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Magnetic fields in hot stars and what we can learn for cool stars

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The talk reviews the key observational facts but will focus on the theoretical understanding of magnetic fields in radiative zones of intermediate-mass stars. The results for magnetic instabilities and dynamo effect are then seen in view of their effects in cool stars of which in fact many also have a radiative part. We now have a detailed understanding of these magnetic instabilities and can draw first conclusions about the dynamo effect expected from them. The activity of cool stars may actually be only understandable if the dynamo process is seen in conjunction with magnetic instabilities.

1 Introduction

Dynamo processes are most likely the drivers of the magnetic phenomena observed on low-mass stars. The only viable option for this type of stars possessing convection zones is a turbulent dynamo. The understanding of how the small-scale motions generate a coherent large-scale field is not fully established yet. The concept of mean-field magnetohydrodynamics led to a possibility to describe the dynamo process by the induction equation alone (Krause & Rädler 1980). On the one hand, global dynamo models become numerically easy in this concept, while on the other hand, hydrodynamic and magnetic instabilities are no longer visible in this treatment.

There is good reason to investigate the stability of magnetic fields in intermediate-mass stars or, more precisely, in radiation zones which many cool stars also possess. One of the problems is the existence of main-sequence stars of type late B to early F exhibiting surface magnetic fields of a few hundred gauss to a few tens of kilogauss. However, roughly 90% of stars of the same type do not show substantial magnetic fields. The magnetic ones come along with chemical peculiarities on their surfaces. We will call them Ap stars here collectively. Intermediate-mass stars are characterized by large radiative envelopes over 70–80% of their radius. The envelopes are not capable of driving a convective dynamo as we know it from cool stars. Yet a fraction of these stars shows magnetic fields on their surfaces which are virtually constant in time. A dynamo in their convective core alone is not capable of delivering kilogauss fields all way up to the surface. There is also a very thin top layer convection zone which is not likely to host an efficient dynamo either.

A second problem concern the angular momentum transport through radiation zones in general. The problem is rather dramatic in the evolution of intermediate-mass stars. As soon as they start to expand into their giant stadium, the cores shrink and gain rotational velocity. Observed white dwarfs, however, show at least two orders of magnitude lower specific angular momenta than these stellar cores would be expected to possess, if no angular momentum removal takes place (Suijs et al. 2008).

A similar problem is encountered when looking at low-mass stars which tend to spin at periods of ten and more days (e.g. Meibom et al. 2011 for a 1-Gyr stellar cluster), while their original periods must have been days (e.g. Irwin et al. 2009 for a 130-Myr cluster). If there was just be the microscopic viscosity of the radiative interior the coupling with the convective envelope would be too weak and the stellar core must rotate at its initial period practically during the entire main-sequence life. Helioseismology can now exclude such a fast core for the Sun down to about 0.15 solar radii (Eff-Darwich & Korzennik 2012). Sufficient removal of angular momentum must have taken place somewhen during the early life of the Sun.

Under certain conditions, large-scale magnetic fields can change a differential rotation into a uniform one directly through Maxwell stresses. Alternatively magnetic fields can excite instabilities whose generated turbulence transport angular momentum effectively. We will look at those instabilities in the comparison below.

2 Magnetic instabilities in radiation zones

We are distinguishing current-driven instabilities and shear-driven instabilities in the following. Both classes may require the presence of magnetic fields, but the main energy source is different. Instabilities involving thermal conductivity are not considered here. We are not claiming that this is the only way of categorizing instabilities in well coupled

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plasmas either. In the end of this paper, we will even see that these classes are not disjunct groups.

2.1 Current-driven instability

The fact that most magnetic field configurations are unstable in three dimensions, if only the magnetic field strength is large enough, has been known for decades. The works by Vandakurov (1972) and Tayler (1973) were particularly detailed on the stability of toroidal magnetic fields which we encounter so often in stellar interiors. The prerequisite of the fields to become unstable above a critical field strength is the presence of currents. A potential field is stable against this type of instability but many destabilize shear flows as shown in Sect. 2.2.

In many cases, non-axisymmetric perturbances are the ones that grow at fastest rates. They may eventually turn into turbulence through nonlinear coupling and lead to turbulence effects like turbulent angular-momentum transport or drive a dynamo. While many studies have been published based on cylindrical geometry, we will be concerned with results obtained in spherical geometry.

