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Solar and stellar cycles and variability on century timescale

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Sunspot cycles and Grand Minima

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Abstract. Observational data concerning the long-term history of cyclic solar activity as recorded in sunspot and isotopic data are discussed in the context of solar dynamo theory. In particular, a simple dynamo model based on differential rotation and the mirror asymmetry of convection with random fluctuations of dynamo governing parameters is shown to reproduce some basic features of the solar magnetic activity evolution.

Keywords. Sun: magnetic fields, (Sun:) sunspots, stars: magnetic fields.

1. The general form of the record of solar activity

The solar activity cycle is usually considered as a quasiperiodic process with cycles relatively similar to each other. The standard physical explanation for the solar activity cycle is that the phenomenon underlying the cycle is the propagation of a wave of quasi-stationary magnetic field which in turn is excited by the solar dynamo. The classical explanation of the solar dynamo (Parker, 1955) is that solar differential rotation produces toroidal magnetic field from poloidal, while mirror-asymmetric convective flows produce poloidal field from toroidal (the "α-effect"), thus closing the chain of magnetic field self-excitation. This scheme results in latitudinal propagation of a wave of toroidal magnetic field (the dynamo wave). Appropriate tuning of the dynamo governing parameters allows equatorialward propagation of the dynamo wave that fits various observational feature of the activity wave. However, the link between the concepts of solar dynamo and helioseismological data remains an area of intensive discussion in scientific community (e.g. Kosovichev, 2008). Another disputable point here is to what extent a particular parametrization in terms of the α-effect, originating from Parker's paper and developed as a physical concept by Steenbeck, Krause and Rädler (Krause & Rädler, 1980), covers the range of possible physical dynamo mechanisms. For example, should the idea of a transport dynamo based on meridional circulation (e.g. Dikpati & Charbonneau, 1999) be considered as an independent option? In any case, the physical nature of solar activity on the time-scales of several activity cycles can be considered as a relatively understood phenomenon.

Of course, successive solar cycles differ slightly from one another, in amplitude, length and other characteristics. A physical explanation of the corresponding dynamics as well as the ability to predict the forthcoming cycle based on knowledge of recent cycles is being addressed by a number of approaches (see e.g. Kitiashvili & Kosovichev, 2008). This remains however a challenge for solar physics, but it looks plausible that the topic can be developed based on available ideas from solar dynamo theory and helioseismology.

Solar activity is a much more complicated phenomenon than just a quasi-regular cycle. The famous Maunder Minimum interrupted the normal sequence of activity cycles from the mid-XVIIth to the beginning of the XVIIIth century. Fortunately, it was the age of the birth of modern astronomy, and the Maunder Minimum was relatively well observed, and the records survived in the astronomical archives. A personal important scientific contribution from the King of France, Louis XIV, should be mentioned here as a rare example of this kind. For a review of the archive findings in the context of the solar dynamo see e.g. Sokoloff (2005). It is important that cyclic activity is visible, to some extent at least, at the end of the Maunder Minimum and that the activity wave then becomes substantially asymmetric, being visible almost solely in the Southern solar hemisphere, in contrast to the symmetric modern strong cycles. Another occurrence of an asymmetric activity wave is known from Gassendi's observations for the epoch just before the Maunder Minimum. Solar activity looks asymmetric also during the lost cycle however sunspots occurred that time preferably in the Northern hemisphere.

In an analysis of the cosmogenic isotope data, which provides a standard proxy for solar activity in the past, Usoskin *et al.*, 2007 showed that phenomena comparable with the Maunder Minimum occurred from time to time during the history of solar activity. A general name for the phenomenon is now Grand Minima of solar activity. Cosmogenic isotope data give some hints that, apart from Grand Minima, epochs of Grand Maxima of solar activity can be also observed, a typical example being the high solar activity in the second half of the 20th century. Of course, the further back in time a Grand Minimum is, the less precise our knowledge concerning its properties. As far as can be seen from the data available (Miyahara *et al.*, 2006) the Maunder Minimum represents a typical Grand Minimum in its general properties.

