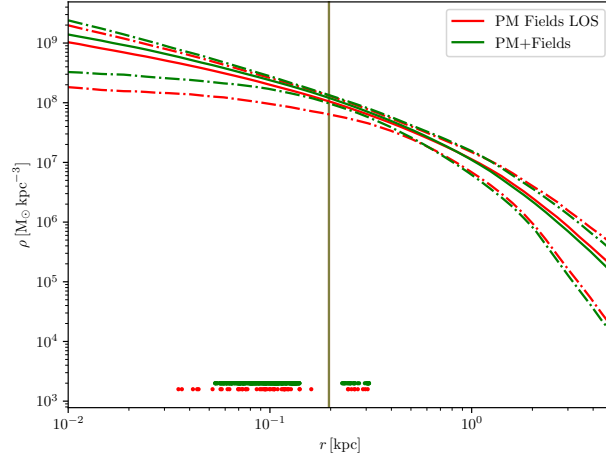


Figure 4.4.2: Comparison of the density profile for PM+Fields (green) and PM Fields LOS (red) data. The vertical line indicates the half-light radius and the dashed lines the 1σ -environments. The stellar positions are displayed in corresponding colour as dots at corresponding radius at arbitrary height.

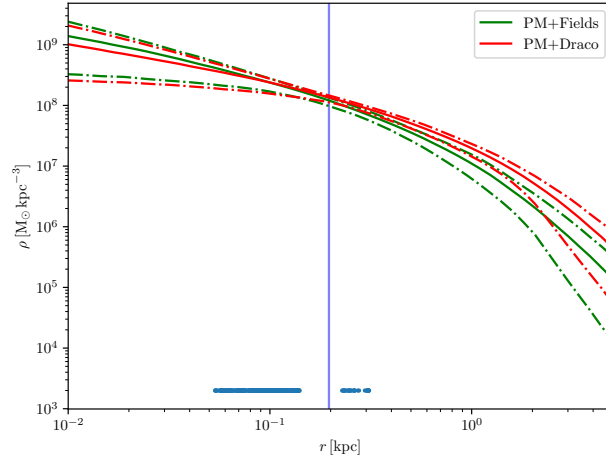


Figure 4.4.3: Comparison of the density profile for PM+Draco (red) and PM+Fields (green) data. The vertical line indicates the half-light radius and the dashed lines the 1σ -environments. The stellar positions are displayed as dots at corresponding radius at arbitrary height.

$\log M_{200}$ is the Virial mass and has been discussed in previous sections in detail. All values agree well with each other as well as the literature values.

c is the concentration. All values agree with each other though they also have large upper errors close to the value itself.

$\log r_c$ is the core radius. While all values agree well with each other, in part again due to very large upper and lower errors, PM+Draco has a noticeably lower mean value.

n describes the slope of the dark matter density profile and is between 0 and 1. The results agree well with each other at only a difference of 0.04 between the largest value of 0.46 for PM+Draco and 0.42 for PM+Fields, PM Fields LOS, and Comparison Fields LOS.

$\log r_t$ is the tidal radius. Similarly to n , all results are close together with a difference of 0.08 between 0.68 for Draco LOS and 0.60 for PM+Draco.

	PM+Draco	PM+Fields	Draco LOS	PM Fields LOS	Comparison Field LOS
$\log M_{200}$	$9.70^{+0.58}_{-0.31}$	$9.12^{+0.45}_{-0.30}$	$9.54^{+0.59}_{-0.37}$	$8.84^{+0.55}_{-0.35}$	$9.54^{+0.60}_{-0.37}$
c	$16.40^{+17.60}_{-7.32}$	$20.53^{+17.93}_{-7.76}$	$15.20^{+14.20}_{-6.82}$	$17.37^{+14.03}_{-8.34}$	$18.01^{+14.68}_{-8.46}$
$\log r_c$	$-0.15^{+0.71}_{-1.01}$	$-0.43^{+0.83}_{-1.03}$	$-0.63^{+1.02}_{-0.92}$	$-0.75^{+1.13}_{-0.85}$	$-0.95^{+1.01}_{-0.95}$
n	$0.46^{+0.29}_{-0.26}$	$0.42^{+0.34}_{-0.28}$	$0.44^{+0.35}_{-0.29}$	$0.42^{+0.36}_{-0.28}$	$0.42^{+0.36}_{-0.28}$
$\log r_t$	$0.60^{+0.48}_{-0.37}$	$0.61^{+0.46}_{-0.42}$	$0.68^{+0.43}_{-0.41}$	$0.61^{+0.45}_{-0.41}$	$0.67^{+0.43}_{-0.41}$

Table 4.5.1: Summary of physical properties as produced by GravSphere for all relevant datasets.

Chapter 5

Conclusion

In this thesis I discussed whether 1) including proper motions improved the dark matter profile calculated by GravSphere and 2) proper motions help solve the core-cusp problem.

Given that 1) the current proper motions data, while similar in size to the line-of-sight data, is also not well spatially distributed across the galaxy, and 2) all analysis has been done using one code, GravSphere, which assumes spherical symmetry, it is important to note that all conclusions can only be made with those constraints in mind.

- Adding proper motions to a large, well distributed line-of-sight data set (Draco Line-of-sight) does not improve the fit of the dark matter mass profile. It does, however, reduce the error at the radii where there is proper motions data when adding it to a smaller, centrally distributed line-of-sight set (PM Fields LOS) by up to 0.38.
- The physical properties fit by GravSphere do not have better boundaries when including proper motions.
- Adding proper motions to GravSphere does not impact the dark matter density profiles and does not help toward solving the core-cusp problem.
- Compared to previous proper motions analysis by Vitral et al.(2024) ([43]) where the proper motions data was published ([42]), they used both spherical Jeans modeling (MAMPOSST-PM) and axisymmetric Jeans modeling (JAMPY) and arrived at a core-like profile for the spherical model and a cusp-like profile for the axisymmetric model. The version of GravSphere I

used forces a core-like density profile, despite assuming spherical symmetry for both line-of-sight and proper motions.

As part of this work I have also confirmed that a smaller data set results in larger errors, but is still consistent with larger data sets, and that having significantly fewer stars in comparison to the bin size at a significantly different radius does not negatively impact the dark matter mass fit and physical parameters.

5.1 Future Work

Currently, proper motions data for Draco is only available in two fields close to the centre on one side of the galaxy. Given that radii further away from data points are more uncertain given the necessary extrapolation, this analysis can be repeated once proper motions data that is well distributed across a galaxy is available.

References

- [1] A. Aparicio, R. Carrera, and D. Martínez-Delgado. “The star formation history and morphological evolution of the Draco dwarf spheroidal galaxy”. In: *ApJ* (2001).
- [2] W. Baade and H. H. Swope. “The Draco system, a dwarf galaxy”. In: *The Astronomical Journal* (1961).
- [3] A. M. Brooks et al. “A baryonic solution to the missing satellites problem”. In: *The Astrophysical Journal* (2013).
- [4] D. V. Bugg. “MOND - a review”. In: *Canadian Journal of Physics* (2015).
- [5] J. S. Bullock and M. Boylan-Kolchin. “Small-Scale Challenges to the CDM Paradigm”. In: *Annual Review of Astronomy and Astrophysics* (2017).
- [6] S. Capozziello and M. De Laurentis. “Extended theories of gravity”. In: *Physics Reports* (2011).
- [7] CERN. *Estimated matter-energy content of the Universe*. 2019. URL: <https://cds.cern.ch/record/2665176/>.
- [8] Corinne Charbonnel. “A baryometer is back”. In: *Nature* (2002).
- [9] D. Clowe et al. “A Direct Empirical Proof Of The Existence Of Dark Matter”. In: *ApJ* (2006).
- [10] M. L. M. Collins et al. “Andromeda XXI - a dwarf galaxy in a low-density dark matter halo”. In: *Mon. Not. R. Astron. Soc.* (2021).
- [11] Jan Conrad. *Indirect Detection of WIMP Dark Matter: a compact review*. 2014. arXiv: 1411.1925 [hep-ph]. URL: <https://arxiv.org/abs/1411.1925>.
- [12] J. de Swart, G. Bertone, and J. van Dongen. “How Dark Matter Came to Matter”. In: *Nature Astronomy* 1.3 (2017).