Importantly for stars, rotation stabilizes magnetic fields against the Tayler instability. A simple estimate of the limiting magnetic field strength under rotation is that instability sets in when $\Omega_\mathrm{A} > \Omega$ (see e.g. Pitts & Tayler 1985). In this relation, Ω_A is the Alfvén angular velocity of the background magnetic field strength B_0, $\Omega_\mathrm{A} = B_0/\sqrt{\mu_0\,\rho}R$ with μ_0 being the permeability, ρ the bulk density of the stellar gas and R the cylindrical distance from the rotation axis. The other quantity, Ω, is the stellar angular velocity.

Toroidal magnetic fields come typically into play when differential rotation is present. Poloidal fields of any geometry are sheared and azimuthal fields are generated. In axisymmetry, toroidal fields are equal to azimuthal fields.

In a numerical experiment, the onset of the Tayler instability was studied as it may occur in young A stars (Arlt & Rüdiger 2011a) which are assumed to provide some differential rotation after surface braking effects have taken place in the pre-main-sequence phase. In view of the emergence of surface fields that resemble Ap star fields in topology and strength, this study was extended by Arlt & Rüdiger (2011b). A typical surface pattern from such a simulation is shown in Fig. 1. The structures are rather large-scale and do not develop an actual turbulence. The magnetic Reynolds number for this run was 10^4.

Searches for extremely slowly evolving "end-states" of magnetic fields in Ap stars were also performed by Braithwaite & Nordlund (2006) and Braithwaite (2009). Random fields left over from the early convective phase of the star lead to non-axisymmetric surface fields that are changing on a time-scale as long as the magnetic diffusion time of the radiation zone. Similar in both attempts is the time it takes to get to these configurations. In both cases, it is the Alfvén time, i.e. essentially the time for Lorentz forces to change the flow, which is typically between a few tens of rotations and a few thousand years, depending on which field

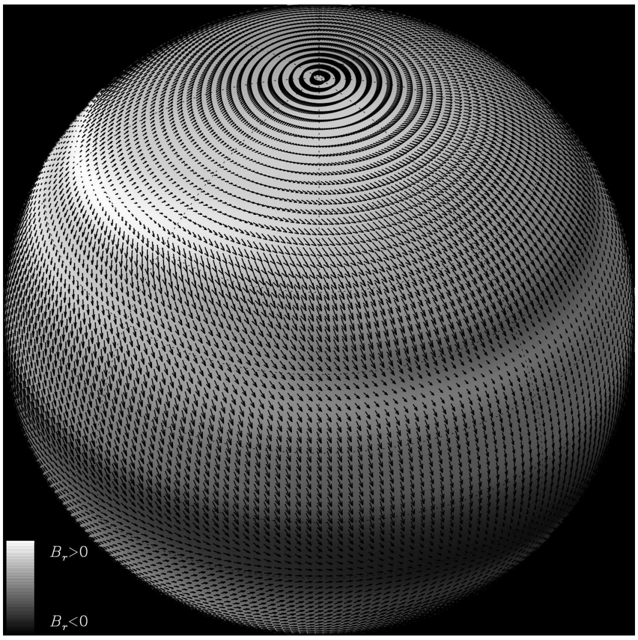

Fig. 1 Surface plot of the magnetic field perturbations growing from a current-driven instability and emerging at the surface. Grey shades represent the radial component, while the arrows show the azimuthal and latitudinal components.

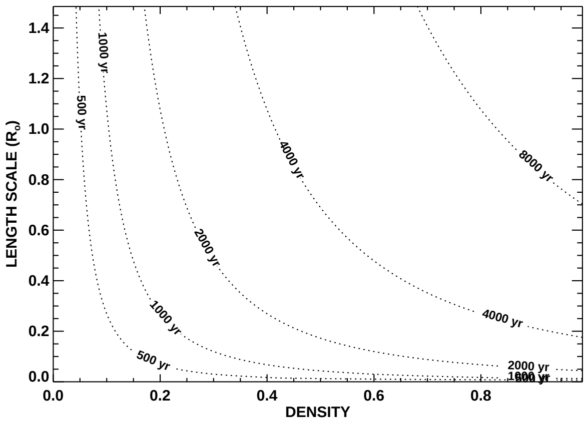

Fig. 2 Alfvén time in years for a magnetic field strength of 1 kG. The density is given in g/cm^3, the length is normalized with the solar radius. The entire radius of a 1-Myr-old $3M_\odot$ star is roughly $2R_\odot$.