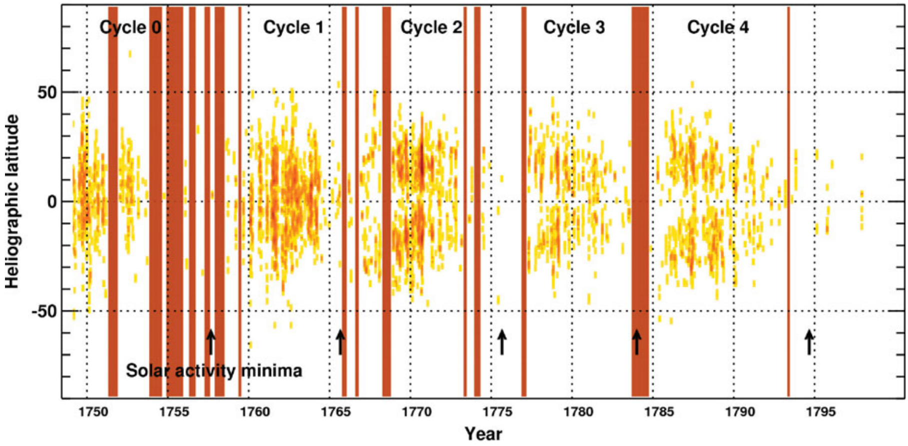

Figure 1. The historical butterfly diagram for XVIII century obtained from the Staudacher data. Sunspot density is given in scales of gray. Black vertical strips indicates epochs for which the data are insufficient to get the butterfly diagram.

The physical nature of Grand Minima in general, and the Maunder Minimum in particular, is much less clear than the nature of the solar cycle itself. Indeed, solar dynamo models (Brandenburg *et al.*, 1989, Jennings & Weiss, 1991, Covas *et al.*, 1998) as well as first laboratory dynamo experiments (Ravelet *et al.*, 2008) show that activity minima occur from time to time in the temporal evolution of particular dynamos, although a physical scenario for such events that is more or less generally accepted in the solar physics community is still lacking. An important point here is that as far as it follows from the available archive data the beginning of the Maunder Minimum was quite abrupt. It remains unclear to what extent a prediction, based on the previous record of solar activity, of a future Grand Minimum can be made. We stress that the prediction of the parameters a new solar cycle and those of a new Grand Minimum may be physically different undertakings. The role of a *local* minima of solar activity such as the Dalton Minimum, as possible precursors of Grand Minima, appears an attractive topic in this field.

One more point is that the temporal sequence of Grand Minima appears quite random; at least it is difficult to isolate a periodic behaviour. On the other hand, the intervals between successive minima vary too strongly to be referred as a realization of a Poisson random process (Usoskin *et al.*, 2007).

In the above context, the XVIIIth century was usually considered as a normal epoch with a typical record of solar activity. Some doubts in this respect came from the concept of a lost activity cycle which seems to have occured at the very end of the century (Usoskin *et al.*, 2003, Usoskin *et al.*, 2009b). Recent archive findings (Arlt, 2009) have confirmed that this impression of normality needs to be revised substantially. First of all, the butterfly diagram (Fig. 1) reconstructed from the Staudacher data for the XVIIIth century shows some features which looks peculiar to this century and are absent in the contemporary record of solar activity. In particular, butterflies for some cycles are concentrated near to the solar equator while others are separated from the equator by a strip of low activity. We discuss below how these features can be embedded in the framework of solar dynamo theory.

2. Dipolar versus quadrupolar modes of solar dynamo

A magnetic field of dipole-like symmetry is not the only magnetic configuration which can be excited by solar and stellar dynamos. Brandenburg *et al.* (1989) were the first to demonstrate that alpha-quenched mean field dynamos in spheres can move through regions with stable dipole-like and quadrupole-like, or even mixed, parity solutions as the dynamo parameters are changed. A rich dynamical behaviour of solar dynamos was discussed in context of zero or one dimensional models by e.g. Weiss *et al.* (1984) and Tobias *et al.* (2005), while experiments with more realistic solar dynamos show that the preferred symmetry of solutions near marginal excitation can be sensitive to quite small changes in the model (e.g. Moss, 1999, Dikpati & Gilman, 2001, Bonnano *et al.*, 2002, Jouve & Brun, 2007), or at least the excitation conditions for even and odd modes are similar (e.g. Moss & Brooke, 2000).

Recently Moss *et al.* (2008) summarized evidence that neither dynamo theory nor the observational data give strong support to the idea that stellar magnetic fields must have dipolar rather than quadrupolar symmetry with respect to the stellar equator. They demonstrated the spontaneous transition of the dynamo-excited magnetic field from one symmetry type to another and explored observational tests to distinguish between the two types of magnetic field symmetry, and thus detect the presence of quadrupolar magnetic symmetry in stars. A complete absence of quadrupolar symmetry would present a distinct challenge for contemporary stellar dynamo theory.