- [13] Q. Decant et al. “Simulation-based inference on warm dark matter from HERA forecasts”. In: *Journal of Cosmology and Astroparticle Physics* (2025).
- [14] ESA. *ESA Gaia*. URL: <https://www.cosmos.esa.int/web/gaia/home>.
- [15] A. Genina et al. “To β or not to β : can higher order Jeans analysis break the mass-anisotropy degeneracy in simulated dwarfs?” In: *Mon. Not. R. Astron. Soc.* (2020).
- [16] H. Haekstra and B. Jain. “Weak gravitational lensing and its cosmological applications”. In: *Annu. Rev. Nucl. Part. Sci.* (2008).
- [17] E. Halley. “Considerations of the change of latitude of some principal fixes stars”. In: *Phil. Trans. Roy. Soc.* (1718).
- [18] P. W. Hodge. “Distribution of stars in the Draco dwarf galaxy”. In: *The Astronomical Journal* (1964).
- [19] W. Huggins. “Further observations on the spectra of some of the stars and nebulae with an attempt to determine therefrom whether these bodies are moving towards or from the Earth”. In: *Phil. Trans. Roy. Soc.* (1868).
- [20] J. H. Jeans. “The motions of stars in a Kapteyn universe”. In: *Mon. Not. R. Astron. Soc.* (1922).
- [21] F. Jiang and F. C. van den Bosch. “Comprehensive assessment of the too big to fail problem”. In: *Mon. Not. R. Astron. Soc.* (2015).
- [22] A. Lipnicky and S. Chakrabarti. “Is the Vast Polar Structure of Dwarf Galaxies A Serious Problem For Lambda CDM?” In: *Mon. Not. R. Astron. Soc.* (2016).
- [23] E. W. Maunder. “The motions of stars in the line of sight”. In: *The Observatory* (1885).
- [24] N. U. Mayall and L. H. Aller. “The Rotation of the Spiral Nebula Messier 33”. In: *The Astrophysical Journal* (1942).
- [25] A. W. McConnachie. “The observed properties of dwarf galaxies in and around the local group”. In: *ApJ* (2012).
- [26] M. Milgram. “Dynamics with a non-standard inertia-acceleration relation: an alternative to dark matter”. In: *Ann. Phys.* (1994).
- [27] Julio F. Navarro, Carlos S. Frenk, and Simon D. M. White. “The Structure of Cold Dark Matter”. In: *The Astrophysical Journal* (1996).

- [28] Q. L. Nguyen et al. “3-3-1 Self interacting dark matter and the galaxy core-cusp problem”. In: *Bulletin of the American Physical Society* (2018).
- [29] A. B. Pace. *The Local Volume Database: a library of the observed properties of nearby dwarf galaxies and star clusters*. 2024. arXiv: 2411.07424 [astro-ph.GA]. URL: <https://arxiv.org/abs/2411.07424>.
- [30] M. Pawlowski. “Phase-Space Correlations Among Systems of Satellite Galaxies”. In: *MDPI* (2021).
- [31] J. L. Pawsey. “Observation of a Line in the Galactic Radio Spectrum: The Interstellar Hydrogen Line at 1,420 Mc./sec., and an Estimate of Galactic Rotation”. In: *Nature* (1951).
- [32] M. Perryman. “The history of Astrometry”. In: *EPJ* (2012).
- [33] J. I. Read and D. Erkal. “Abundance matching with the mean star formation rate: there is no missing satellites problem in the Milky Way above $M_{200} \sim 10^9 M_{\odot}$ ”. In: *Mon. Not. R. Astron. Soc.* (2019).
- [34] J. I. Read and P. Steger. “How to break the density-anisotropy degeneracy in spherical stellar systems”. In: *Mon. Not. R. Astron. Soc.* (2017).
- [35] J. I. Read, M. G. Walker, and P. Steger. “The case for a cold dark matter cusp in Draco”. In: *Mon. Not. R. Astron. Soc.* (2018).
- [36] V. Rubin and K. Ford. “Rotation of the Andromeda Nebula from a Spectroscopic Survey of Emission Regions”. In: *The Astrophysical Journal* (1970).
- [37] V. Rubin, K. Ford, and N. Thonnard. “Rotational Properties of 21 Sc Galaxies with a Large Range of Luminosities and Radii, from NGC 4605 ($R=4\text{kpc}$) to UGC 2885 ($R=122\text{kpc}$)”. In: *The Astrophysical Journal* (1980).
- [38] J. D. Simon. “The faintest dwarf galaxies”. In: *Annual Review of Astronomy and Astrophysics* (2019).
- [39] S. T. Sohn et al. “Space motions on the dwarf-spheroidal galaxies Draco and Sculptor on HST proper motions with a ~ 10 yr time baseline”. In: *ApJ* (2017).
- [40] A. Szentgyorgyi. “Hectochelle: high resolution multiobject spectroscopy at the MMT”. In: *New Astronomy Reviews* (2006).
- [41] M. Viel et al. “Constraining warm dark matter candidates including sterile neutrinos and light gravitinos with WMAP and the Lyman- α forest”. In: *The American Physical Society* (2005).

- [42] E. Vitral et al. *Data for "HSTPROMO Internal Proper Motion Kinematics of Dwarf Spheroidal Galaxies: I. Velocity Anisotropy and Dark Matter Cusp Slope of Draco"*. 2024.
- [43] E. Vitral et al. "HSTPROMO Internal Proper Motion Kinematics of Dwarf Spheroidal Galaxies: I. Velocity Anisotropy and Dark Matter Cusp Slope of Draco". In: *ApJ* (2024).
- [44] M. G. Walker, E. W. Olszewski, and M. Mateo. "Bayesian analysis of resolved stellar spectra". In: *Mon. Not. R. Astron. Soc.* (2015).
- [45] M. G. Walker, E. W. Olszewski, and M. Mateo. "Bayesian analysis of resolved stellar spectra: application to MMT/Hectochelle observations of the Draco dwarf spheroidal". In: *Mon. Not. R. Astron. Soc.* (2015).
- [46] M. G. Walker et al. "Magellan/M2FS and MMT/Hectochelle spectroscopy of dwarf galaxies and faint star clusters within the Galactic halo". In: *ApJS* (2023).
- [47] A. G. Wilson. "Sculptor-type systems in the local group of galaxies". In: *Publ. Astron. Soc. Pacific* (1955).
- [48] F. Zwicky. "Die Rotverschiebung von extragalaktischen Nebeln". In: *Helvetica Physica Acta* (1933).

Appendices

Appendix A

Data Sets Additional Information

	Data	$R_{max}[\text{kpc}]$	$M(< R_{max})[M_{\odot}]$	$M_{max}[M_{\odot}]$	$M_{min}[M_{\odot}]$	Multiplier
LOS	Draco	0.85	1.05	1.17	0.89	10^8
	PM Fields	0.31	0.12	0.15	0.07	10^7
	Inner	0.14	0.20	0.27	0.08	10^7
	Comp	0.16	0.46	0.61	0.23	10^7
PM	+Draco	0.31	0.15	0.16	0.13	10^8
	+Fields	0.31	0.14	0.15	0.11	10^8

Table A.0.1: Values for radius of outermost star R_{max} and the dark matter mass at this radius M times the multiplier.