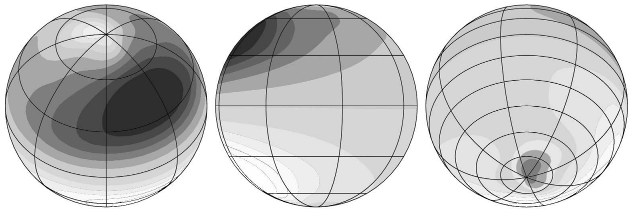

Fig. 3 Radial component of the surface magnetic field after the Tayler instability has set in (adapted from Szklarski & Arlt 2013).

strengths and length scales are considered. The only way how magnetic fields can change over such short times is by flows (assuming that Hall effect and ambipolar diffusion are negligible in a normal star).

Typical Alfvén times for an example magnetic field strength of 1 kG is plotted in Fig. 2 as a function of the density and the length-scale on which the change of magnetic-field configuration should take place. In reality, these times will be the result of a field emergence through a range of densities and of nonlinearities. But the numbers illustrate that there is a time-scale between the rotation period and the magnetic diffusion time of 10^{10}–10^{11} yr.

In an attempt to combine fully compressible MHD with differential rotation and the generation of azimuthal magnetic fields, Szklarski & Arlt (2013) computed the emergence of surface magnetic fields in an isothermal spherical shell. The resulting threshold for unstable azimuthal fields leads to roughly 300 G surface fields, which is (maybe accidentally) exactly what has been proposed to be the lower limit for Ap star fields by Aurière et al. (2007). The surface field from one of these simulations is shown in Fig. 3. Strong radial components are distributed quite unevenly with even one view angle under which practically no field is visible. We did not find growth of axisymmetric modes as proposed by Bonanno & Urpin (2006) – despite both setups being isothermal –, perhaps because the properties of spherical and cylindrical geometries are too different.

For the Tayler instability to run as a continuous process, the non-axisymmetric perturbation have to deliver an induction effect on the large-scale, axisymmetric field which is the one becoming unstable again. A continuous process acts like a dynamo fed by differential rotation as proposed by Spruit (2002). In principle, such a self-excitation is possible through the turbulent electromotive force, $\langle \boldsymbol{u'} \times \boldsymbol{B'} \rangle$, which contains inductive and diffusive parts. The dynamo effect from the Tayler instability has been found to be small or absent though in actual simulations (Brun & Zahn 2006; Zahn et al. 2007; Gellert et al. 2008; Arlt & Rüdiger 2011a). As for now, we cannot recommend using enhanced viscosities in stellar evolution codes motivated by current-driven instabilities, because they have not proven to work as a continuous process.

2.2 Shear-driven instability

A very similarly looking instability, at first glance, is the magneto-rotational instability (MRI) which also requires the presence of a magnetic field but needs an additional differential rotation. It taps the rotational energy rather than the magnetic energy for the growth of the perturbations in its pure form (see Balbus & Hawley 1998 for a review). A homogeneous, vertical magnetic field (which has no currents) and a rotation profile which decreases with the distance from the rotation axis has been shown to deliver sufficient velocity and magnetic field fluctuations to transport angular momentum efficiently.

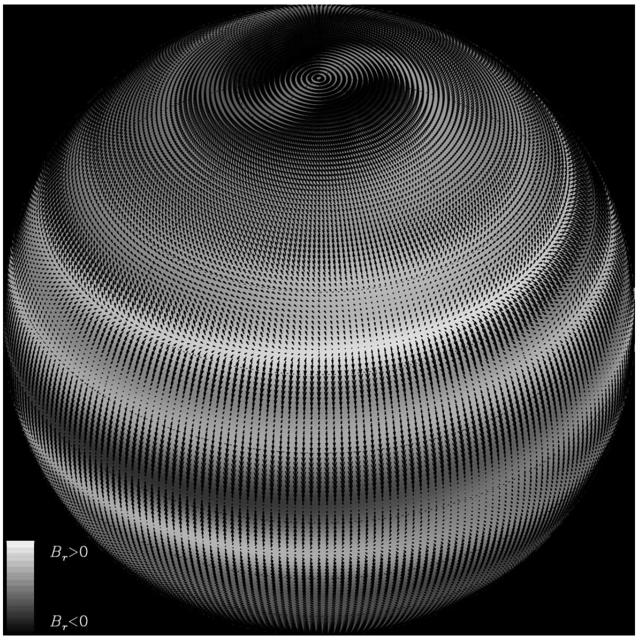

Fig. 4 Surface plot of the magnetic field perturbations growing from magnetorotational instability and emerging at the surface. As in Fig. 1, arrows show the horizontal components, while the greyscale represents the radial component.