In our opinion, the archival butterfly diagram reconstructed by Arlt (2009) for the XVIIIth century from the Staudacher data (Fig. 1) gives a reasonably meaningful hint that the solar magnetic configuration demonstrates from time to time substantial excursions towards quadrupole-like parity. Of course, the historical data do not contain information concerning the polarity of sunspot groups, and so a direct verification of the Hale polarity law is impossible. The point is, however, that any toroidal magnetic field with dipole-like symmetry has to vanish at the solar equator, while a quadrupole-like field usually has a maximum of toroidal field at the equator, and is very unlikely to have zero amplitude there (Fig. 2). Such a maximum can be compared with the maximum at the solar equator which is visible in Fig. 1 for cycle I (and possibly for cycle 0), in contrast to a clear minimum at the equator visible for cycles 3 and 4.

Moss *et al.* (2008) argue that information on stellar cycle properties may offer another window on magnetic symmetry. Dynamos operating in stars in significantly supercritical regimes can give rise to complex phenomena, such as multiply periodic solutions and beating between modes with different symmetry properties, for example. Beating between dipolar and quadrupolar components might explain the apparent multiple periods observed in some stars. A cyclic quadrupolar component could show up as an amplitude and/or period modulation superimposed on a main dipole cycle period, or as the primary period itself if the quadrupolar symmetry dominates. Using cycle information, there is some limited evidence that fields with quadrupolar symmetry *may* have already been observed in some stars. A subclass of moderate-activity cool dwarfs exhibit multiple cycle periods in long-term Ca II HK measurements (Baliunas *et al.*, 1995). Similar secondary periods $P_{\rm cyc}(2)$ are seen in long-term photometric data of active single dwarfs as well (e.g. Messina & Guinan, 2002, Oláh & Strassmeier, 2002. These $P_{\rm cyc}(2)$ *may* represent the cycle period of the quadrupolar dynamo component $P_{\rm cyc}({\rm Q})$ (if the dipole is dominant), a beat frequency between $P_{\rm cyc}({\rm Q})$ and $P_{\rm cyc}({\rm D})$, the dipole period (if the quadrupole is dominant), or merely a long-term modulation of the amplitude of $P_{\rm cyc}({\rm D})$ due to non-linear effects.

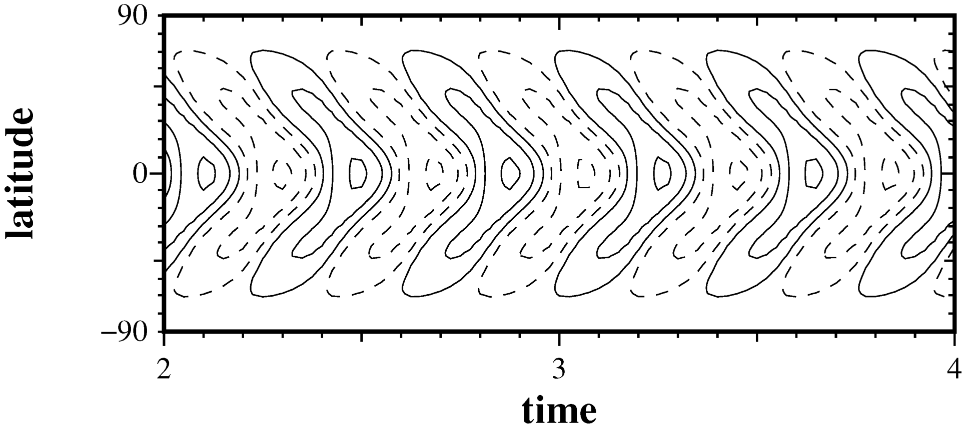

Figure 2. A simulated butterfly diagram for the toroidal magnetic field in a simple Parker migratory dynamo with algebraic α-quenching and quadrupole-like symmetry. Contours show toroidal field strength, with solid contours indicating positive values and broken contours negative values. The contours clearly show a maximum at the solar equator. Time is given in arbitrary dimensionless units not years.

3. Fluctuations in the solar governing parameters and Grand Minima

We stressed above that dynamo theory provides several options that might explain the features observed in the long-term evolution of solar activity including Grand Minima, asymmetric and quadrupole magnetic configurations, beating, lost cycles etc. One could expect that each particular phenomenon of that kind would require a specific explanation in terms of this theory.