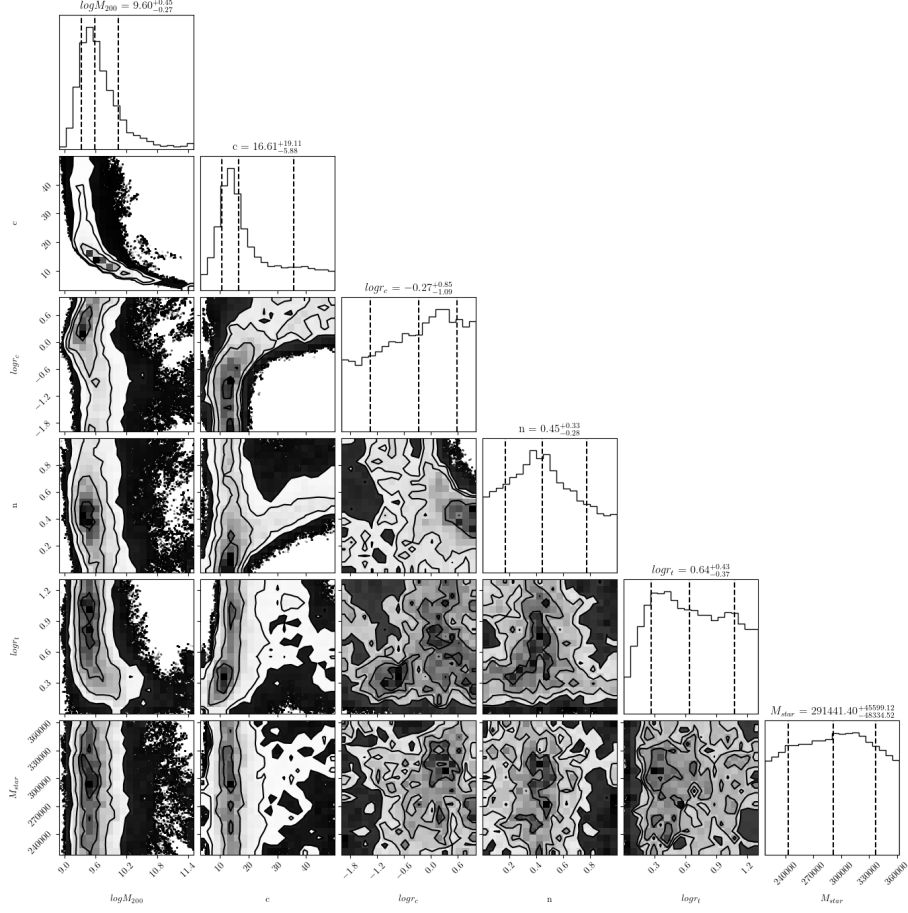


Figure A.0.1: All physical parameters of Draco Line-of-sight. In order: M_{200} : Virial mass, c : concentration, r_c : core radius, n : describes shape of the dark matter density profile, r_t : tidal radius, M_{star} : stellar mass

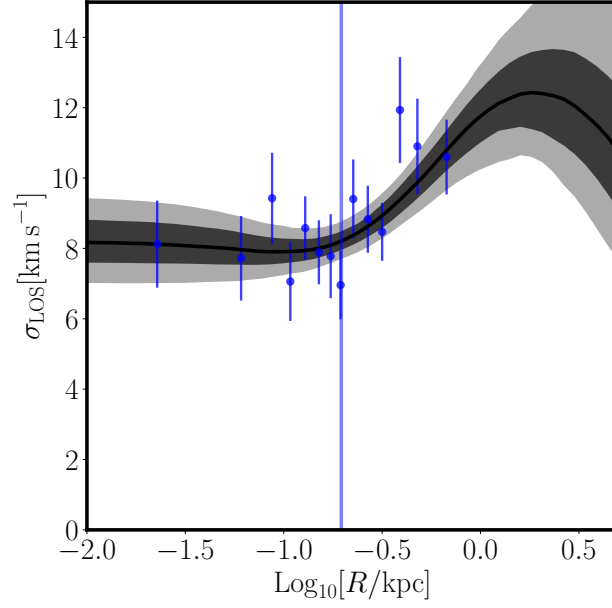


Figure A.0.2: The stellar projected velocity dispersion σ_{LOS} profile for Draco Line-of-sight.

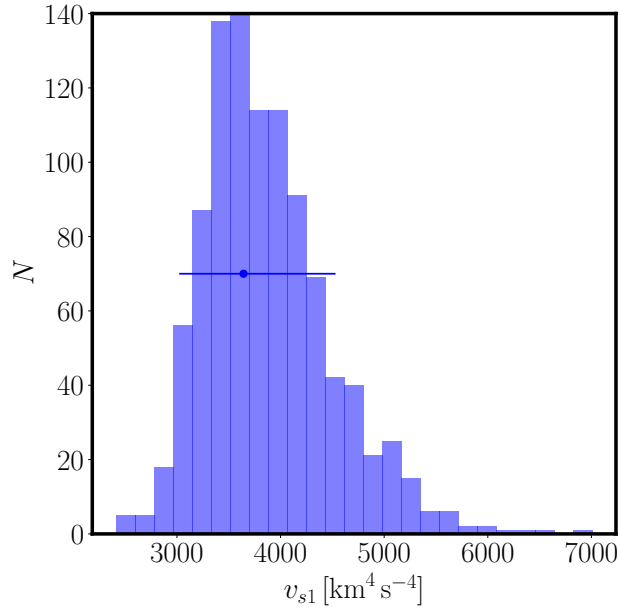


Figure A.0.3: The Virial shape parameter v_{s1} parameter for Draco Line-of-sight (more detail in Read and Steger(2017) ([34]), Equation 21).

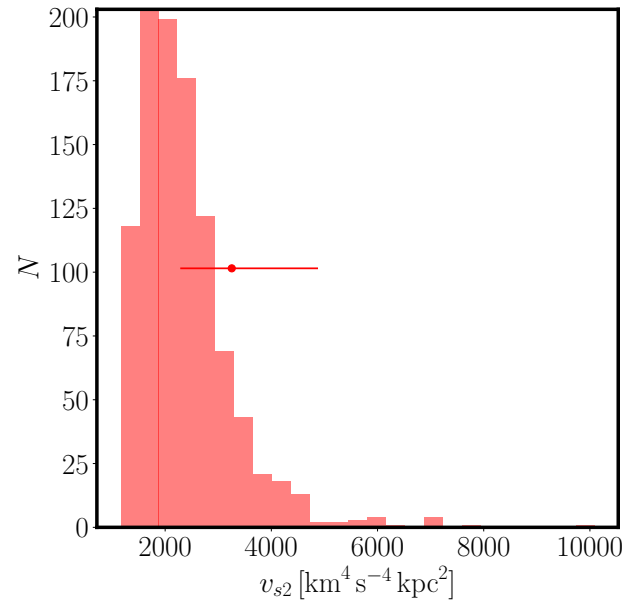


Figure A.0.4: The Virial shape parameter v_{s2} parameter for Draco Line-of-sight (more detail in Read and Steger(2017) ([34]), Equation 23).

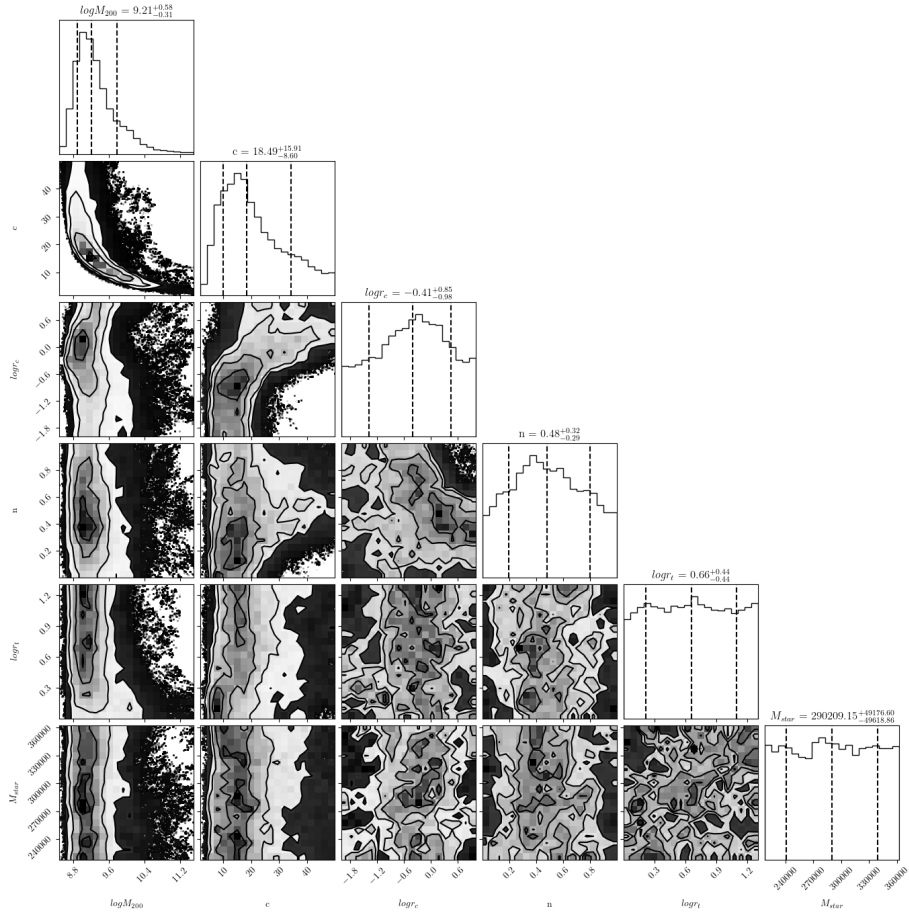


Figure A.0.5: All physical parameters of PM Fields LOS.