The onset of the MRI in spherical geometry was studied by Arlt et al. (2003) in view of stars and by Petitdemange et al. (2008) in view of planetary cores. The MHD turbulence emerging transports angular momentum primarily by Maxwell stresses (magnetic field fluctuations) rather than Reynolds stresses (velocity fluctuations). This has also been observed in simulations of MRI turbulence in accretion disks. Arlt et al (2003) attempted to determine the time it takes to turn an initially differential rotation into a uniform one. The measurement is difficult in any time-dependent global simulation because the viscosity (be it physical or numerical) is orders of magnitude larger than the microscopic viscosity of the stellar plasma in a radiation zone. Alternatively, one may assume the viscosity in the code is correct, but then ends up with an extremely slow rotation of the model star. The difficulty consists of deciding if the time-scale of reaching uniform rotation is coupled to the viscous time-scale, to the rotational time-scale, or to the Alfvén time-scale.

In contrast to the fields emerging from the Tayler instability, the MRI typically delivers small-scale, often banded structures on the stellar surface. Figure 4 shows an example from recent simulations. A homogeneous background field parallel to the rotation axis was combined with a differential rotation profile where, again, the rotation period only depends on the distance to the axis. This combination does practically not generate any azimuthal fields and is not Tayler unstable because there are no currents. Yet if small perturbations are added to the system, an instability grows. A test with a rotation profile that is increasing with the dis-

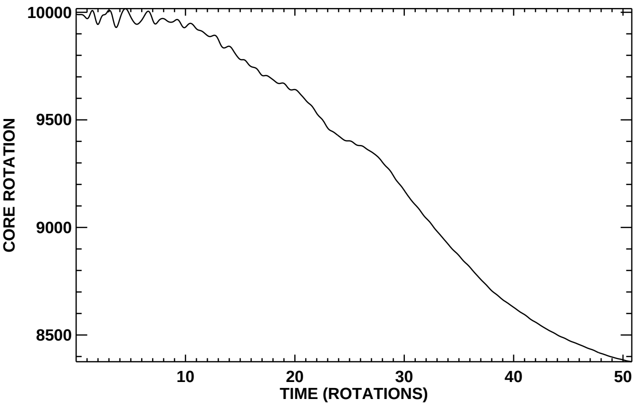

Fig. 5 Magnetic Reynolds number of the angular velocity at the inner boundary of the simulation domain versus time. It shows the reduction of the rotation frequency due to angular-momentum removal.

tance to the axis showed no instability. Stars that are spun-up by external torques are not affected by the MRI.

Figure 5 demonstrates how the angular velocity of the core diminishes in such a calculation, here in terms of the magnetic Reynolds number of the rotation speed at the inner boundary of the computational domain. The graph is plotted versus the time in units of rotation periods. Note that these times are relatively short, due to the “low” magnetic Reynolds number of 10^4 as compared to real stellar values of roughly 10^{12}. We show the actual limit of numerical computations here (and any global, time-dependent simulation of today) in that the time-scales of diffusion and rotation are much closer than in reality. Any attempt to convert times into years tends to hides this fact.

The spin-down shown in Fig. 5 only reaches rotation frequency in the middle between the initial core and surface rotations, since we do not extract angular momentum from the surface as it would be available through stellar winds or just further expansion of the envelope. More sophisticated simulations will show in the future, whether the entire reduction of the specific angular momentum by two orders of magnitude is achievable, as it is anticipated by Suijs et al. (2008).