The most straightforward idea here is to recognize that the α-effect, being the result of the electromotive force averaged over turbulent vortices, can contain a fluctuating contribution (Hoyng, 1993, Hoyng *et al.*, 1994, Ossendrijver & Hoyng, 1996). The idea can lead to events similar to the Maunder Minimum on the timescale of centuries (see e.g. Brandenburg & Spiegel, 2008).

Moss *et al.* (2008), Usoskin *et al.* (2009a) investigated the long-term dynamics of solar activity by confronting the predictions of a Parker migratory dynamo model containing a random contribution to the α-coefficient with the available data concerning the sequence of Grand Minima and Maxima, to recognize that this simple model provides in a proper parametric range all bulk of the phenomena under discussion similar at least qualitatively to that one observed on the Sun. The intention was to test whether a simple physical model can reproduce the basic phenomena of the long-term solar dynamics. Of course, a detailed explanation of the phenomena needs a much more realistic model, including at least a 2D description of the solar magnetic field, realistic solar rotation curve, etc. Moreover, it is not excluded *a priori* that the phenomena could have some alternative explanation. The analysis was based on a simple illustrative model, rather than on something more realistic, in order to isolate physical phenomena and to take into account the quite limited status of the actual observational information.

Random fluctuations of the dynamo governing parameters can be instructive in explaining the stochastic features of short-term dynamics of solar activity, on the timescale of a few solar cycles (e.g. Moss *et al.*, 1992, Hoyng *et al.*, 1994). In contrast, Moss *et al.* (2008), Usoskin *et al.* (2009a) considered global fluctuations of α on the temporal scales of order the cycle length and spatial (latitudinal) scale of the whole solar hemisphere. The analysis did not include variations of short time and latitudinal extent on the scales

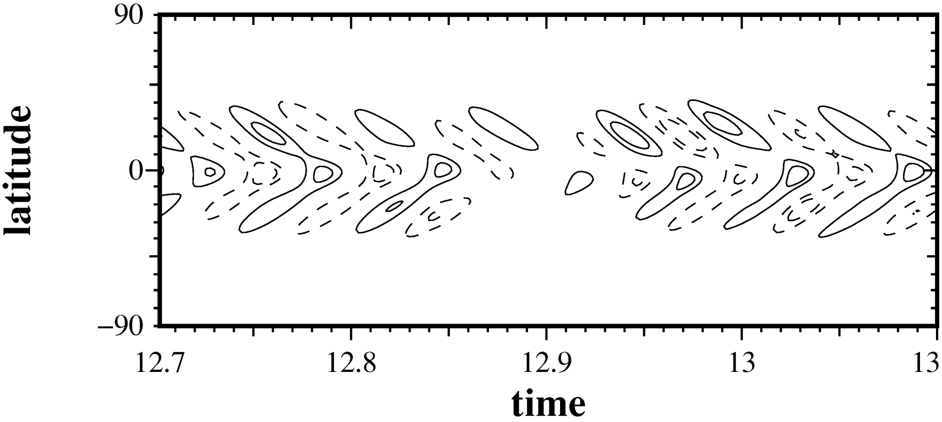

Figure 3. A simulated butterfly diagram for the toroidal magnetic field in a simple Parker migratory dynamo with algebraic α-quenching and fluctuating $\alpha(\theta, t)$ and basically quadrupole symmetry. Contours show toroidal field strength, with solid contours indicating positive values and broken contours negative values. The diagram suggests the presence of various types of complicated dynamics, such as transition to mixed parity and quadrupole-like configurations, lost cycles etc. Time is given in arbitrary dimensionless units not years.

of turbulent vortices, which are obviously important for the short-term dynamics of solar activity. The presence of long-term variations in the alpha-coefficient has been reported from analysis of direct numerical simulations by Brandenburg & Sokoloff (2002) and Otmianowska-Mazur *et al.* (2006).

Here we present some new results for solar dynamo models with fluctuating dynamo parameters, that are intended to be directly related to the current observational situation. Fig. 3 presents the butterfly diagram for a model for a particular choice of parameters which demonstrates various transitions in parity, quadrupole-like configurations and lost cycles. This example gives the evolution of a model which demonstrates strict quadrupole-like symmetry in the absence of fluctuations. Fig. 4 gives a similar example for a model with basically dipole-like symmetry. Fig. 5 shows the corresponding evolution of parity and magnetic energy.