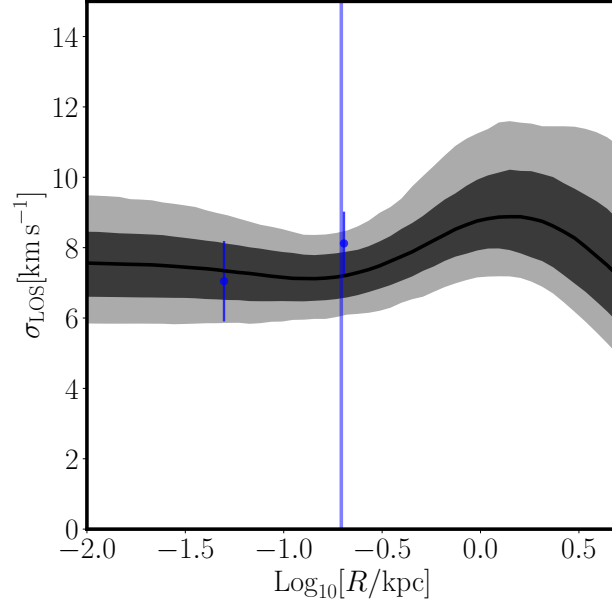


Figure A.0.6: σ_{LOS} profile for PM Fields LOS.

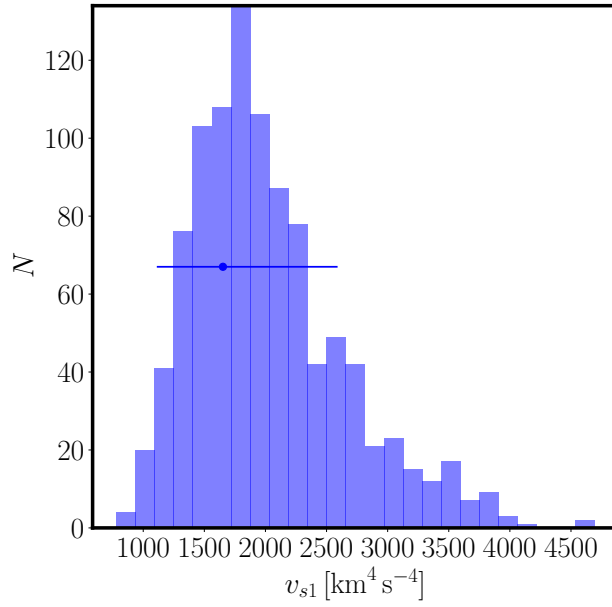


Figure A.0.7: v_{S1} parameter for PM Fields LOS.

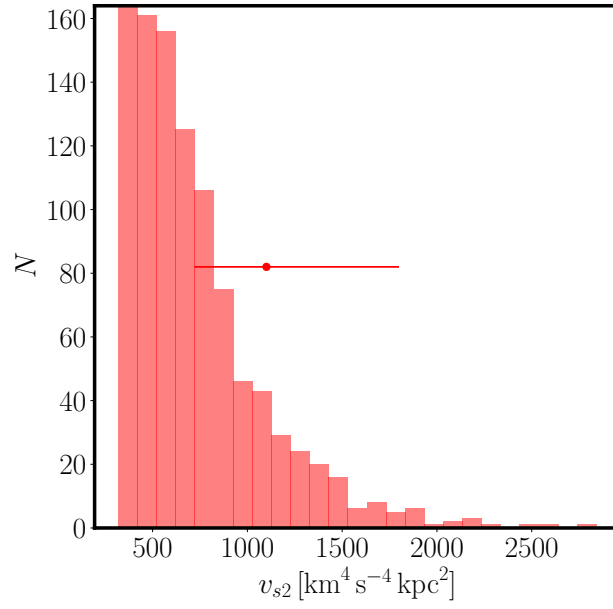


Figure A.0.8: v_{s2} parameter for PM Fields LOS.

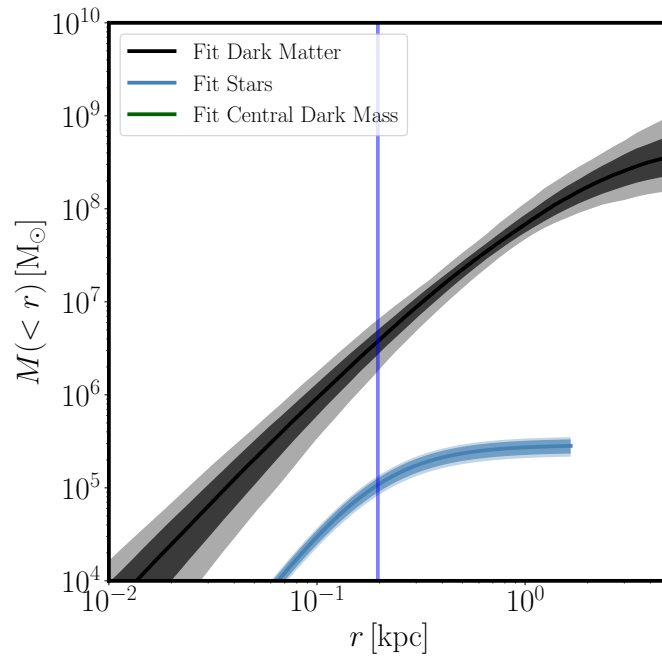


Figure A.0.9: Mass profile of Inner Field LOS.

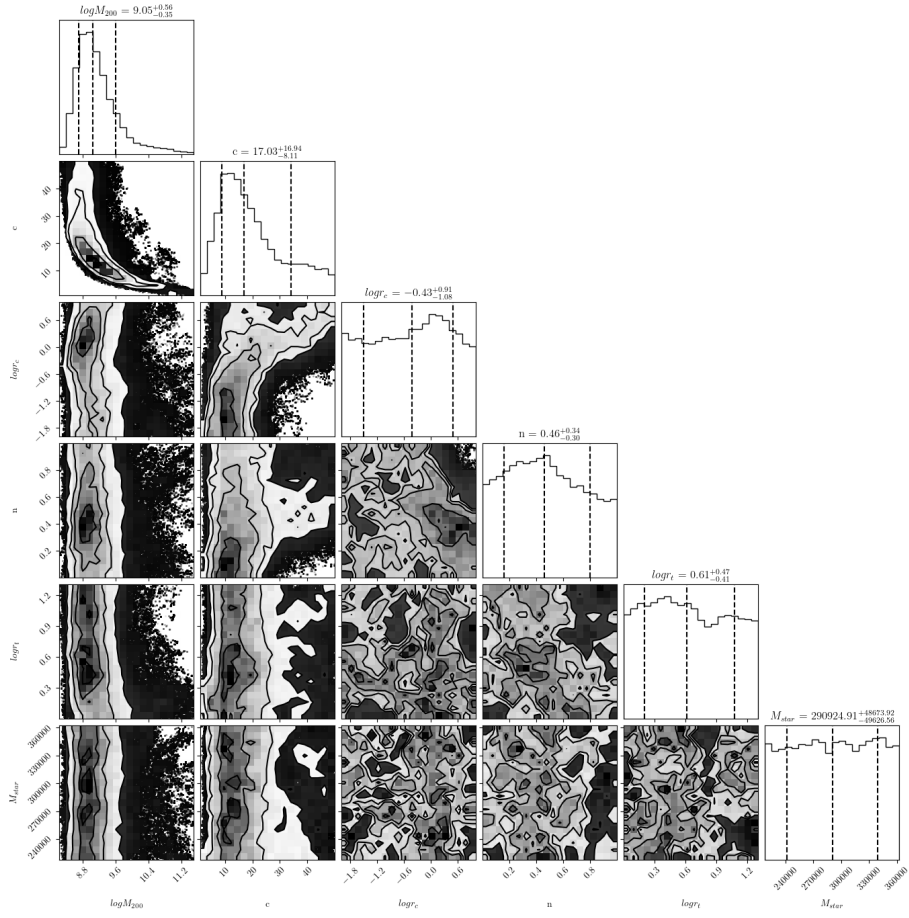


Figure A.0.10: All physical parameters of Inner Field LOS.