3 Transition from current-driven to shear-driven

Since there is an instability for strong fields which also exists with differential rotation, and there is an instability for weak fields plus differential rotation, there may actually be a smooth transition between the two. This was found for a fluid rotating between two cylinders by Rüdiger et al. (2011). If the outer cylinder rotates more slowly than the inner cylinder, and if there is a current through the fluid, the Tayler instability is seen at low Reynolds number, where the magnetic field dominates, and the MRI is seen at high Reynolds numbers, where the differential rotation dominates the magnetic field. Such a mixed situation was very likely seen by various authors before, for example the solar-setup computations by Gilman & Fox (1997) – who called it ‘joint instability’ – and by Dikpati et al. (2004) in an extensive parameter study, as well as in nonlinear simulations by Cally (2001), all using a two-dimensional approximation. In the context of intermediate-mass stars, combinations of differential rotation and relatively strong fields were recently employed by Arlt & Rüdiger (2011) and Szklarski & Arlt (2013), for example, who used 3D setups.

4 Application to cool stars

Many attempts of simulating the solar cycle with mean-field models rely on the amplification of substantial toroidal fields just below the convection zone. MHD instabilities are not visible in this treatment. So it is wise to compute the limiting magnetic field strengths for toroidal fields for the tachocline of the Sun. The simple comparison of the Alfvén frequency and the rotational frequency may give as an estimate. Using solar model densities and the rotation profile determined by helioseismology, we find maximum field strengths of $2 \cdot 10^5$ G at the equator and $5 \cdot 10^4$ G near the poles.

Since we know already, that also lower field strengths are potentially unstable, simulations were employed to study the stability of typical toroidal-field geometries. The limit on the magnetic field strength can be as low as a few hundred Gauss then, at the expense of much longer growth times (Arlt et al. 2007; Kitchatinov & Rüdiger 2008). Such a limit would be in contrast with the results from flux-tube simulations which indicate that about 10^5 G are necessary for an emergence of magnetic flux at low latitudes (see e.g. Choudhuri 1989; Caligari et al. 1995; Granzer et al. 2000). Two possible solutions are seen here: the low-latitude emergence of magnetic flux is either possible at much lower field strengths in a 3D, realistic environment, or the location of the strongest fields of the dynamo is not in the tachocline.

A nearly uniform rotation in the solar radiation zone can be achieved by an internal, weak magnetic field, presumably of fossil origin (Rüdiger & Kitchatinov 1997; Gough & McIntyre 1998; but see also the pilot work by Mestel & Weiss 1987). It basically says where differential rotation is present, azimuthal fields emerge and Lorentz forces act against this generation by diminishing the shear. This explanation bears the problem that the field may not be confined to the interior and is not capable of maintaining the uniform rotation when extending into the tachocline (Garaud 2002; Brun & Zahn 2006), a fact that may be alleviated by the meridional circulation on top of the radiation zone, generated in the solar convection zone (Rüdiger et al. 2005). While this issue needs further investigations, the actual removal of angular momentum cannot be provided by confined magnetic fields.

The presence of lithium at the solar surface poses an upper limit on the turbulence generated in the radiation zone. The turbulent viscosity should not be enhanced by one or two orders of magnitude as compared to the microscopic gas viscosity, unlike the one in the solar convection zone which is at least 10 orders of magnitude above the microscopic one. With such a turbulent viscosity in the radiation zone, all lithium would have been converted at temperatures above $2.5 \cdot 10^6$ K. It will necessary to test the turbulence generated by the instabilities on its turbulent magnetic Prandtl number, i.e. the ratio of turbulent viscosity to turbulent magnetic diffusivity.

Somewhat counter-intuitively, the turbulent viscosity is a function of the velocity and magnetic field fluctuations, while the turbulent magnetic diffusivity is a function of the velocity fluctuations only (Vainshtein & Kitchatinov 1983). That means it is the turbulent magnetic diffusivity which describes the mixing by velocity, while the turbulent viscosity is a representative of the angular momentum transport, which lives from both Reynolds and Maxwell stresses. Hence, the turbulent magnetic Prandtl number should be larger than unity.

In the present Sun, there is little place for MRI, since Ω increases with axis distance in the largest fraction of its interior. Considerable negative gradients are only found in the tachocline between about 40° and 80° latitude and very close to the solar surface below about 50° latitude. Ogilvie (2007) found though that it is the latitudinal shear rather than the gradient over axis distance that matters for the MRI in stably stratified zones. There is no place with a negative latitudinal gradient in Ω in the Sun.

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