We stress that the above figures significantly exploit the concept that sunspot formation is a threshold phenomenon; the butterfly diagrams from toroidal field contours are only plotted for fields above a certain threshold strength. Cyclic behaviour absent in a particular hemisphere during an activity cycle in the above figures persists still in form of a very weak magnetic field oscillations which are not strong enough to appear in the butterfly diagrams. Possibly, this is an option to explain that some tracers of cyclic sunspot behaviour (Frick *et al.*, 1997), and even of apparent solar diameter (Nesme-Ribes *et al.*, 1995), are recognizable from the archive data for the Maunder Minimum.

4. The record of solar activity on the time-scale of millennia

Moss *et al.* (2008) discuss the possibility that solar activity cannot be considered as a stationary random process even on the timescale of [tens of thousands of] years, as suggested by the following test. Let us consider a time series $f_n = f(t_n)$ where the instants of observations $t_i = n\tau$. If the length of the time series is sufficient to consider it as a stationary random process then the random quantity $g_n = \Sigma_0^n f_i$ is expected to behave as $g_n = \langle f \rangle n + h(n)$ where $\langle f \rangle$ is the mean of the random process f_n and $h(n)$ is a function that grows more slowly then n. By plotting g_n against n and comparing the results with a linear trend we can check whether the time series is long enough to be consider as a realization of a stationary random process (for applications of this

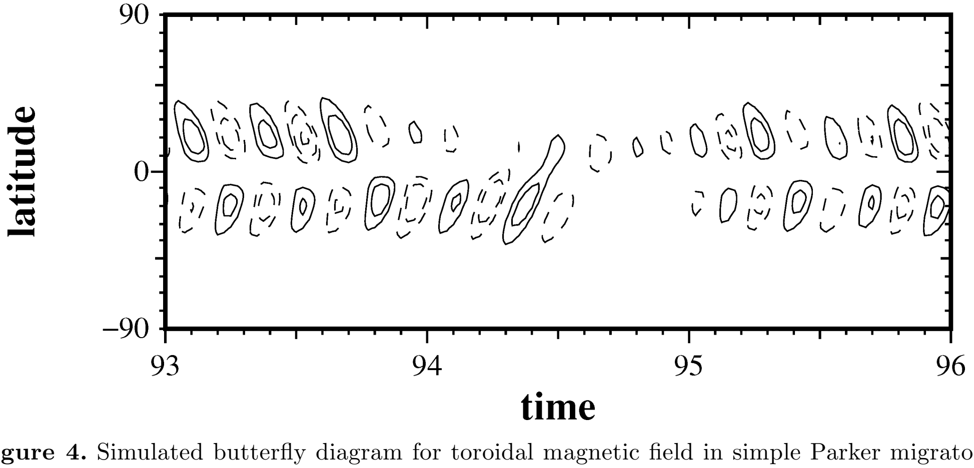

Figure 4. Simulated butterfly diagram for toroidal magnetic field in simple Parker migratory dynamo with algebraic α-quenching and fluctuating $\alpha(\theta, t)$ and basic dipole-like symmetry. Contours show toroidal field strength, with solid contours indicating positive values and broken contours negative values. Time is given in arbitrary dimensionless units not years.

test in solar physics, see Kutvitskii *et al.*, 2009). The test when applied to the record of solar activity that was exploited by Usoskin *et al.* (2009a) to provide a sequence of Grand Minima, demonstrates substantial deviations from [a linear behaviour]. The corresponding plot for dynamo models with fluctuating dynamo governing parameters give similar deviations from a straight line (Moss *et al.*, 2008). This result appears quite unexpected because the underlying physics do not contain any timescale longer than the cycle length, and thus we are faced with a nontrivial behaviour of a noisy system. Unfortunately, the solar activity record based on isotopic data is too short to claim that these dynamics inevitably follow from the observations.

Conclusion of this paper can be summarized as follows. Observational data concerning cyclic solar activity as recorded in sunspot and isotopic data demonstrate a complicated long-term history which includes Grand Minima, asymmetric cycles or even cycles of quadrupole symmetry and lost cycles. A simple dynamo model based on differential rotation and the mirror asymmetry of convection with random fluctuations of dynamo