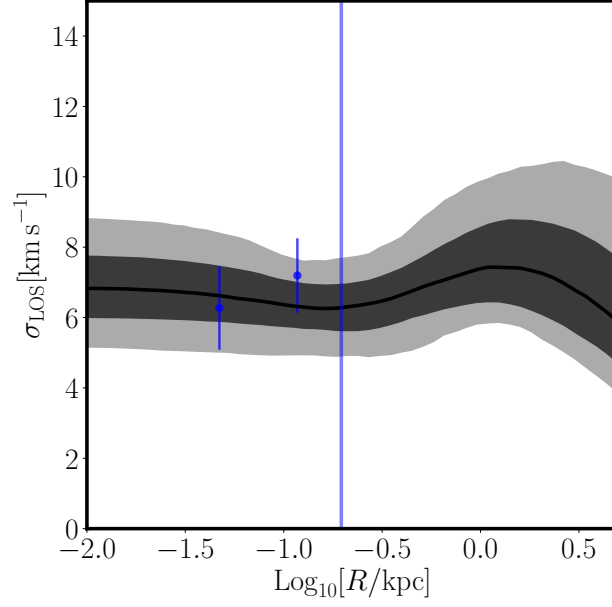


Figure A.0.11: σ_{LOS} profile for Inner Field LOS.

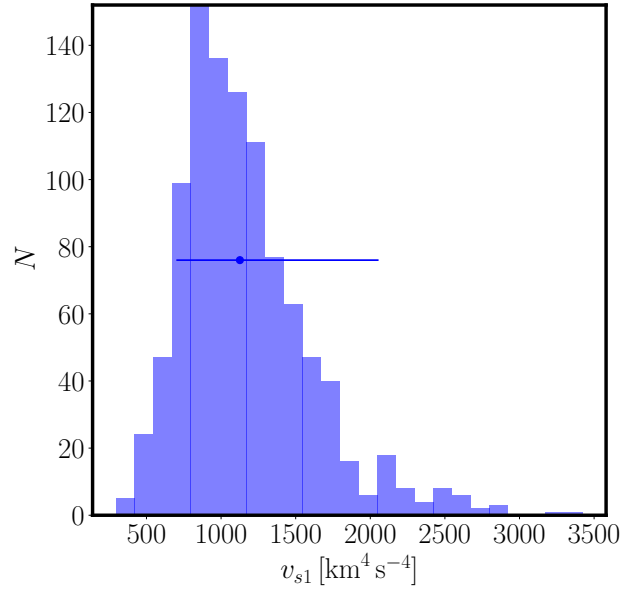


Figure A.0.12: v_{s1} parameter for Inner Field LOS.

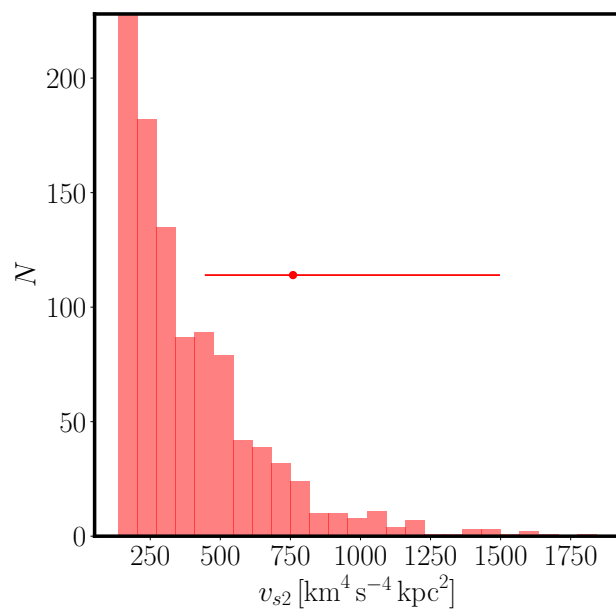


Figure A.0.13: v_{s2} parameter for Inner Field LOS.

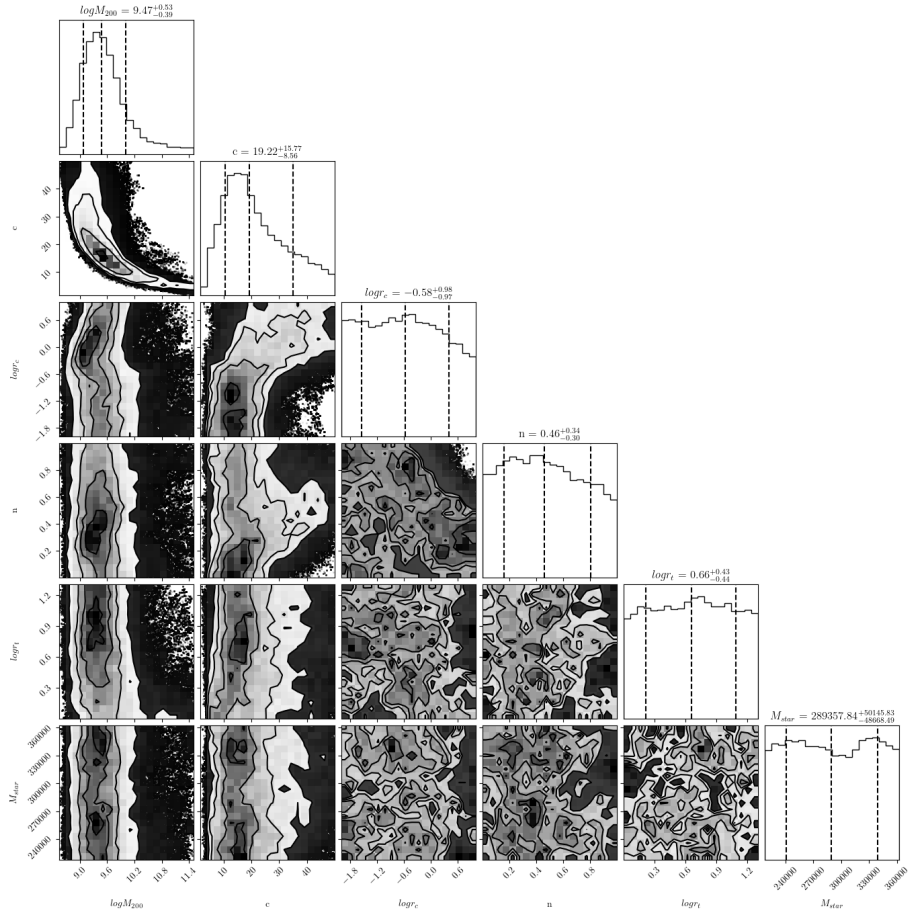


Figure A.0.14: All physical parameters of Comparison Field LOS.

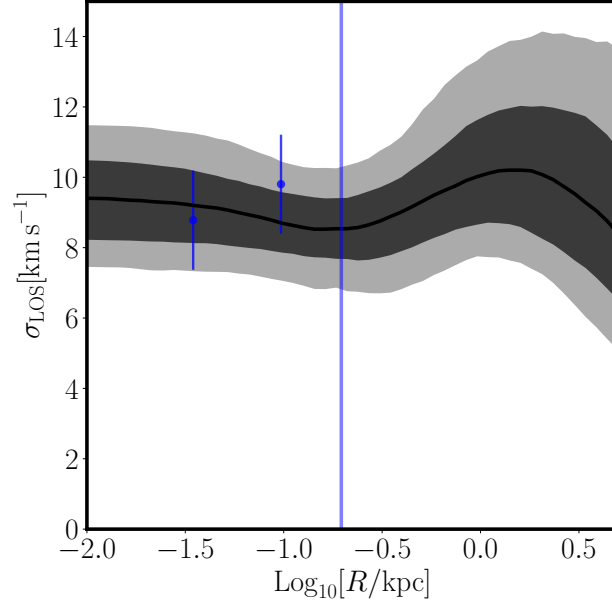


Figure A.0.15: σ_{LOS} profile for Comparison Field LOS.

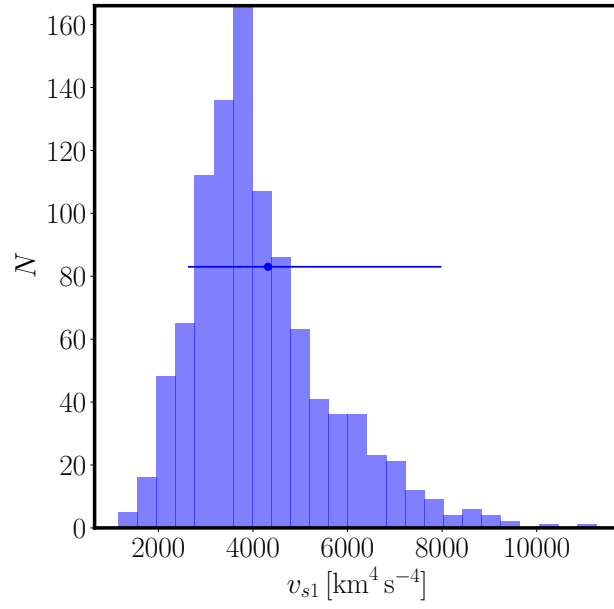


Figure A.0.16: v_{S1} parameter for Comparison Field LOS.

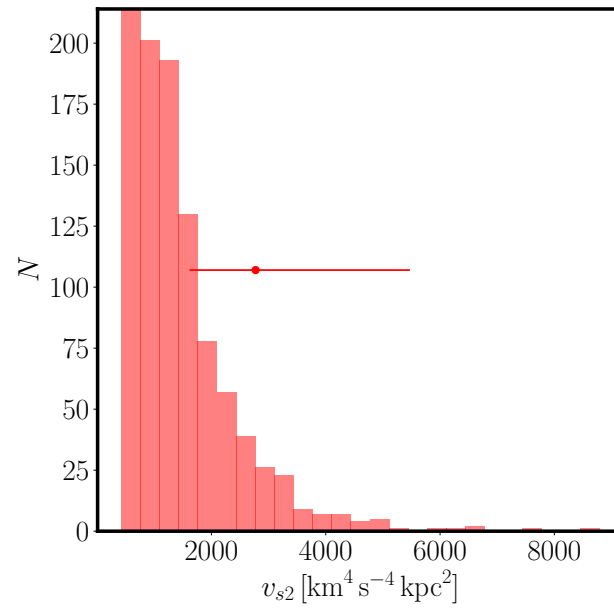


Figure A.0.17: v_{s2} parameter for Comparison Field LOS.

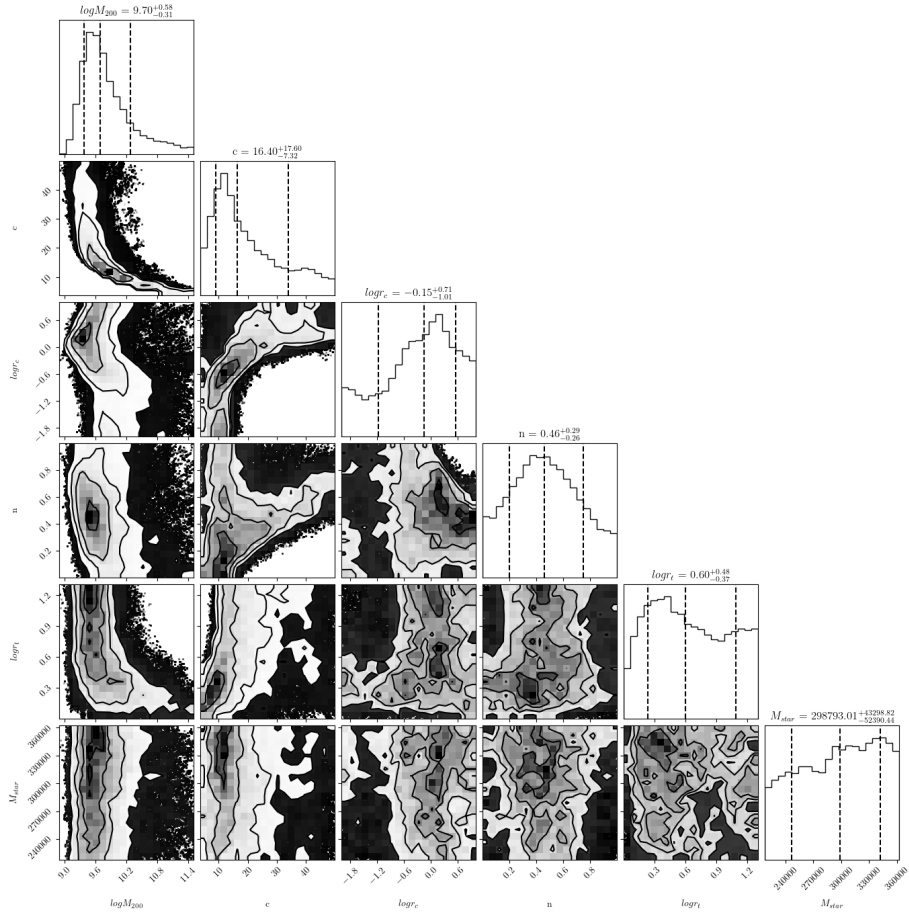


Figure A.0.18: All physical parameters of PM+Draco.

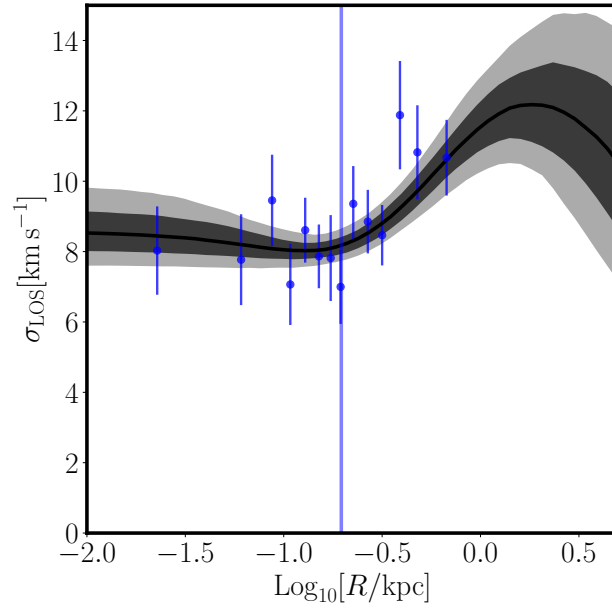


Figure A.0.19: σ_{LOS} profile for PM+Draco.

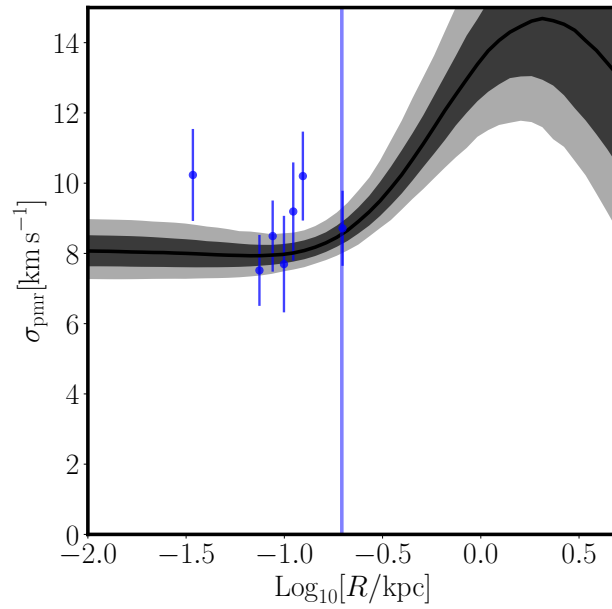


Figure A.0.20: The stellar proper motions dispersion in radial direction σ_{PMr} profile for PM+Draco.

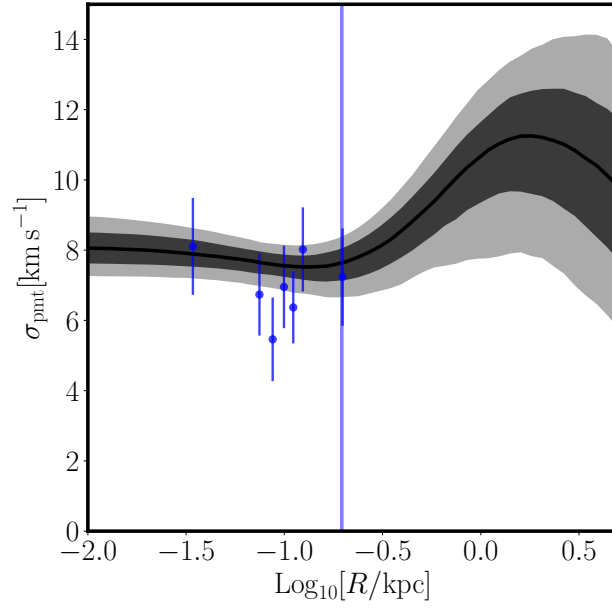


Figure A.0.21: The stellar proper motions dispersion in tangential direction σ_{PMt} profile for PM+Draco.