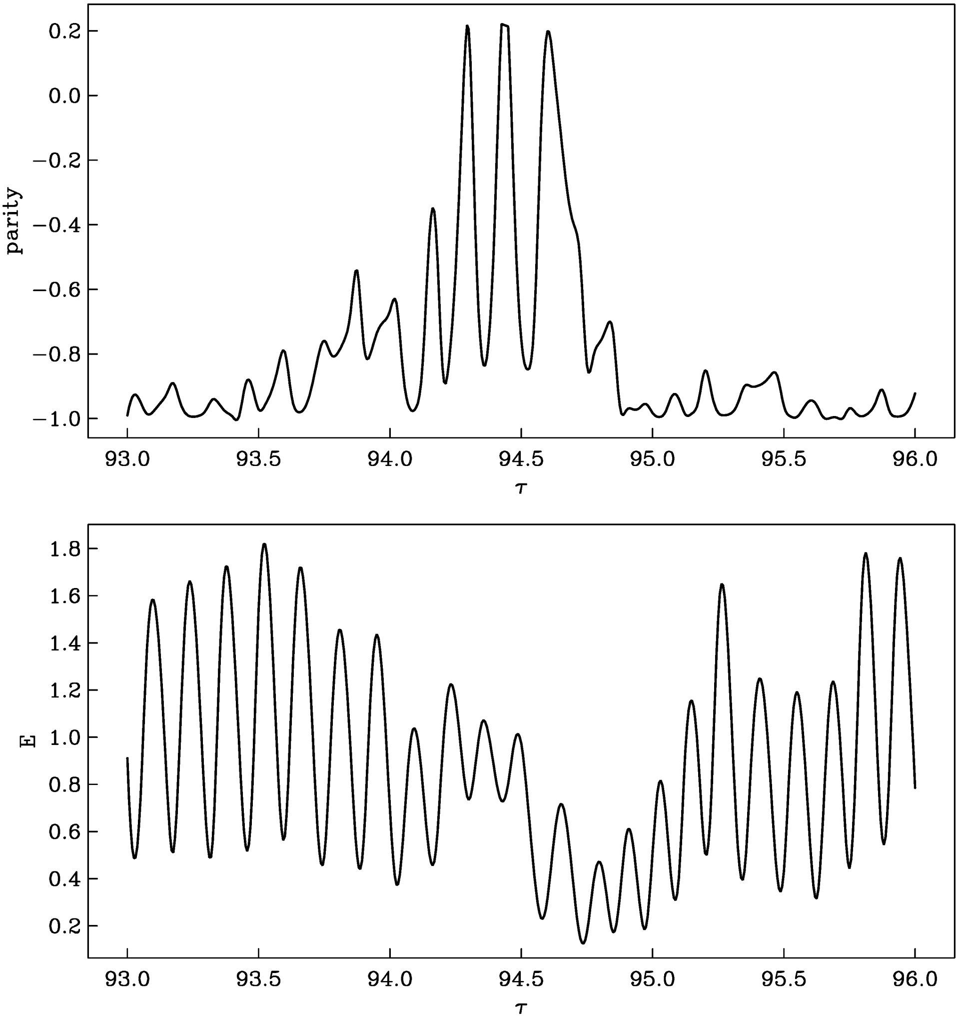

Figure 5. Temporal evolution of magnetic field parity (upper) and energy (lower panel) for the model illustrated in Fig. 4. Parity $P = -1$ means a dipole-like configuration and $P = +1$ corresponds to the quadrupole-like case. Energy is measured in units of equipartition energy.

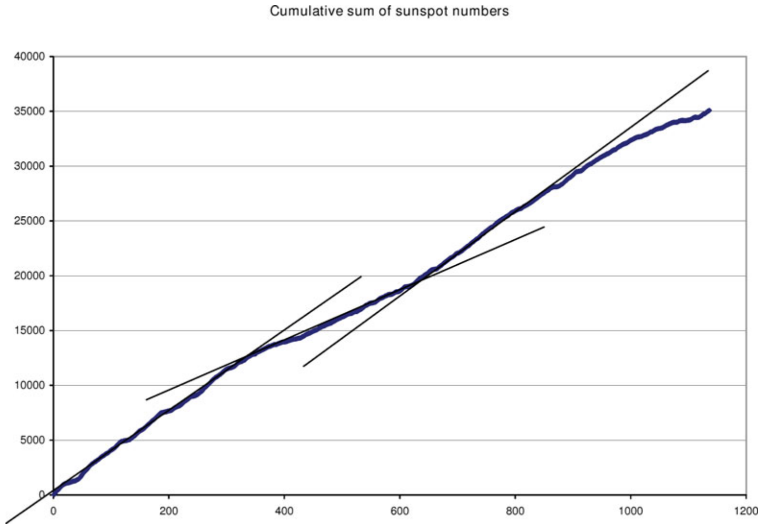

Figure 6. Cumulative sunspot number against time. The straight lines approximate the data locally.

governing parameters is able to reproduce some basic features of the solar magnetic activity evolution.

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