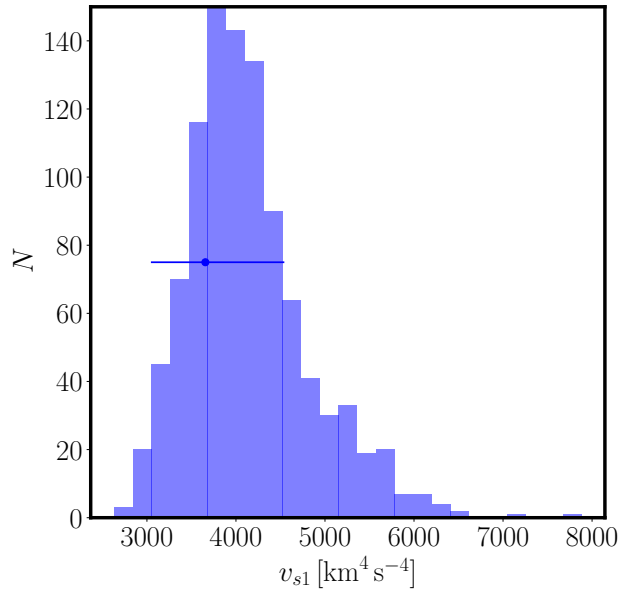


Figure A.0.22: v_{s1} parameter for PM+Draco.

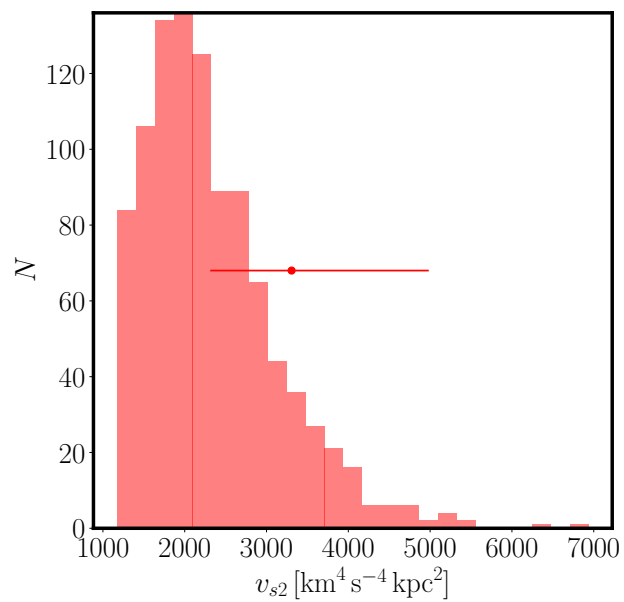


Figure A.0.23: v_{s2} parameter for PM+Draco.

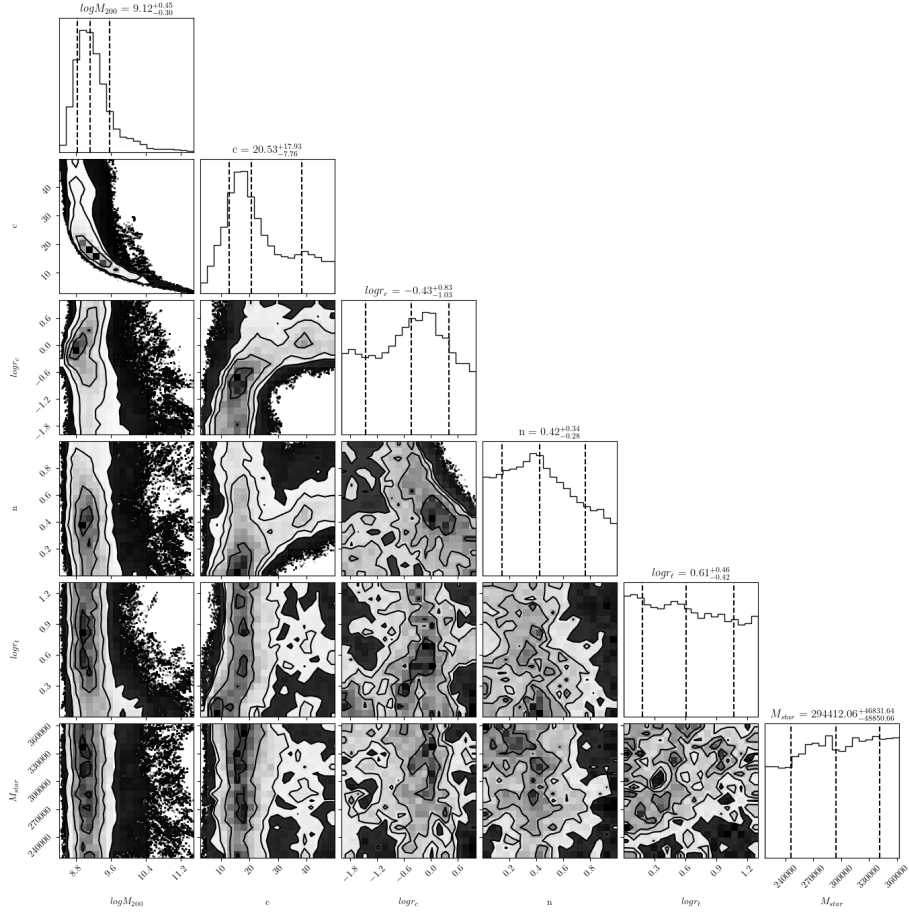


Figure A.0.24: All physical parameters of PM+Fields.

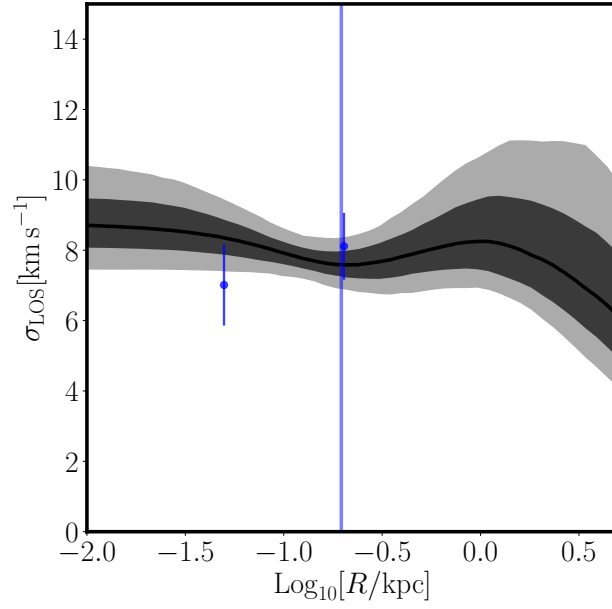


Figure A.0.25: σ_{LOS} profile for PM+Fields.

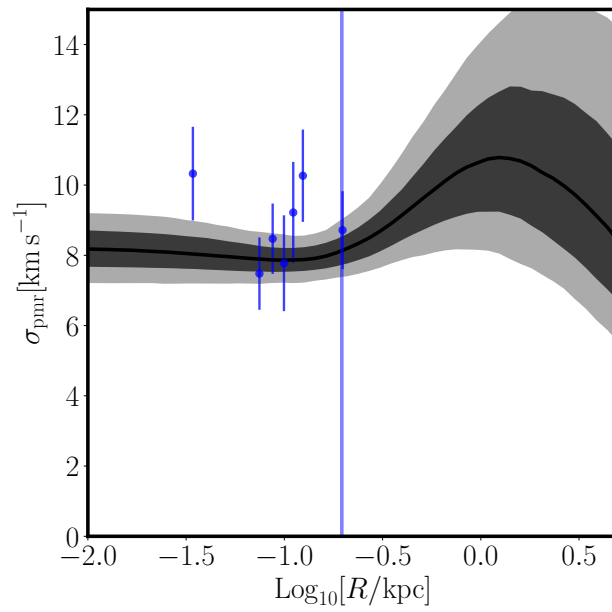


Figure A.0.26: σ_{PMr} profile for PM+Fields.

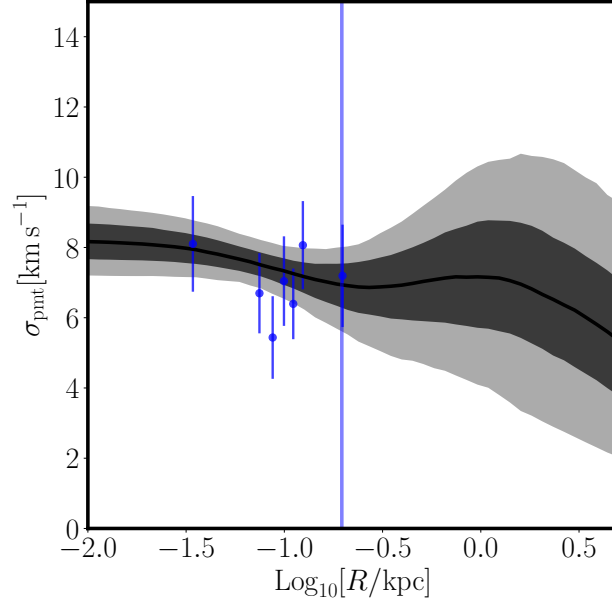


Figure A.0.27: σ_{PMt} profile for PM+Fields.

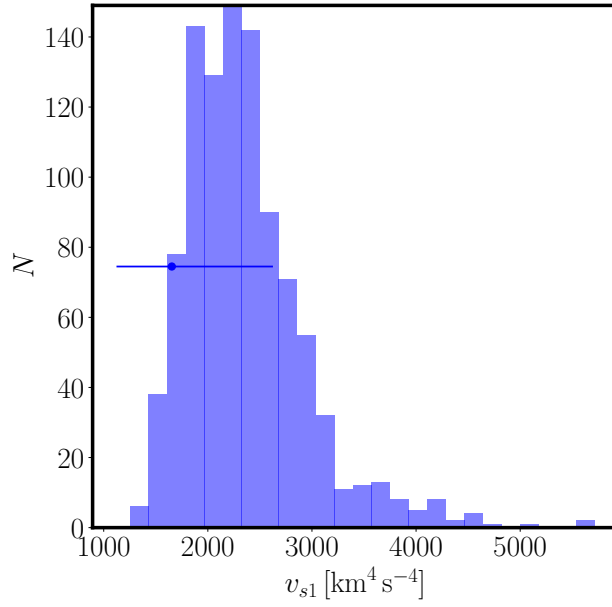


Figure A.0.28: v_{S1} parameter for PM+Fields.

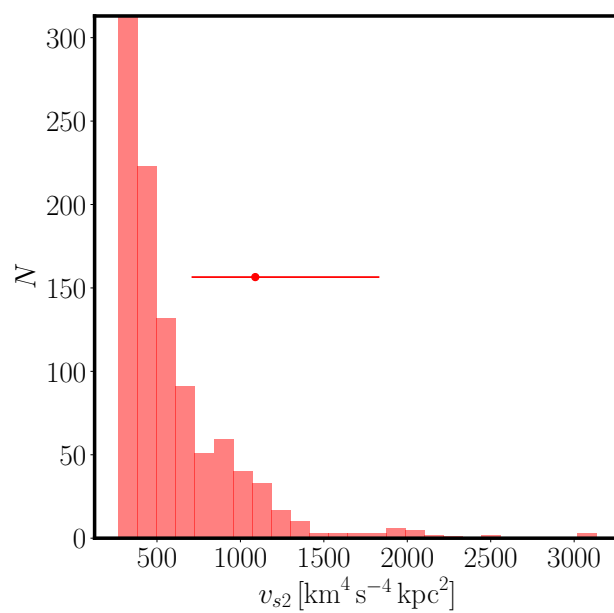


Figure A.0.29: v_{s2} parameter for PM+Fields.

Appendix B

Code

APIs to run GravSphere for Draco with and without proper motions using the format of data used in this project.

Draco Proper Motion API

```
import numpy as np
from astropy.coordinates import SkyCoord
from astropy import units as u

def draco_propmotion_api(data_prop):

    data = np.genfromtxt(data_prop, dtype='f8')

    draco_center = SkyCoord('17_20_12.4_+57_54_55', unit=(u.hourangle, u.deg))
    dgal_kpc = 76.0 #kpc

    #distance
    ra = data[:,0]
    dec = data[:,1]
    coords = SkyCoord(ra, dec, unit=u.deg)
    distance = coords.separation(draco_center).rad*dgal_kpc #kpc

    #to x-y frame
    coord_frame = draco_center.skyoffset_frame()
    star_pos_frame = coords.transform_to(coord_frame)
    x = star_pos_frame.lon.to(u.rad).value*dgal_kpc #kpc
    y = star_pos_frame.lat.to(u.rad).value*dgal_kpc #kpc

    #proper motions to km/s
    pm_ra = data[:,2] #mas/yr
    pm_dec = data[:,4] #mas/yr
    pm_ra_err = data[:,3] #mas/yr
```

```

pm_dec_err = data[:,5] #mas/yr

#conversion
mas_rad = 4.8481e-9
kpc_km = 3.086e+16
yr_s = 365*24*3600

#distance_km = distance*kpc_km
distance_km = dgal_kpc*kpc_km
pm_ra_rad = pm_ra*mas_rad
pm_dec_rad = pm_dec*mas_rad
pm_ra_err_rad = pm_ra_err*mas_rad
pm_dec_err_rad = pm_dec_err*mas_rad

vx = (pm_ra_rad*distance_km)/yr_s
vy = (pm_dec_rad*distance_km)/yr_s
vx_err = (pm_ra_err_rad*distance_km)/yr_s
vy_err = (pm_dec_err_rad*distance_km)/yr_s

#membership prop
ms_prop = np.zeros(len(ra))+1

return x,y,vx,vx_err,vy,vy_err,ms_prop

```

Draco API

```

from distance import *
import pandas as pd
import numpy as np
from astropy.coordinates import SkyCoord
from astropy import units as u
from scipy.integrate import simps as integrator
from constants import *
from functions import *
from figures import *
import pylab as plt

def draco_api(data_all, data_match, dgal_kpc, Nbin):

    data_stars = np.genfromtxt(data_all, dtype='f8')
    data_kin = np.genfromtxt(data_match, dtype='f8')
    infile_phot = './Data/Walker_dwarfs/dra_justin1_phot.dat'

    #Surface density:

    #calculate distance

```

```

df = get_distance(dgal_kpc, arcmin)
R_kin = pd.to_numeric(df['radial_distance'])

#First surface density:
data_phot = np.genfromtxt(infile_phot, dtype='f8')
R = data_phot[:,4]*dgal_kpc/arcmin
ms = data_phot[:,10]
print('Total effective no. of tracers (photometric):', np.sum(ms))
rbin, surfden, surfdenerr, Rhalf = \
    binthedata(R,ms,Nbin)
print('Data Rhalf:', Rhalf)

#Velocity data

vz = data_kin[:,29]
vzerr = data_kin[:,30]
mskin = np.zeros(len(data_kin[:,0]))+1
vsys = np.sum(vz*mskin)/np.sum(mskin)
print('Systemic velocity:', vsys)
vz = vz - vsys
print('Total effective no. of tracers (kinematic):', np.sum(mskin))

return rbin, surfden, surfdenerr, \
    Rhalf, R_kin, vz, vzerr, mskin, vsys

```

Distance Calculation

```

import numpy as np
from astropy.coordinates import SkyCoord
from astropy import units as u
import pandas as pd

def get_distance(dgal_kpc, arcmin):
    #same data set as in api
    file_path = './Data/Walker_dwarfs/draco_id_match.dat'

    df = pd.read_csv(file_path, delim_whitespace=True)
    ra = df['gaia_sigparallax']
    ra = pd.to_numeric(ra, downcast='float')
    dec = df['ra']
    dec = pd.to_numeric(dec, downcast='float')
    coords = SkyCoord(ra, dec, unit=u.deg)

    c = SkyCoord('17_20_12.4_+57_54_55', unit=(u.hourangle, u.deg))

    distance = coords.separation(c)

```

```
separation_value = distance.rad  
new_col_series = pd.Series(separation_value)  
new_col_units = pd.Series(np.zeros(len(separation_value))+dgal_kpc)  
df['radial_distance']=new_col_series*new_col_units  
return df
```